Shree H.N. Shukla College of I.t. & Mgmt, Rajkot

BCA Sem- 01 (CS- 07 : Mathematics in Ancient India : Exploring the Rich Heritage of Vedic Matheatics)

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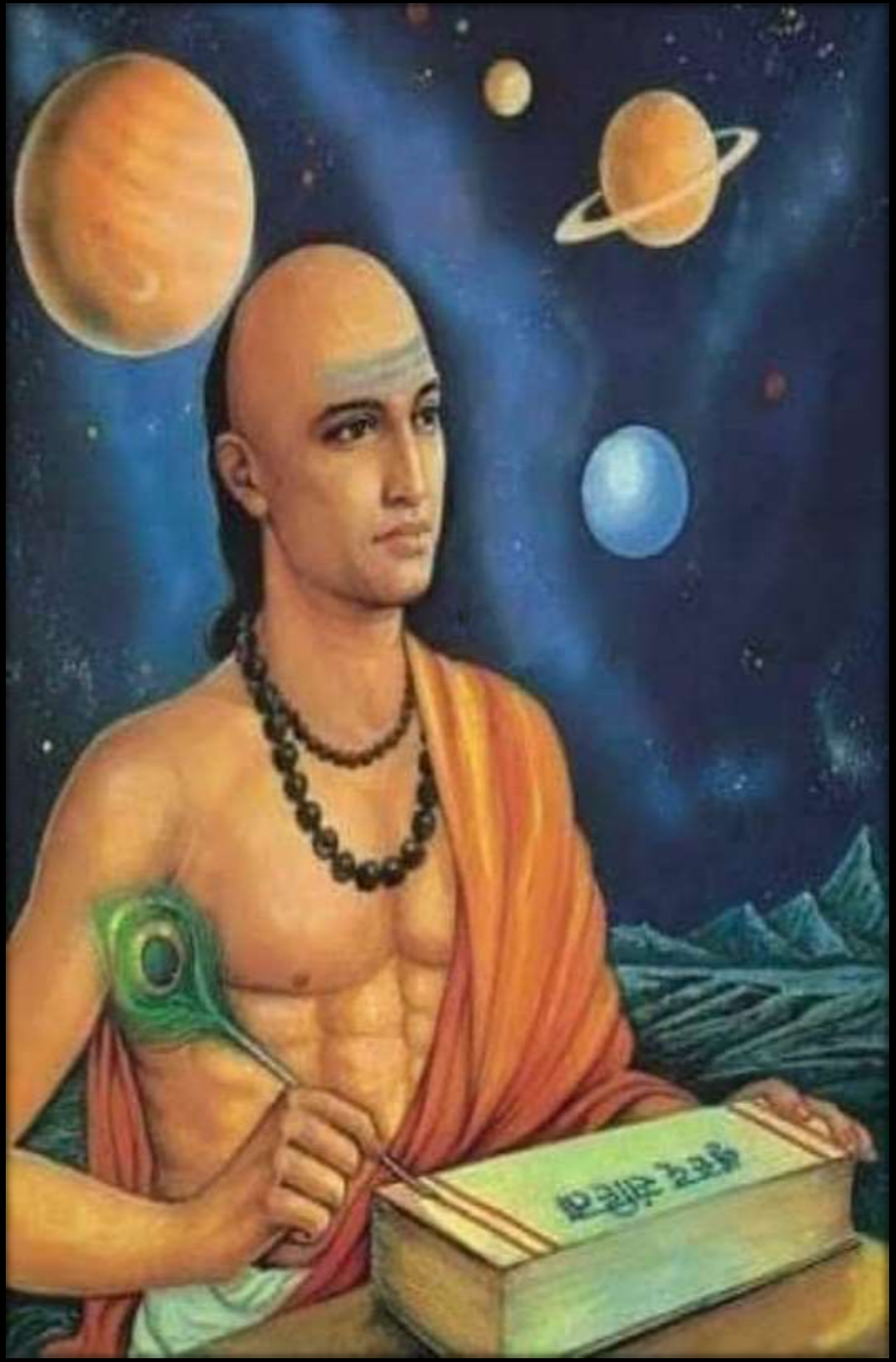
Unit 1

Biographies of Ancient Indian Mathematicians

A brief introduction to the lives and information on the works of the following mathematicians:



Introduction to Lives and Works of Aryabhata

Aryabhata's Background and Significance

- Aryabhata, an esteemed Indian mathematician and astronomer, revolutionized ancient knowledge with his remarkable contributions. Discover the incredible life and works of this visionary genius.

- Aryabhata (476 – 550 CE) was the first of the major mathematician, astronomers from the classical age of Indian mathematics and Indian astronomy.

Aryabhata's Background and Significance

- His works include the Aryabhatiya (499 CE, at the age of 23 years old) and the Arya-Sidhhanta.
- Aryabhatta is also reputed to have set up an observatory at the sun temple in Taregana, Bihar.

Time and Place of Birth

- Aryabhatta mentions in the Aryabhatiya that it was composed 3,600 years into the Kali yuga, when he was 23 years old.
- Aryabhatta provides no information about his place of birth.
- The only information comes from Bhaskara I (his student) , who describes Aryabhatta as asmakiya. During the budhha's time a branch of the asmaka people settled in the region between the Narmada & Godavari rivers in central India.

Time and Place of Birth

- It is fairly certain that at some point he went to Kusumpura for advanced studies & lived there for some time which was known as Pataliputra, modern Patna.

- It is speculated that Aryabhatta might have been the head of the Nalanda University.

Aryabhata's Key Contributions

Aryabhatiya

Explore the profound concepts and theories presented in Aryabhata's influential mathematical treatise, Aryabhatiya.

Arya-siddhanta

Journey through the ancient Indian astronomical system and unravel the intricacies of Arya-siddhanta, another major work by Aryabhata.

Other Notable Works

Discover the lesser-known yet impactful contributions by Aryabhata to the world of mathematics and astronomy.

Works

- Aryabhatta is the author of several treaties on maths & astronomy, some of which are lost. His major work Aryabhatta a compendium of maths & astronomy was extensively referred to in the Indian mathematician literature & has survived to modern times.
- The mathematical part of Aryabhatta covers, arithmetic, algebra, plane trigonometry & spherical trigonometry. It also contains continued fractions, quadratic equations, sums of power series & a table of sines.

Works

- The Arya-Siddhanta, a lost work on astronomical computations is known through the wrutings of mathematians and commentators including Brahmagupta & Bhaskara I

- It also contained a description of several instruments:

- The gnomon (shanku yantra), A shadow instrument (chhaya yantra), Possibly angle-measuring devices, semi circular & circular (dhanur yantra/ chakra yantra), A cylindrical stick yasti yantra, an Umbrella shaped device called the chhatra yantra.

Aryabhata's Impact on Science

Mathematics

Explore the indelible mark left by Aryabhata on the field of mathematics, including his advancements in trigonometry and algebra.

Astronomy

Discover how Aryabhata's groundbreaking theories transformed the understanding of celestial bodies and their movements.

Global Scientific Knowledge

Uncover the wide-reaching influence of Aryabhata's work, transcending geographical and cultural boundaries.

Aryabhatiya

It contains four chapters :

1. Gitikapada

It deals with cosmoplogy, there is also a table of sines, the planetary revolutions, In Mahayuga is mentioned to be upto 4.32 millions years.

2. Ganitapada

Mensuration, Simple quadratic and indeterminate equations, arithmetic and geometric equations

3. Kalakriyapada

It includes astronomy where using varying units of time the count days, weeks and months.

4. Golapada

In this chapter he derives into the causes of days and nights, rising of zodiac signs, eclipse , celestial equators, node and the shape of the earth.

Approximation of π given by Aryabhatta

- Aryanbhatta worked on the approximation for pi.

Aryanbhatta correctly insisted that the earth rotates about its axis daily and that the apparent movement of the stars is a relative motion caused by the rotation of earth, contrary to then Prevailing view that the sky rotated

- He states that Moon and planets shine by reflected sunlight.

Planetary motion

- Considered in modern English units of time, Aryanbhatta calculated the sidereal rotation (the rotation of the earth referencing the fixed stars) as 23 h, 56 m and 4.1 s and the modern values is 23:56:4.091

- Similarly, his value for the length of the sidereal year at 365 days, 6h, 12m & 30 s (i.e., 365.25858 days) is an error of 3m and 20 s over the length of a year
(exact is 365.25636 days).

Death

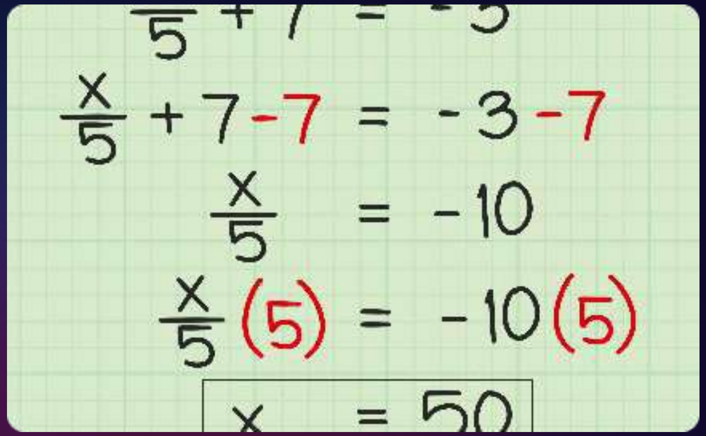
- Aryabhatta died as a successful mathematician, astronomer and a scientist at the age of 74.

- The place and time of death are still unknown.

In his Honor…

- India’s first satellite Aryabatta and the lunar crater Aryabatta are both named his honor, the Aryabatta satellite also featured on the reverse of the Indian 2 rupees note.
- An institute for conducting research in astronomy, astro- physics & astrospheric science (ARIES) near nainital, India.
- There is also Aryabatta maths competition held in school.

Conclusion



Key Takeaways

Summarize the key insights gained from exploring Aryabhata's life and works, appreciating his enduring impact on scientific and mathematical knowledge.

Scientific Legacy

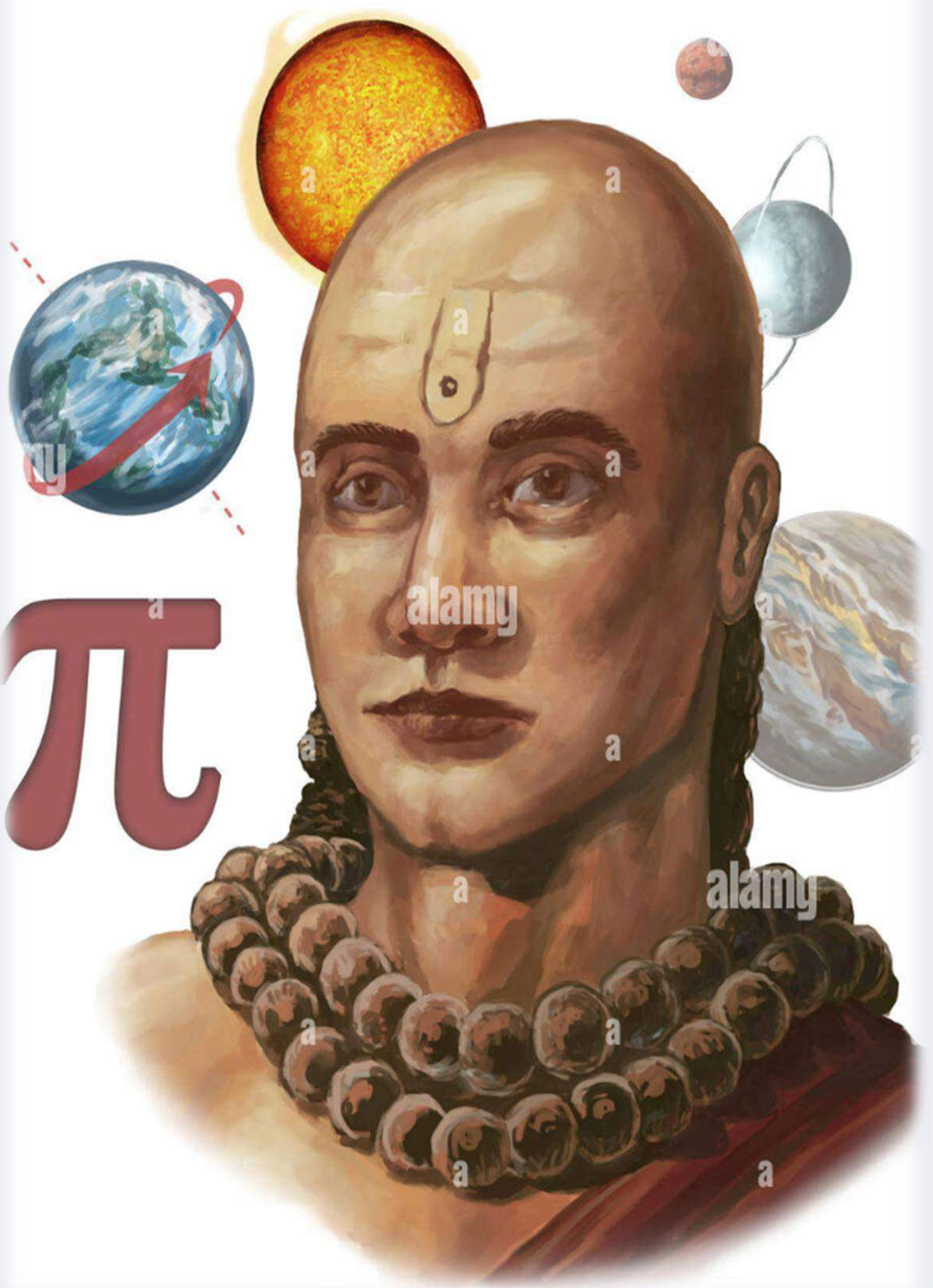
Reflect on how Aryabhata's contributions continue to shape the path of scientific progress, both in India and globally.

Appreciating Aryabhata

Celebrate the brilliance and innovation of Aryabhata, recognizing his immense contributions to our understanding of the universe.

The Life and Contributions of Vrahamihira

Vrahamihira, an ancient Indian mathematician, made significant contributions to the field of mathematics. Join us as we explore his life, writings, and impact on modern mathematics.



Early Life of Vrahamihira

1 Birth and Origins

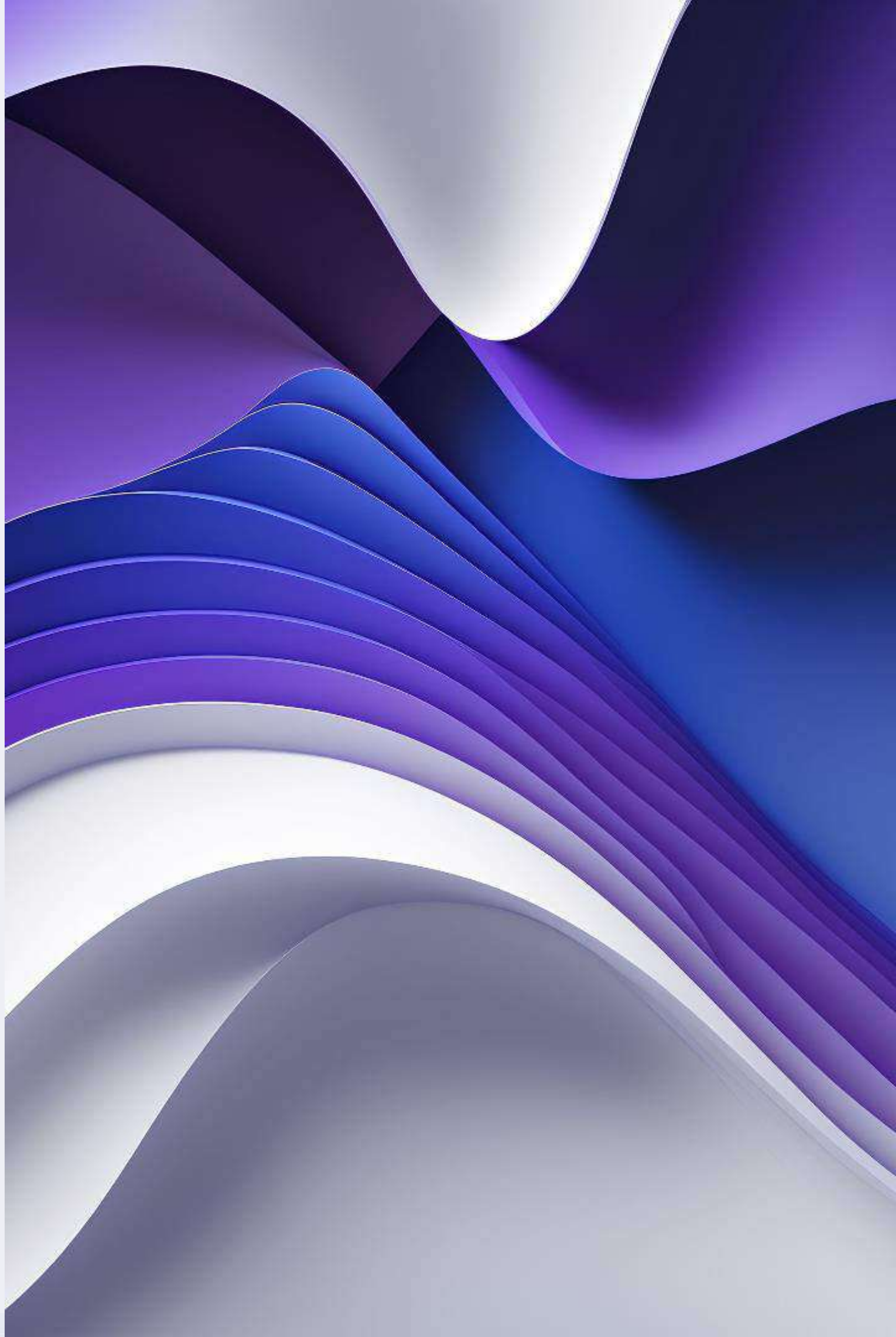
Vrahamihira was born in India during the 6th century AD.

2 Education and Scholarship

He received a comprehensive education in mathematics, astronomy, and astrology.

3 Cultural Influence

Vrahamihira was deeply connected to the rich Indian cultural heritage, which influenced his work.



How Mihira got the title “Varaha”

- King Vikramaditya was aghast at the royal astrologer Mihira’s prediction.

- When Vikramaditya asked Mihira to predict his future then Mihira confirmed the prediction, “ The position of the planets predicts the death of the prince at the age of 18”.

- Though the King controlled his emotions, the Queen could not contain herself and wailed “My lord, you should see to it that this prediction proves false.”

- Though the King had full faith in his astrologer Mihira, he took every precaution to protect and save his son. But on the predicted day, a boar killed the prince.

How Mihira got the title “Varaha”

- When the news reached the King, he summoned Mihira to his court and told him “ I am defeated, you have won”.

- Mihira was as sad as the king and he replied “ My Lord, I have not won. It is the science of astronomy and astrology that has won!”

- “whatever it may be, my respected astrologer,” said the King. “It was convinced that your science is nothing but truth and for your mastery of the subject , I now confer upon you the Magadha Kingdom’s greatest award, the emblem of the varaha (boar).”

- And also included in his 9 gems.

- So from that time Mihira came to be known as Varahmihira.

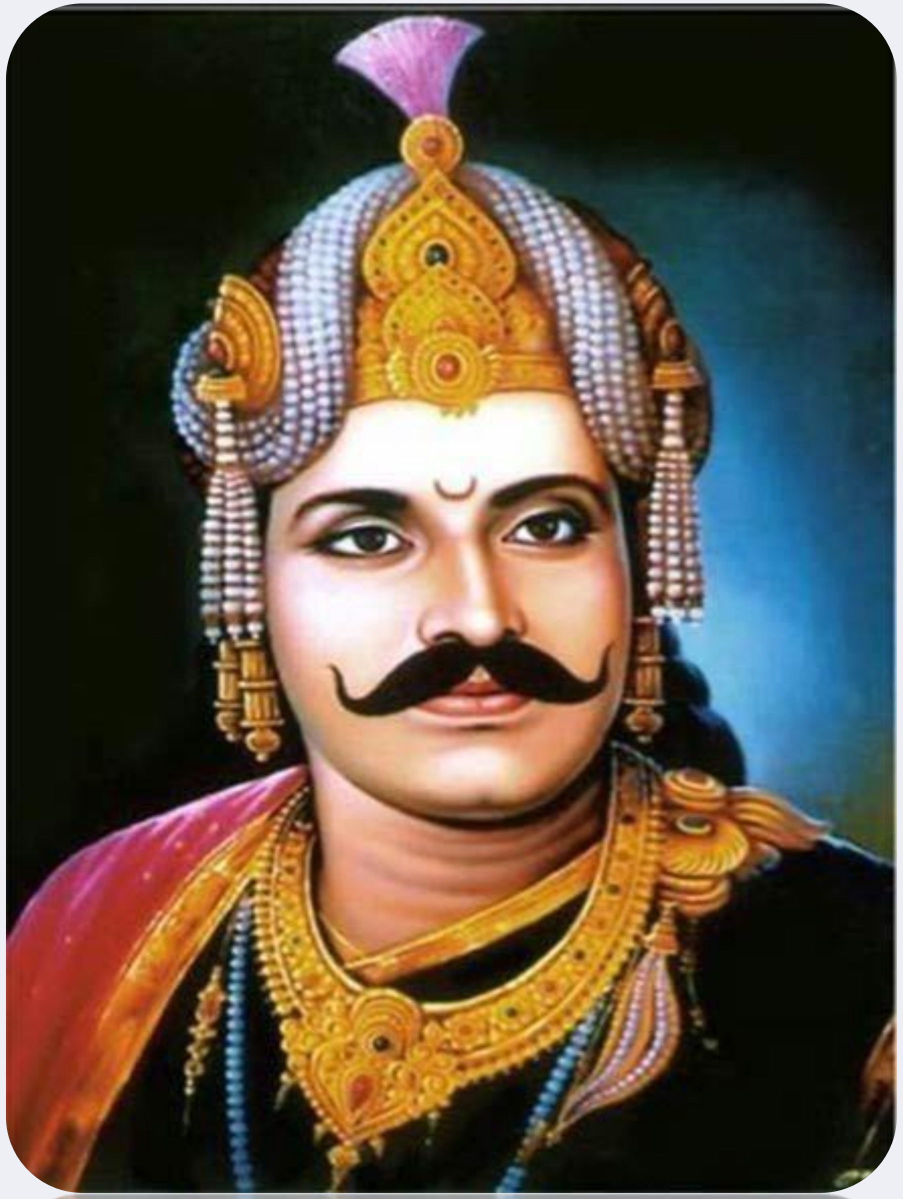
Birth and Origin

- Vrahamihira was born at 499 A.D. into a family of Brahims settled at Kayatha, a village near Ujjain, in the Avanti region, roughly corresponding to modern-day Malwa (part of M.P., India.)

- His father Adityadasa was a worshiper of the sun god and it was he who taught Vrahmihira astrology.

Education

- On a visit to Kusumpura (Patna) young Varahmihira met the great astronomer and mathematician, Aryabhatta.
- The meeting inspired him so much that he decided to take up astrology and astronomy as a lifetime pursuit. At that time, Ujjain was the center of learning, where many schools of arts, science and culture were flourishing in the prosperity of the Gupta region.
- Varahmihira therefore shifted to this city, where scholars from distant lands were gathering.
- In due course, his astrological skills came to the notice of Vikramaditya Chandragupta II, who made him one of the nine gems of his court.



Vikramaditya Chandragupta II

Vrahamihira's Contributions to Mathematics

Trigonometry

Vrahamihira's treatise on trigonometry, known as the "Siddhanta Shiromani," introduced innovative concepts and formulas still used today.

Algebra

His work in algebra laid the foundation for algebraic manipulations and quadratic equations.

Number Systems

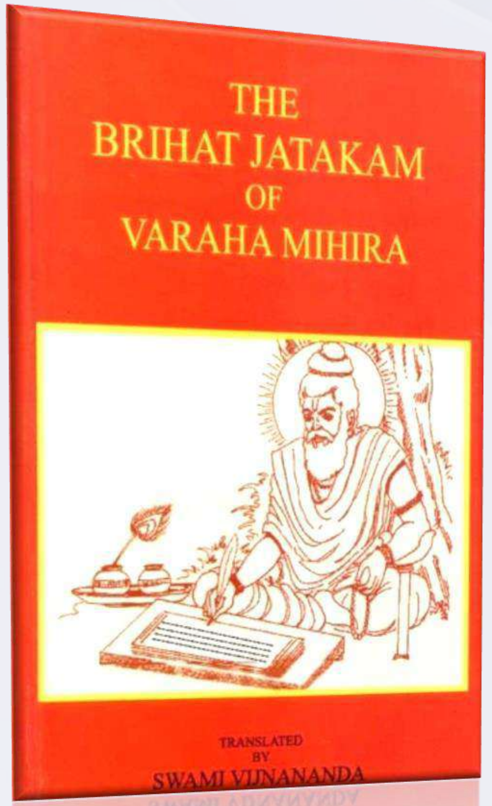
Vrahamihira's contributions to number systems greatly influenced the field.

Varahamihira

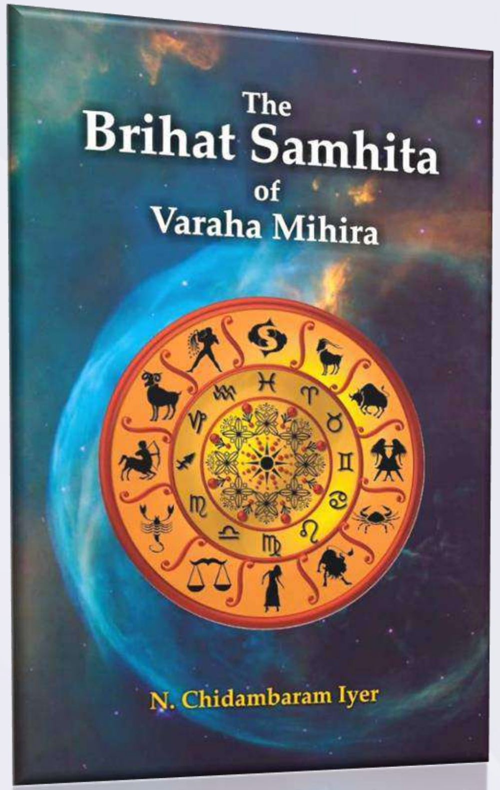
- Varahmihira was honored with a special decoration and status as one of the nine gems in the court of King Vikramaditya in Avanti (Ujjain).
- He discovered a version of Pascal’s triangle and worked on magic squares.
- Varahamihira wrote several important works on Jyotish including but not limited to: Brihat Jataka, Brihat Samhita, Yoga Yantra, Pancha Siddhantika (on astronomy) and Prasna Vallabha.

Notable Writings of Vrahamihira

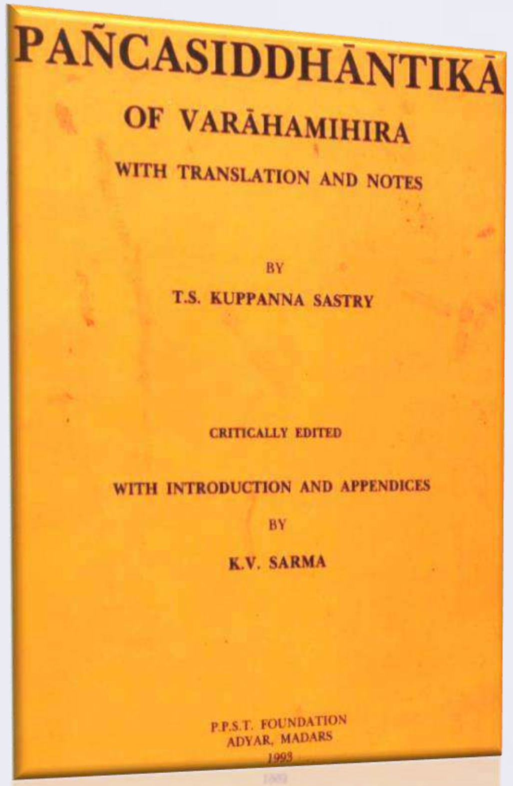
1 **Brihat Jataka**



2 **Brihat Samhita**



3 **Pancasiddhantika**



Influence of Vrahamihira on Modern Mathematics

1 **Legacy of Trigonometry**

Vrahamihira's trigonometric concepts and formulas continue to be taught in mathematics courses worldwide.

2 **Algebraic Advancements**

His contributions to algebra formed the basis of many algebraic techniques.

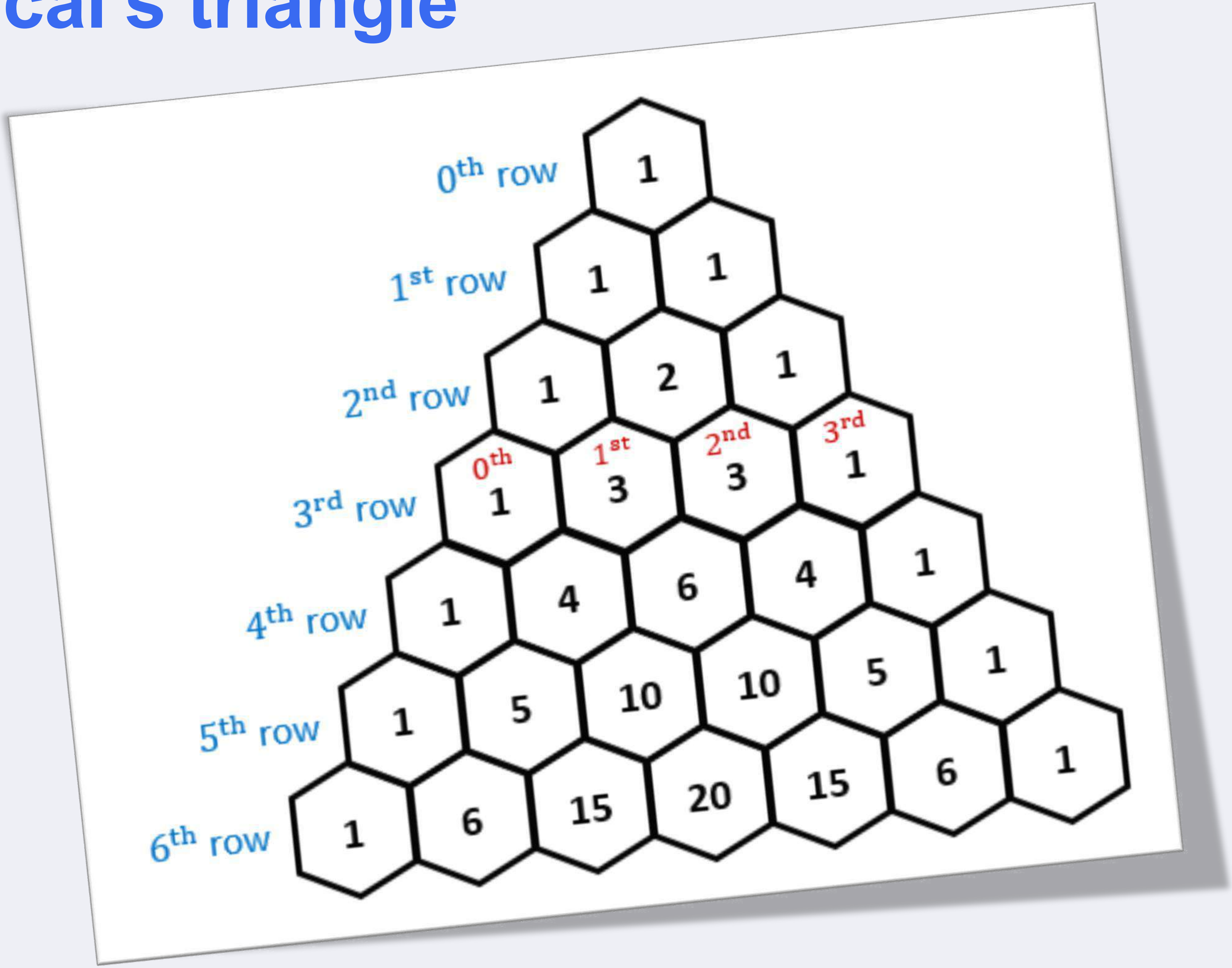
3 **Cultural Preservation**

Vrahamihira's writings helped preserve Indian mathematical knowledge and cultural identity.

Mathematical Work

- Varahmihira’s mathematical work included the discovery of the trigonometric formulas.
- He improved the accuracy of the sine tables of Aryabhata.
- He defined the algebraic properties of zero as well as of negative numbers.
- Furthermore, He was among the first mathematicians to discover a version of what is known as the Pascal’s triangle.
- He used it to calculate the binomial coefficients.

Pascal’s triangle



Vrahamihira's Work in Astronomy and Astrology

1 **Astronomical Observations**

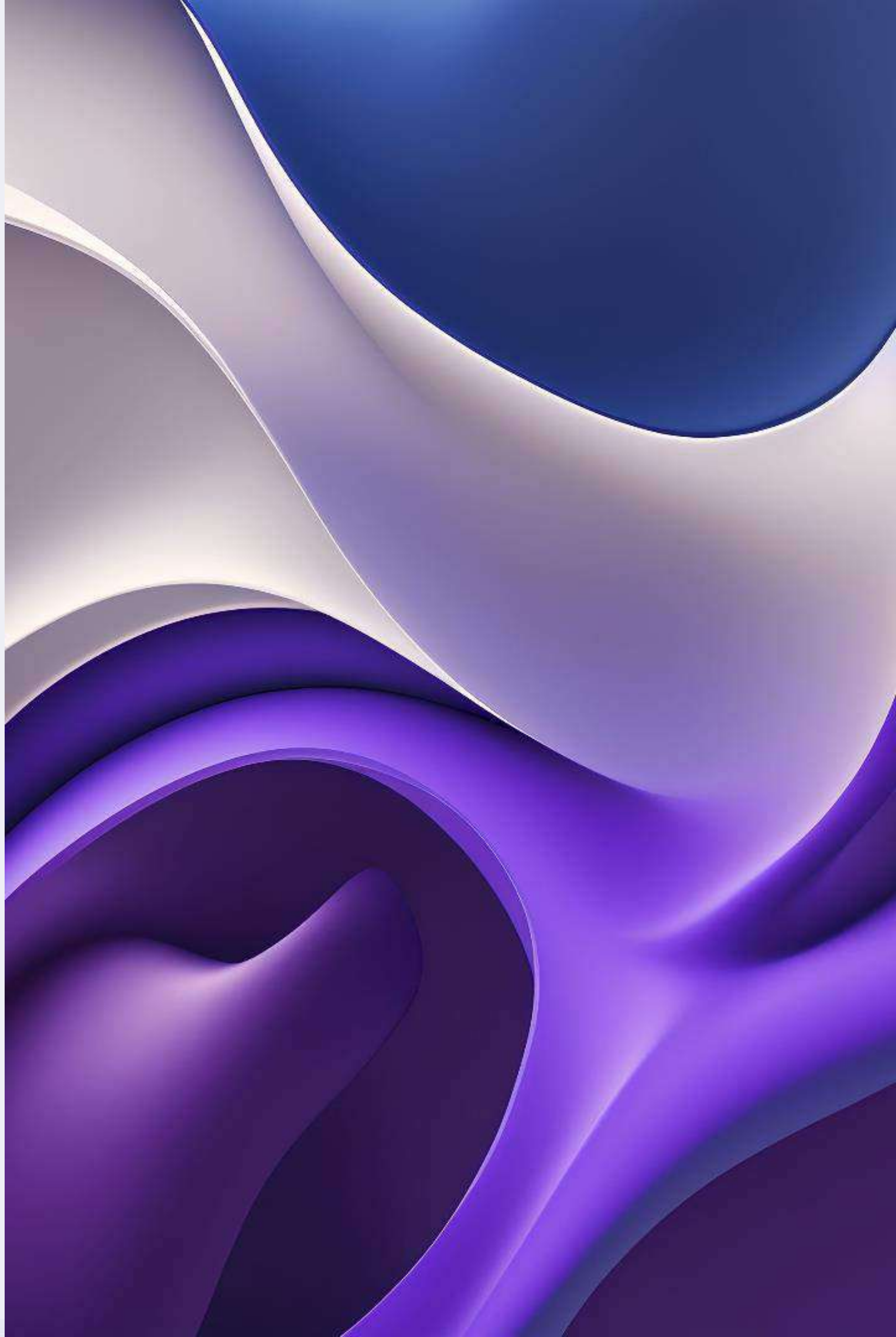
Vrahamihira's precise observations and calculations significantly contributed to advancements in astronomical knowledge.

2 **Astrological Predictions**

His astrological treatises provided valuable insights into horoscopes and predictions, influencing astrology practices.

3 **Cosmic Connections**

Vrahamihira explored the harmonious relationship between astronomy and astrology, connecting the celestial and human realms.



Work

- Varahmihira made some significant observations in the field of ecology, hydrology and geology too.

- He claim that plants and termites serve as indicators of underground water is now receiving attention in the scientific world.

- He was also a prolific writer as his mastery of Sanskrit grammar and poetic meter enabled him to express himself in a unique style.

Death

- Died: 587 AD, Ujjain.

The Legacy of Vrahamihira

1

Acknowledged Influence

Vrahamihira's contributions laid the groundwork for future mathematical and astronomical developments.

2

Global Recognition

His work continues to be widely studied, appreciated, and honored around the world.

3

Preservation of Traditions

His writings preserved and propagated India's ancient mathematical and astrological traditions.

A Brief Introduction to Brahmagupta

Brahmagupta was an ancient Indian mathematician who made significant contributions to the field of mathematics. This presentation explores his life and his remarkable works.

Introduction to Brahmagupta

Brahmagupta was born in the 7th century CE in the city of Bhinmal, located in present-day Rajasthan, India. He was a highly respected mathematician in the court of the ruler of Ujjain.

Background and Early Life

- Brahmagupta's interest in mathematics blossomed at an early age. He received a comprehensive education, studying various branches of mathematics, astronomy, and other sciences.

- Brahmagupta was born in 598 CE according to his own statement.

- He lived in Bhillamala in Gurjaradesa (modern Bhinmal in Rajasthan, India) during the reign of the Chavda dynasty ruler, Vyagrahamukha.

- His father Jisnugupta was an astrologer in the city of Bhinmal (Rajasthan).

- He was a Hindu by religion.

Education

- Bhillamala was the capital of the Gurjaradesa, the second largest kingdom of western India, comprising southern Rajasthan and northern Gujarat in modern-day India.

- It was also centre of learning for mathematics and astronomy.

- Brahmagupta became an astronomer of the Brahmapaksha school, one of the four major schools of Indian astronomy during this period.

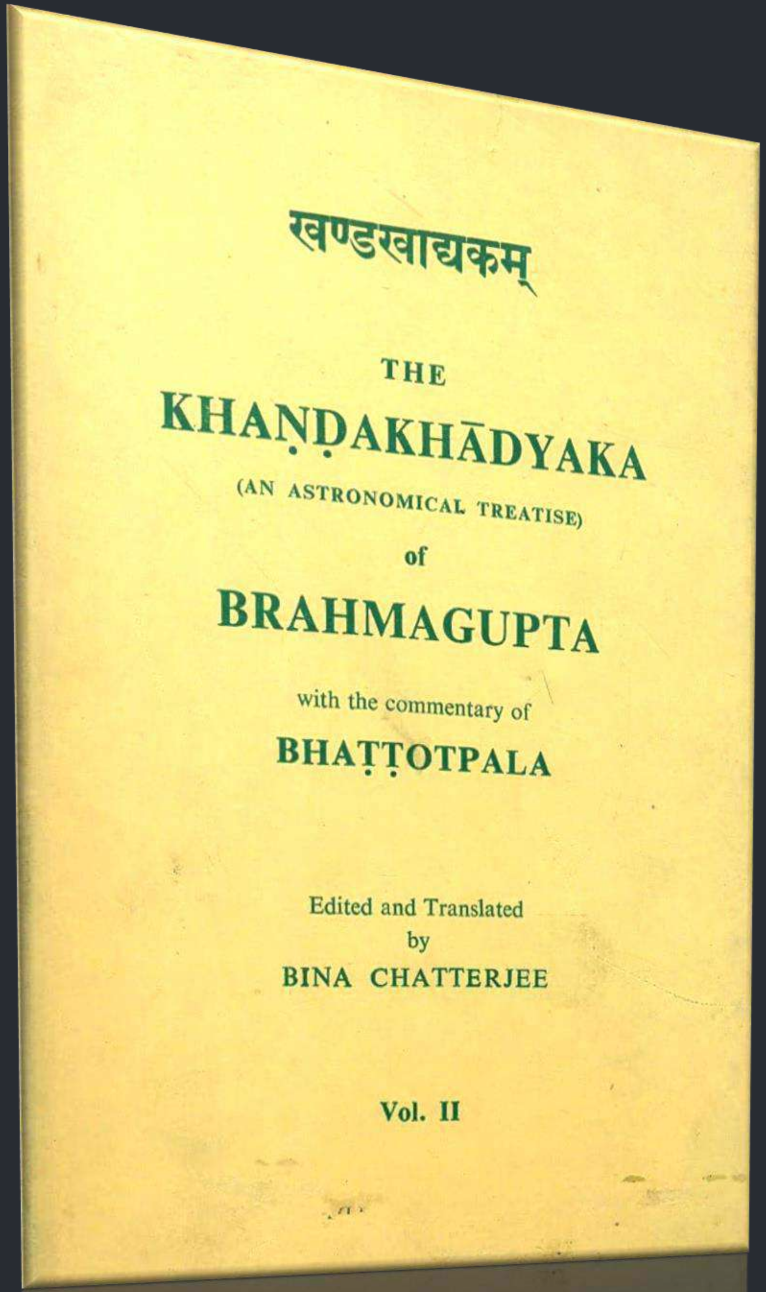
- He studied the five traditional Siddhantas on Indian astronomy as well as the work of other astronomer including Aryabhatta I, Latadeva, Pradyumna, Vrahamihira, Simha, Srisena, Vijayanandin and Vishnuchandra.

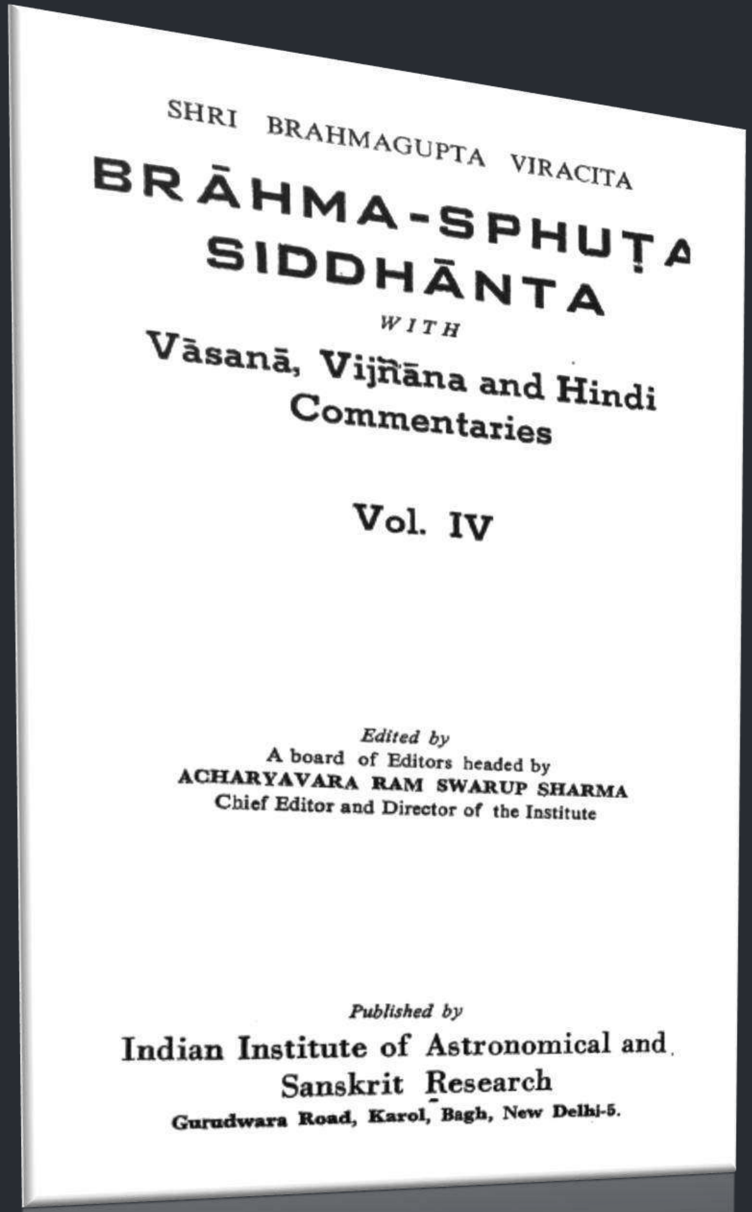
Contribution to Mathematics

Brahmagupta's contributions to mathematics were groundbreaking. He introduced concepts such as zero, negative numbers, and solutions to indeterminate equations. His work laid the foundation of algebra and number systems.

Works and Publications

- Brahmagupta authored several influential treatises on mathematics, including the renowned "Brahmasphutasiddhanta." His works covered a wide range of topics, from arithmetic to algebraic equations.
- Brahmagupta was the first to give rules to compute with zero.
- Brahmasphutasiddhanta, composed in 628 CE.
- Khandakhadyaka, composed in 665 CE.
- Grahanarkanjnana, (ascribed in one manuscript)


खण्डखाद्यकम्
THE
KHAṆḌAKHĀDYAKA
(AN ASTRONOMICAL TREATISE)
of
BRAHMAGUPTA
with the commentary of
BHAṬṬOTPALA
Edited and Translated
by
BINA CHATTERJEE
Vol. II


SHRI BRAHMAGUPTA VIRACITA
BRĀHMA-SPHUṬA
SIDDHĀNTA
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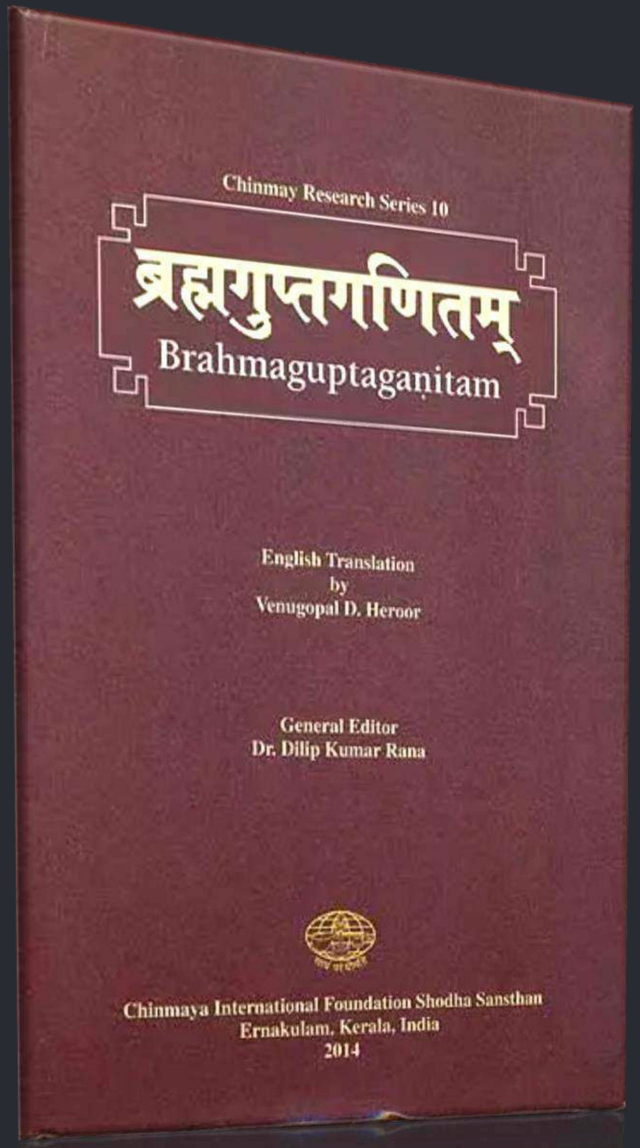
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2014

Brahmagupta’s work on Earth

- Brahmagupta calculated the length of a year is 365 days 6 hours 12 minutes 9 seconds which is very close to today’s calculation.

- He talked about “gravity”. To quote him, “ Bodies fall towards the earth as it is in the nature of the earth to attract bodies, just as it is in the nature of water to flow.”

- Proved that the Earth is a sphere and calculated its circumference to be around 36,000 km (modern 40,075 km).

Zero

- Brahmagupta's Brahmasphutasiddhanta is the first book that provides rules for arithmetic manipulations that apply to zero and to negative numbers.
- The Brahmasphutasiddhanta is the earliest known text to treat zero as a number in its own right, rather than as simply a placeholder digit in representing another number as was done by the Babylonians or as a symbol for a lack of quantity as was done by Ptolemy and the Romans.
- In chapter eighteen of his Brahmasphutasiddhanta, Brahmagupta describes operations on negative numbers. He first describes addition and subtraction.

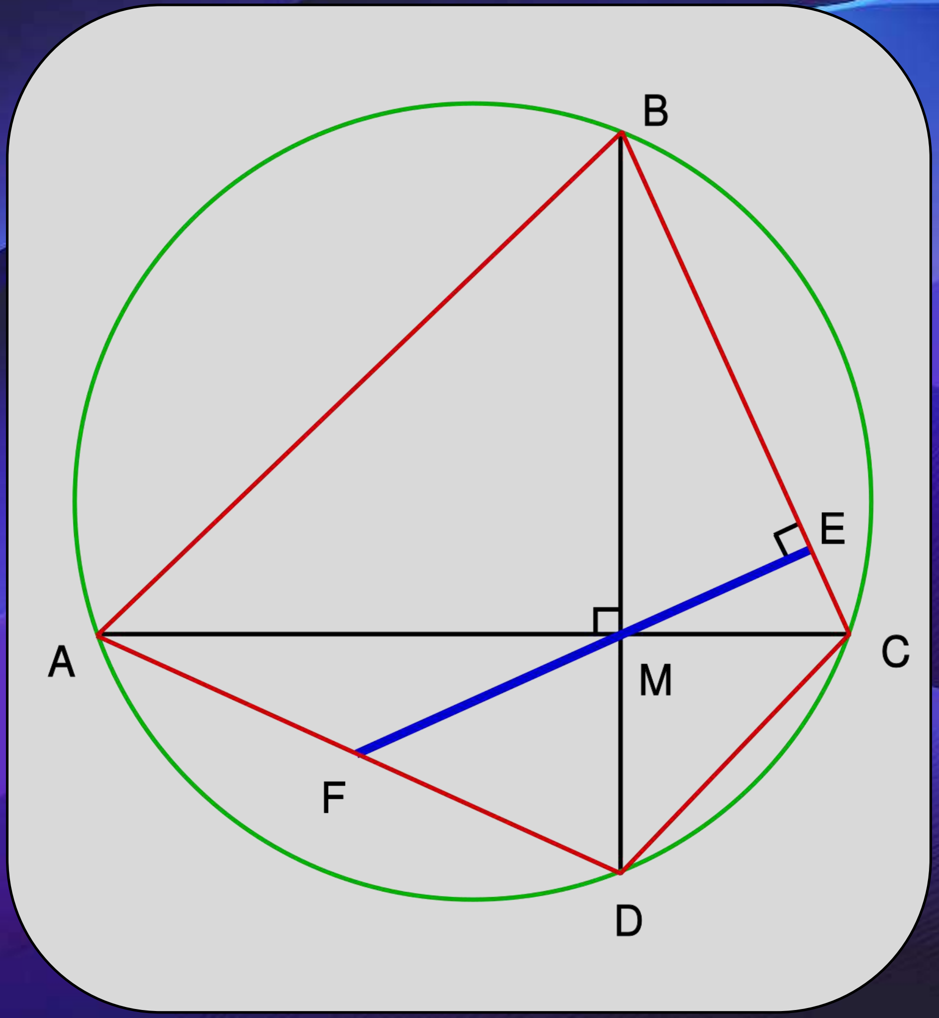
Continue..

- Brahmagupta established the basic mathematical rules for dealing with zero ($1 + 0 = 1$; $1 - 0 = 1$; $1 \times 0 = 0$), although his understanding of division by zero was incomplete (he thought that $1 \div 0 = 0$).

- Almost 500 years later, in the 12th century, another Indian mathematician, Bhaskara II, showed that the answer should be infinity, not zero, an answer that was considered correct for centuries.

- However, this logic does not explain why $2 \div 0$, $7 \div 0$, etc, should also be zero – the modern view is that a number divided by zero is actually “undefined” (i.e., it doesn't make sense).

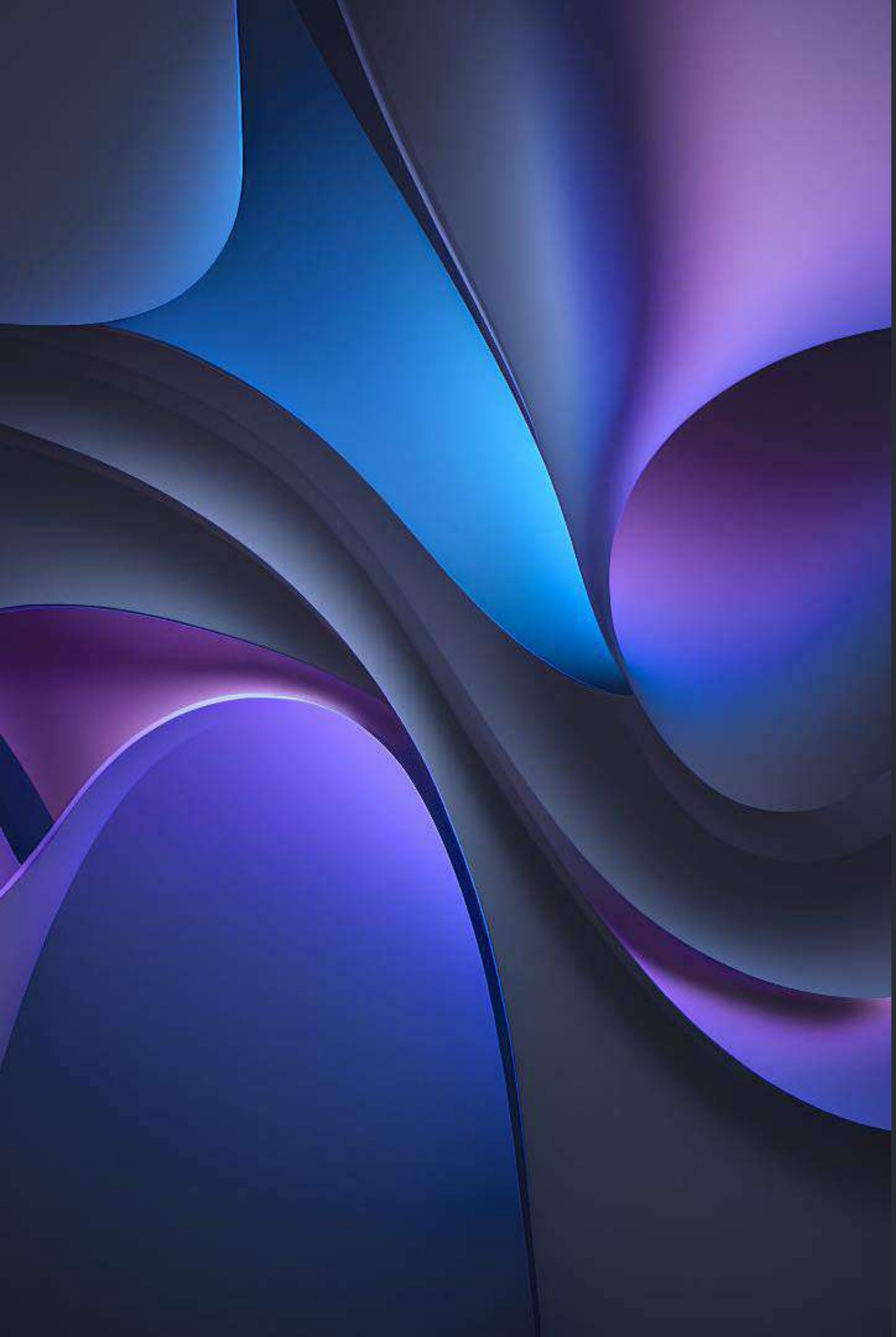
Brahmagupta's Formula



- Brahmagupta is particularly known for his formula to calculate the area of a cyclic quadrilateral. This formula, also known as Brahmagupta's Formula, revolutionized the field of geometry.

$$A_{Cyclic\ Quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- where a, b, c and d are the lengths of the sides of the quadrilateral and s = semi perimeter = $\frac{a+b+c+d}{2}$.
- Brahmagupta dedicated a substantial portin of his work to geometry and trigonometry.
- He established $\sqrt{10}$ (3.162277) as a good practical approximation for π (3.141593), and gave a formula, now known as Brahmagupta’s formula, for the area of a cyclic quadrilateral.



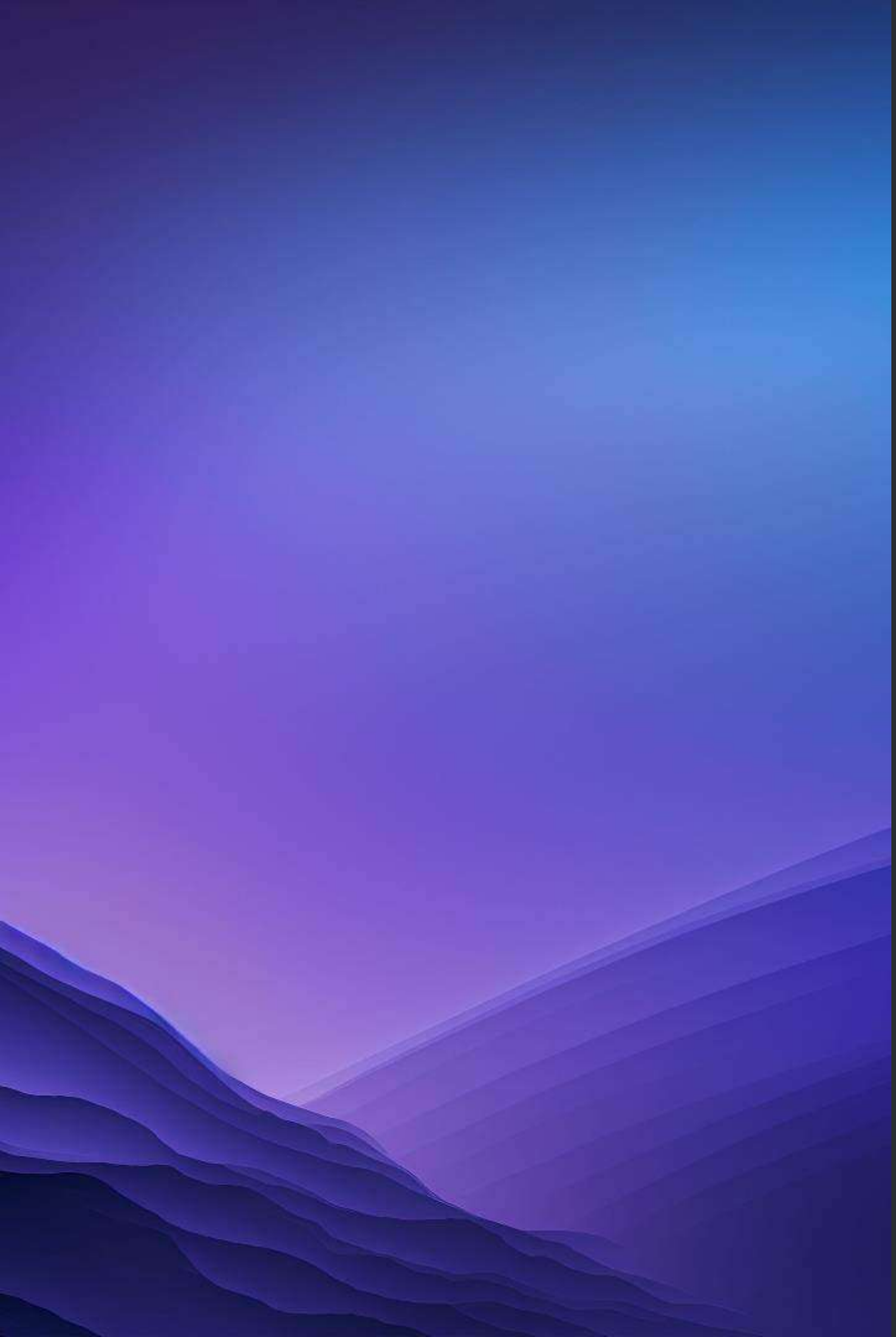
Legacy and Influence

- Brahmagupta's contributions to mathematics influenced scholars from various cultures and laid the groundwork for future advancements. His works spread across the world, impacting fields such as astronomy and architecture.

Death

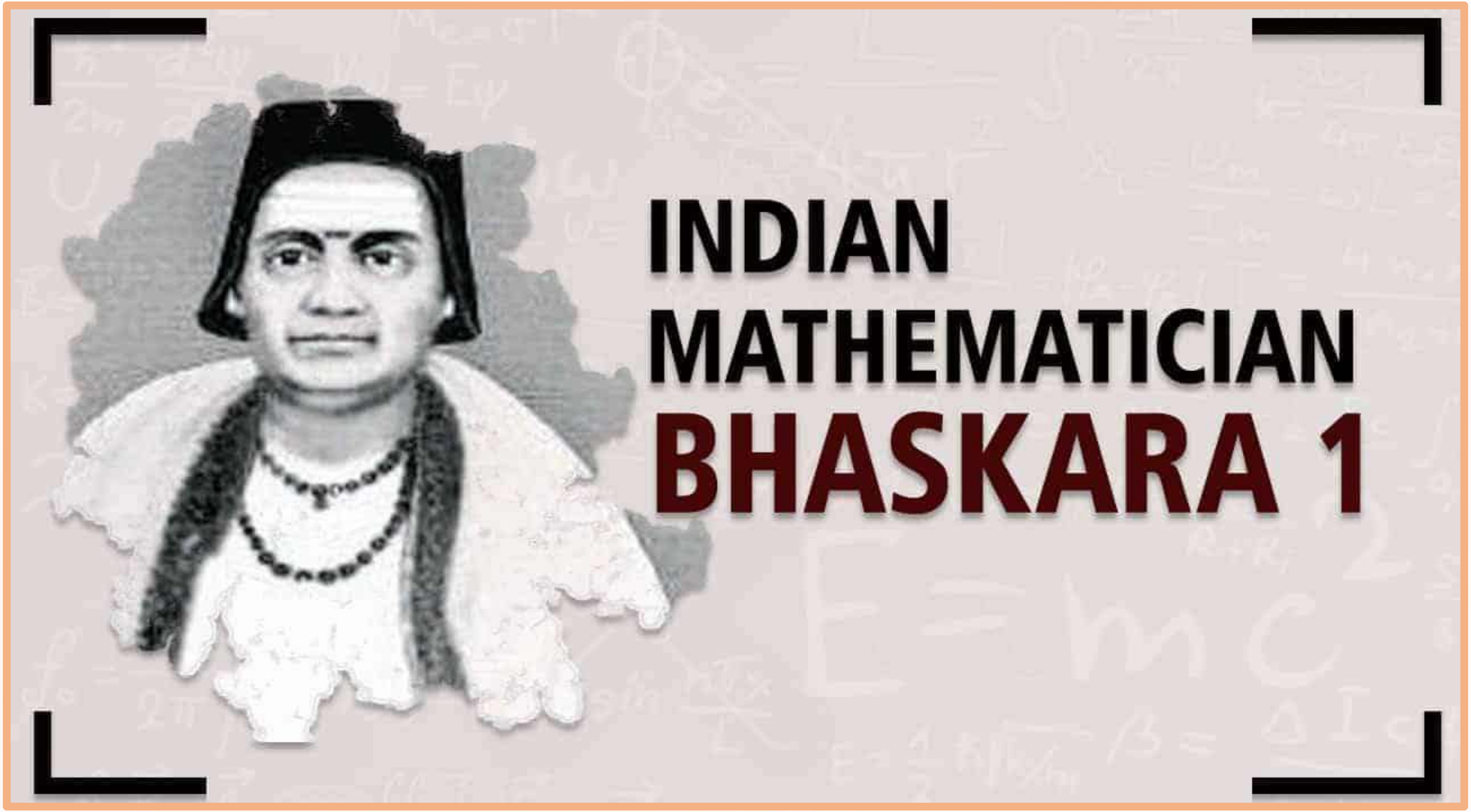
- Brahmagupta died in 668 CE, and he is presumed to have died in Ujjain.

- He was honored by the title given to him by a fellow scientist “Ganita Chakra Chudamani” which is translated as “ The gem of the circle of mathematicians”.



Conclusion

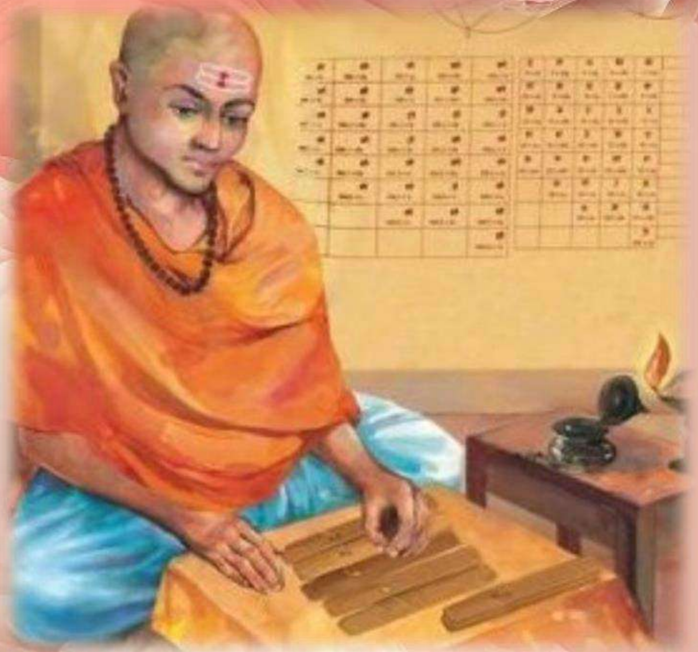
Brahmagupta's genius and innovative ideas continue to inspire mathematicians and scientists worldwide. His pioneering work has left an indelible mark on the history of mathematics.


INDIAN
MATHEMATICIAN
BHASKARA 1

A Brief Introduction to Bhaskara I

Bhaskara I was an influential ancient Indian mathematician and astronomer. His groundbreaking works laid the foundation for advancements in mathematics and astronomy.

Madhava Samgramagrama



Brahmagupta

Who was Bhaskara I?

- He was a 7th- century mathematician and astronomer.
- Who was the first to write numbers in the Hindu decimal system with a circle for the zero, and who gave a unique and remarkable rational approximation of the sine function in his commentary on Aryabhatta's work.
- This commentary, Aryabhatiyabhasya, written in 629 CE, is among the oldest known prose works in Sanskrit on mathematics and astronomy.
- Bhaskara- I is considered to be one of the three pearls of Indian Astronomy and Mathematics along with Brahmagupta and Madhava Samgramagrama.
- He also wrote two astronomical works in the line of Aryabhata's school, the Mahabhaskariya and the Laghubhaskariya.

Who was Bhaskara I?

1 Life and Background

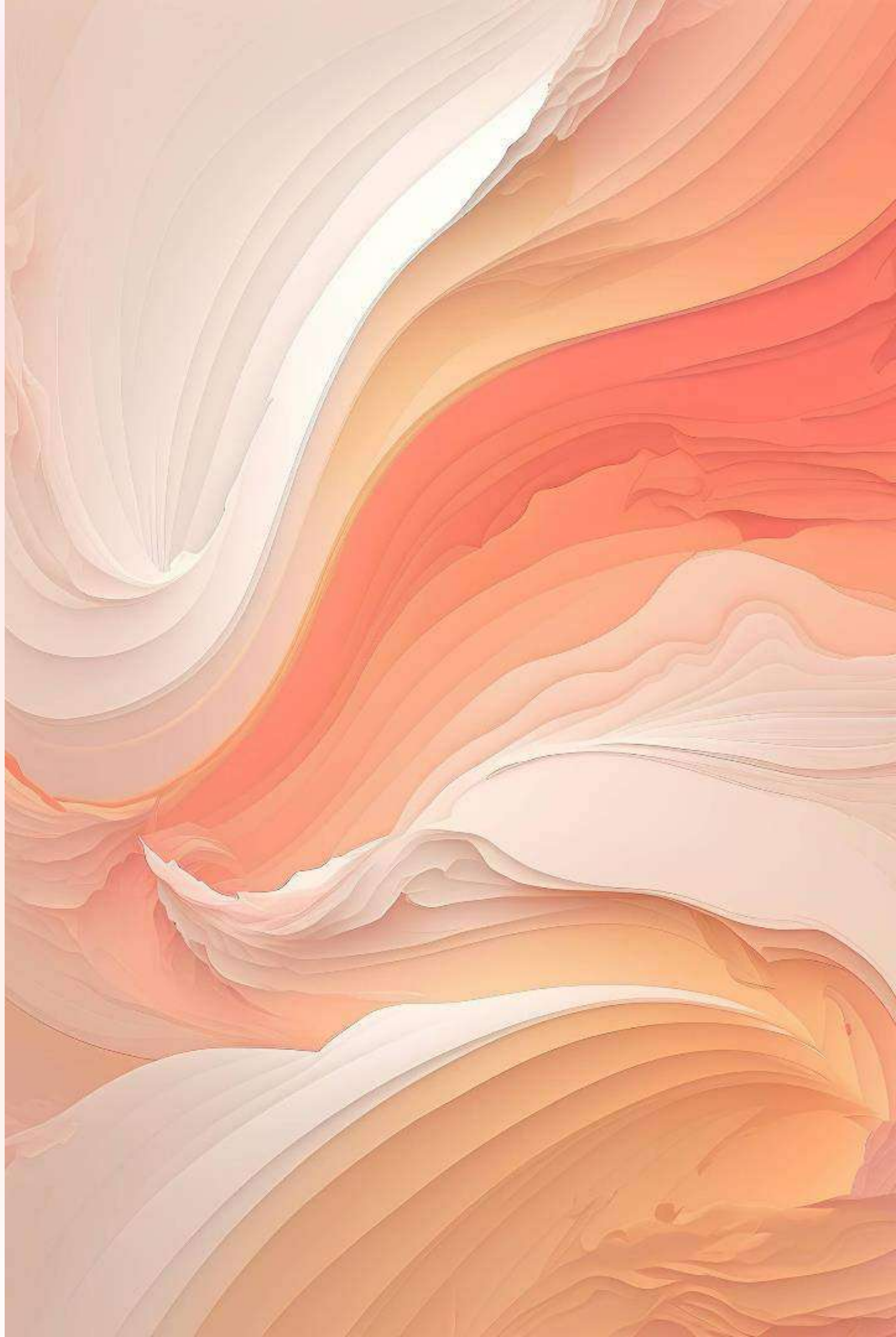
Bhaskara I lived during the 7th century CE in ancient India. He belonged to a family of scholars and received extensive education in mathematics and astronomy.

2 Contributions to Mathematics and Astronomy

Bhaskara I made significant contributions to both mathematics and astronomy. His works revolutionized the understanding of planetary motion, solar and lunar eclipses, and the celestial sphere.

3 Legacy and Influence

Bhaskara I's pioneering ideas and theories had a profound impact on subsequent mathematicians and astronomers. His works became the basis for further advancements in these fields.



Early life of Bhaskara I

- Bhaskara was born in India in the 7th century in Valabhi.
- There are references to places in India in Bhaskara's writings.
- He mentions Valabhi (today vala), the capital of the Maitraka dynasty in the 7th century, and Sivarajapura, which were both in Saurashta which today is the Gujarat state of India.

Bhaskara I

- Also mentioned are Bharuch in southern Gujarat and Thanesar in the eastern Punjab which was ruled by Harsa for 41 years from 606.

- Harsa was the pre-eminent ruler in north India through the first half of Bhaskara I's life.

- A reasonable guess would be that Bhaskara was born in Saurashtra and later moved Asmaka.

Education

- Bhaskara’s astronomical education was given by his father and by his teacher Aryabhatta.
- Bhaskara is considered the most important scholar of Aryabhatta’s astronomical school.
- He and Brahmagupta are two of the most renowned Indian mathematicians who made considerable contributions to the study of fractions.
- Bhaskara I is considered to be a follower of Aryabhatta.

Bhaskara I's Legacy in Mathematics

- As an Indian scholar, Bhaskara I's legacy is deeply rooted in his profound contributions to mathematics. His groundbreaking theories and mathematical techniques laid the foundation for future advancements in trigonometry, algebra, and number theory. Today, we remember him as a trailblazer whose work continues to inspire and shape the world of mathematics.

- Bhaskara I is famous for following works :

- Zero, positional arithmetic, the approximation of sine.

- The three treatises he wrote on the works of Aryabhata (476- 550 CE)

- The Mahabhaskariya (“Great Book of Bhaskara”)

- The Laghubhaskariya (“Small Book of Bhaskara”)

- The Aryabhatiyabhashya (629)

What were Bhaskara I's Main Works?

Aryabhatiyabhashya

Bhaskara I wrote the Aryabhatiyabhashya in 629. Bhaskara I explains in detail Aryabhatta’s method of solving linear equations with illustrative examples.

Mahabhaskariya

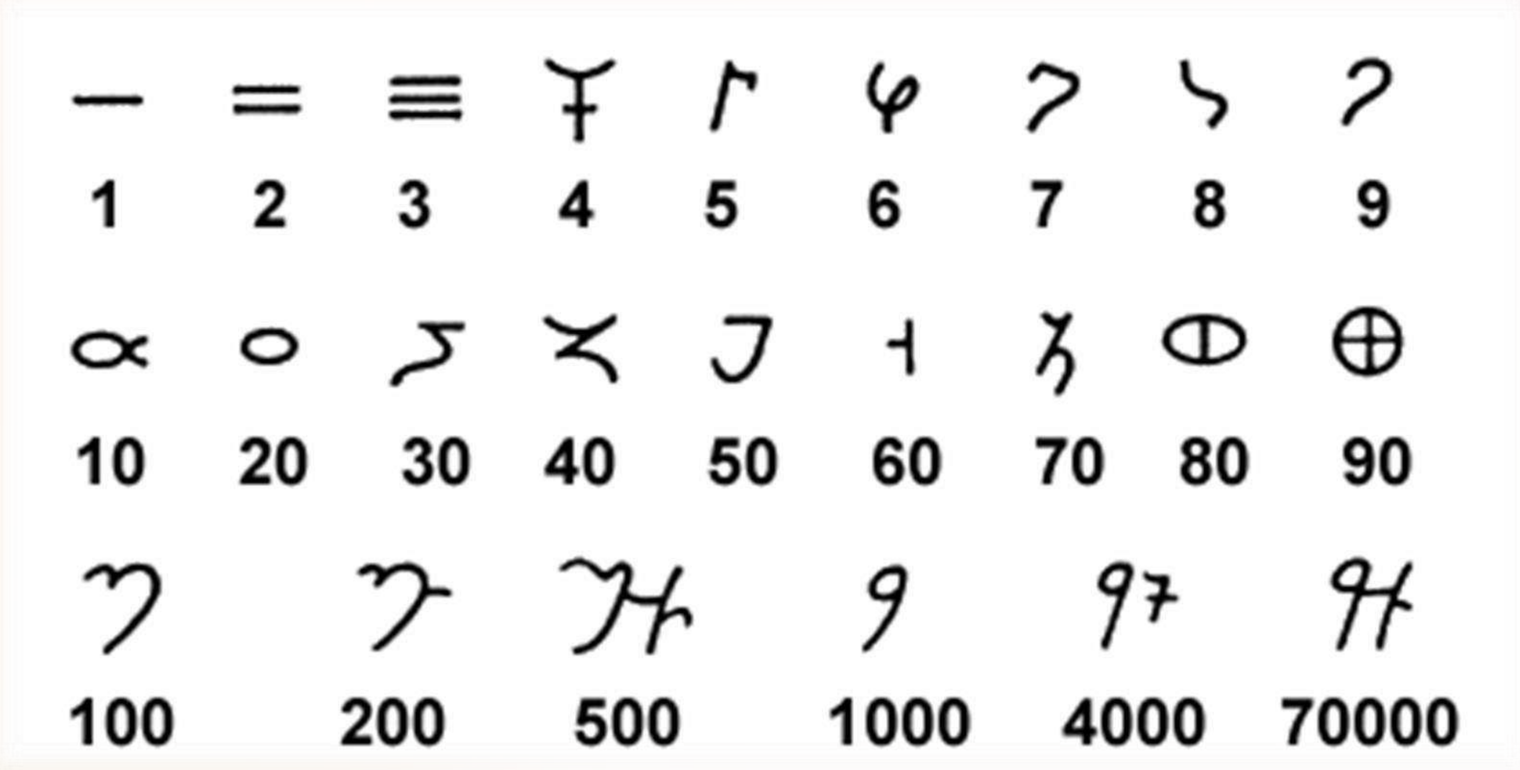
This book deals with the longitudes of the planets, eclipses of the sun and the moon, conjunctions of the planets with each other and with bright stars and rising - setting of the planets.

Laghubhaskariya

It includes such topic as direction, place and time from shadow, Lunar eclipse, solar eclipse etc.

Contribution

- He and Brahmagupta have contributions to the study of fractions.
- He represented the numbers in a positional system.
- He was the first to use the Brahmi numerals in a scientific contribution to Sanskrit.
- He even contributed in prime numbers.



How did Bhaskara I's Theories Advance Knowledge in His Time?

1

Astronomy

Bhaskara I's theories in astronomy brought advancements in understanding planetary motion, predicting solar and lunar eclipses, and explaining the concept of the celestial sphere.

2

Mathematics

Bhaskara I's mathematical theories revolutionized areas such as algebra, geometry, and arithmetic. He made significant contributions to the field of mathematics.

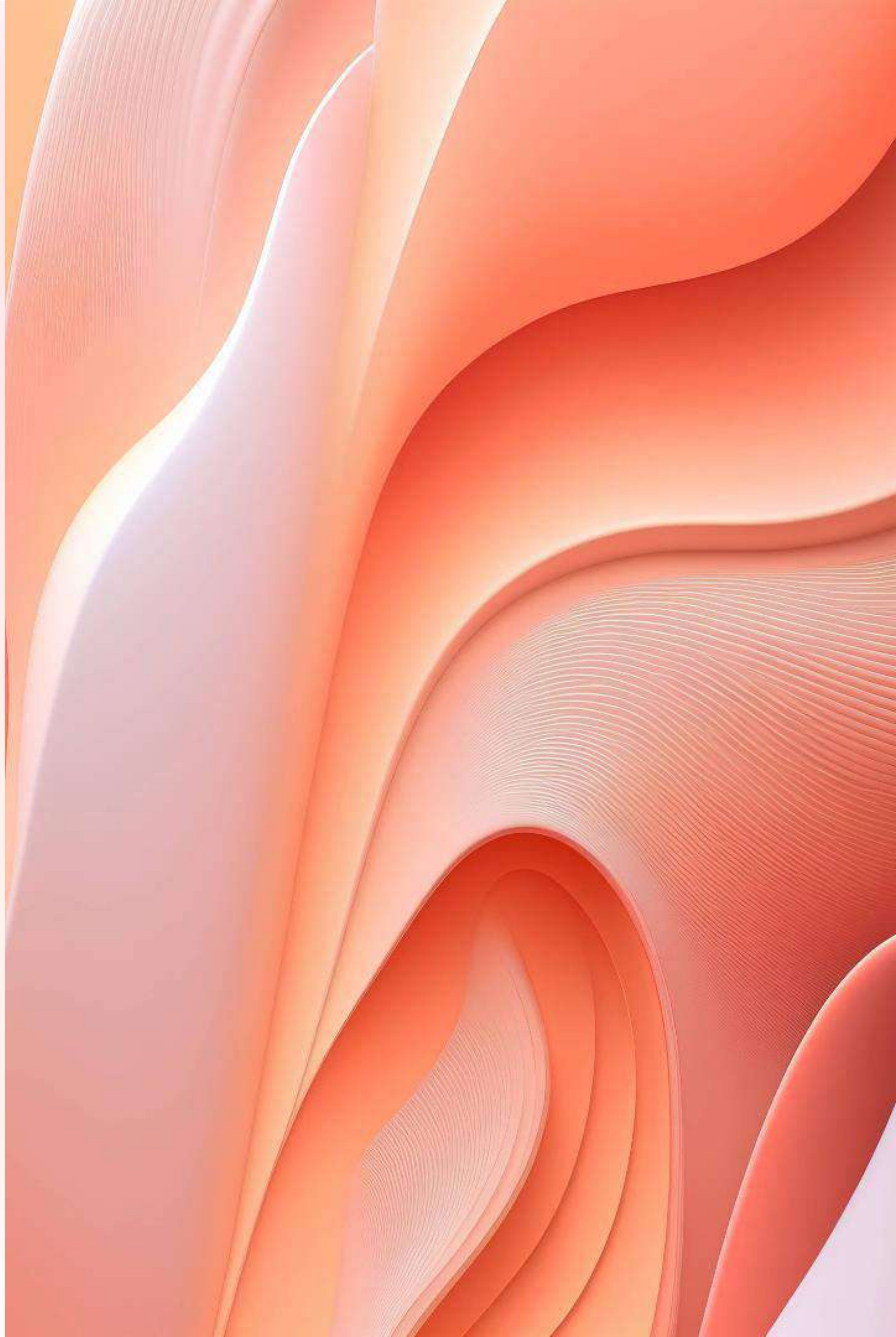
Why is Bhaskara I Important Today?

1 **Historical Context**

Bhaskara I's contributions were part of a vibrant period in Indian mathematics, which influenced developments in the Islamic world and further inspired the European Renaissance.

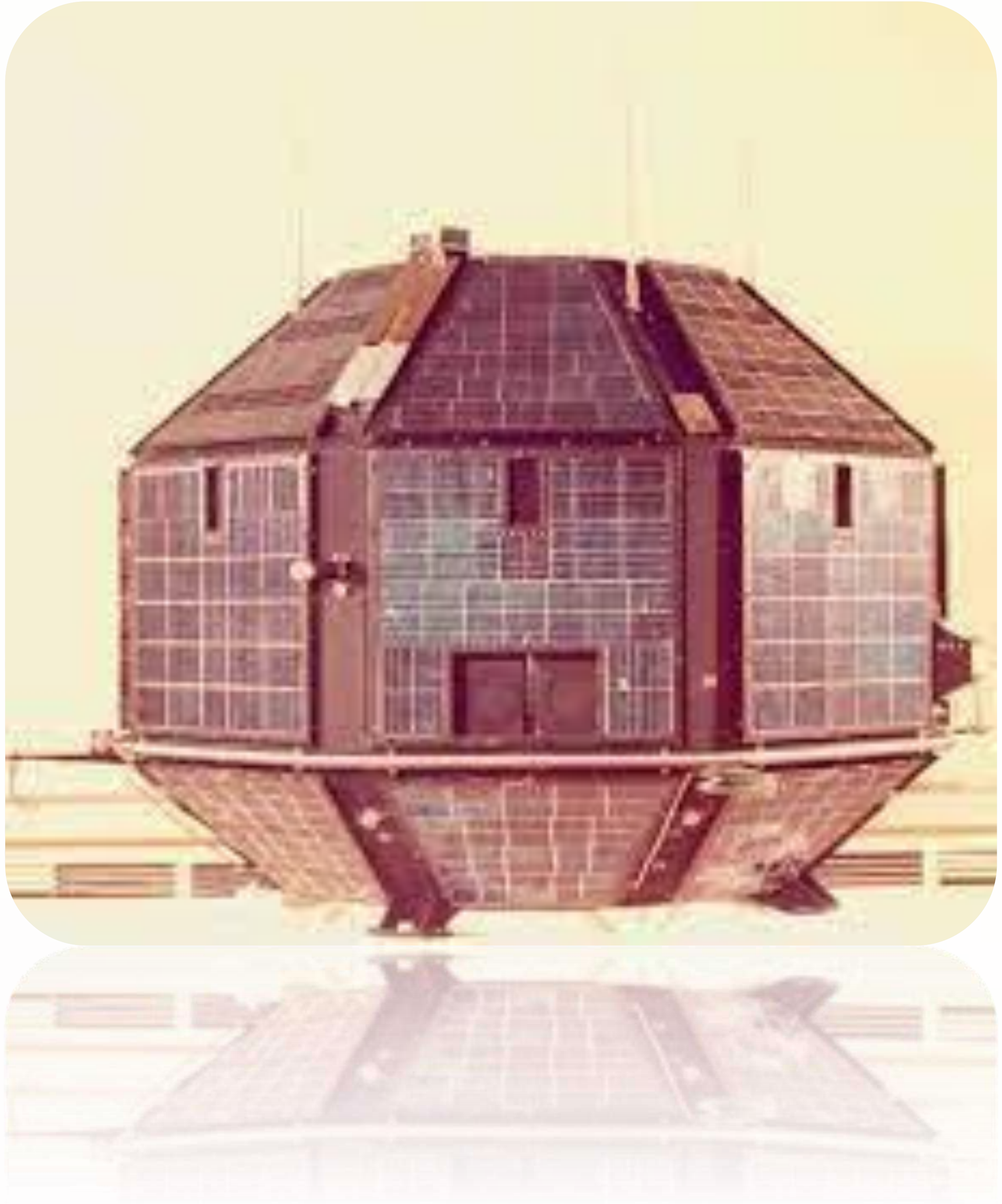
2 **Modern Applications**

Bhaskara I's theories and mathematical concepts find practical applications in various fields today, including calculus, navigation, computer science, and education.



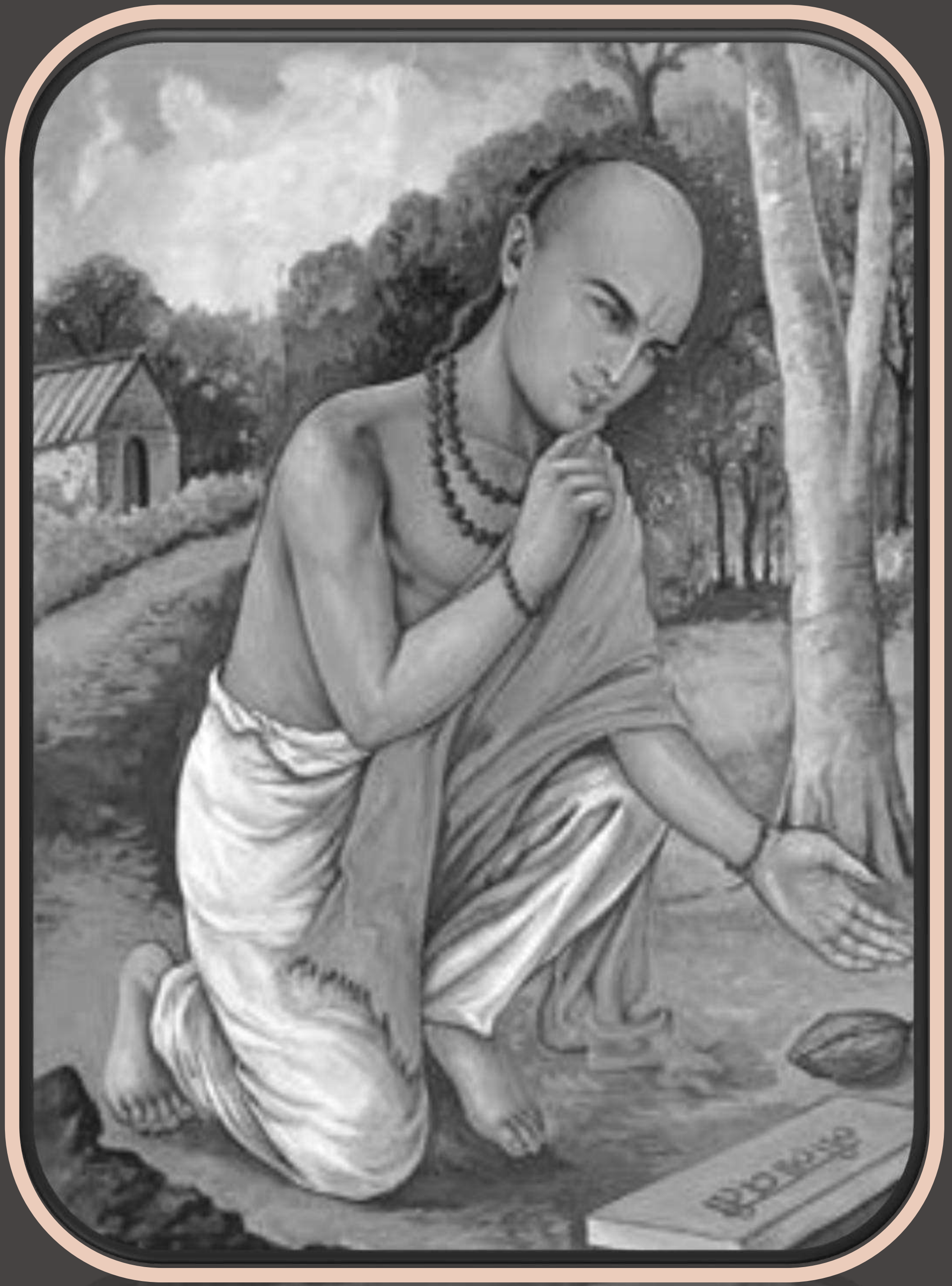
Death & Honors…

- Died in 680 CE.
- On 7 june 1979 the Indian space Reaserch Organisation launched Bhaskara I honouring the mathematician.
- Bhaskara I was the first experimental remote sensing satellite built by India.



Conclusion: Bhaskara I as a Model of Interdisciplinary Thinking and Creative Problem-Solving

- Bhaskara I's ability to bridge mathematics and astronomy exemplifies the power of interdisciplinary thinking. His ingenious solutions to complex problems continue to inspire generations of mathematicians and scientists.



Bhaskara II: The Mathematical Pioneer

Bhaskara II was an ancient Indian mathematician who made significant contributions to the field of mathematics. Let's explore his life and his remarkable achievements.

Who was Bhaskaracharya ?

- Bhaskara II also known as Bhaskara or as Bhaskaracharya, was a 12th century Indian mathematician.

- He was also a renowned astronomer who accurately defined many astronomical quantities, including the length of the sidereal year.

- He made the significant discovery of the principles of differential calculus and its application to astronomical problems and computations centuries before European mathematicians like Newton and Leibnitz made similar discoveries.

- It is believed that Bhaskara II was the first to conceive the differential coefficient and differential calculus.

Early Life and Education

- Bhaskara II was born in the 12th century in India. He received a comprehensive education, studying mathematics, astronomy, and other scientific disciplines.
- He was born in a Desastha Rigvedi Brahmin family near Vijjadavida (believed to be Bijjaragi of Vijayapur in modern Karnataka).
- Bhaskaracharya’s father was a Braman named Mahesvara.
- Mahesvara himself was famed as an astrologer, mathematician, astronomer, who taught him mathematics, which he later passed on to his son Loksamudra.
- Loksamudra’s son helped to set up a school in 1207 for the study of Bhakara’s writings.

Early Life and Education

- Bhaskara is also known as Bhaskara II or as Bhaskaracharya, this latter name meaning " Bhaskara the Teacher".
- It was Bhaskara I's influence that inspired Bhaskara II to join Ujjain, India's "most prestigious mathematical center" at the time.
- Bhaskara is said to have been the head of an astronomical observatory at Ujjain, the leading mathematical center of medieval India.
- It was at Ujjain where Bhaskara II would formulate the ideas which would go on to become his legacy.
- He lived in the Sahyadri region (Patnadevi, in Jalgaon district, Maharashtra).

Earth's Gravity

आकृष्टिशक्तिश्च मही तया यत् खस्थं गुरुस्वाभिमुखं स्वशक्तत्या ।
आकृष्यते तत्पततीव भाति समेसमन्तात् क्व पतत्वियं खे ॥ ६ ॥

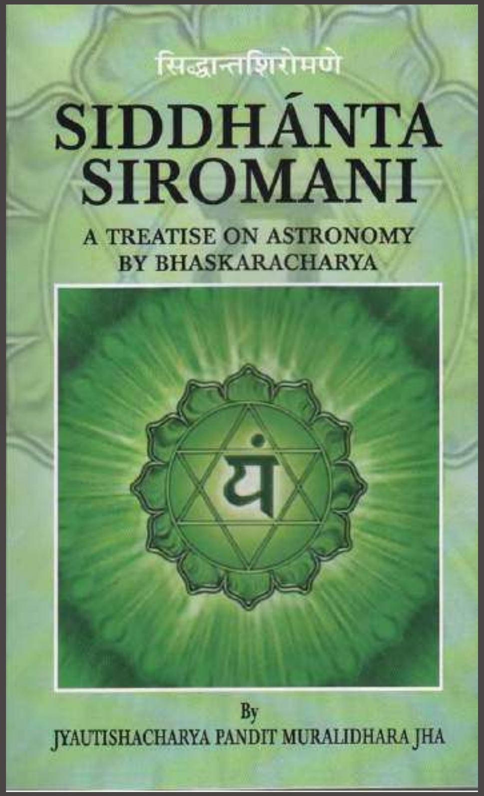
- सिद्धांत शिरोमणि, गोलाध्याय – भुवनकोश

Rough translation –

Earth naturally attracts every object in the space towards itself. Because of this attracting force, all objects fall on the earth. When there is balance in attraction among planets where would they fall ?

Siddhanta-Siromani

- Bhaskaracharya wrote his greatest work Siddhanta Shiromani in 1450 verses and in four parts which includes Lilavati (278 verses), Bijaganita (213 verses), Ganitadhyaya (451 verses) and Goladhyaya (501 verses) which are also sometimes considered four independent works. These all are deal with arithmetic, algebra, mathematics of planets and spheres respectively.



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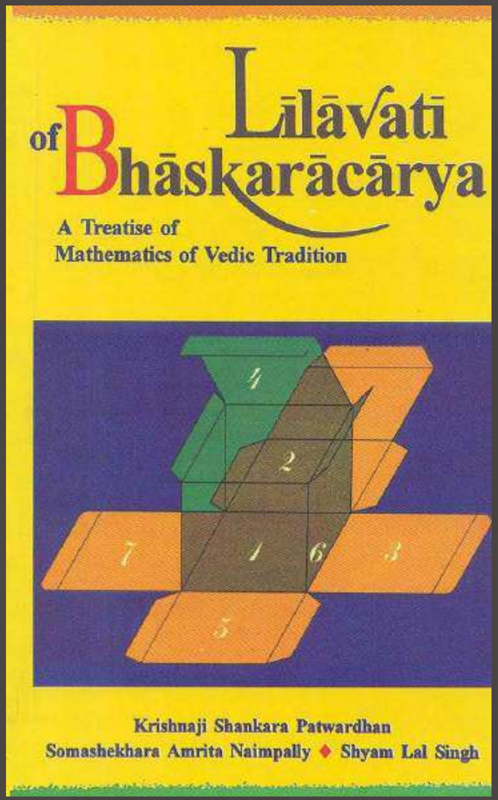
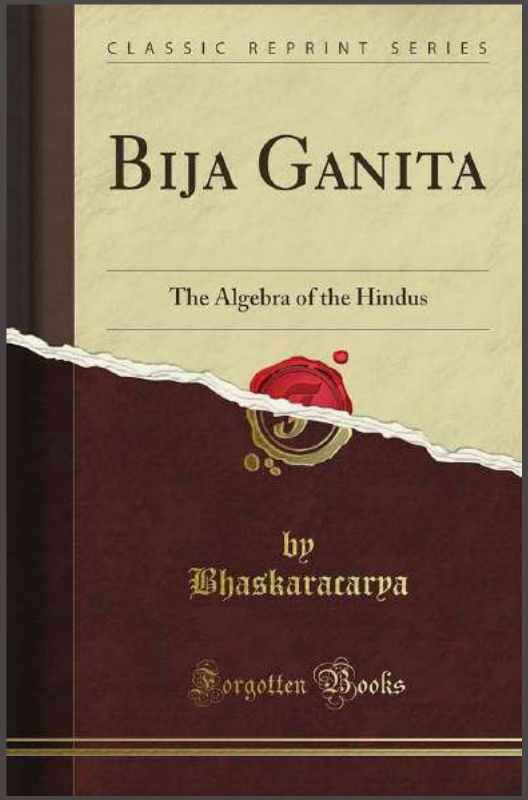
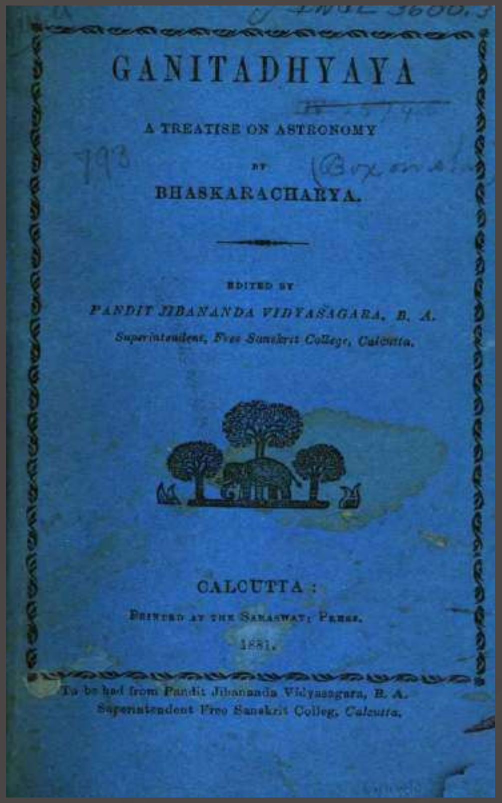
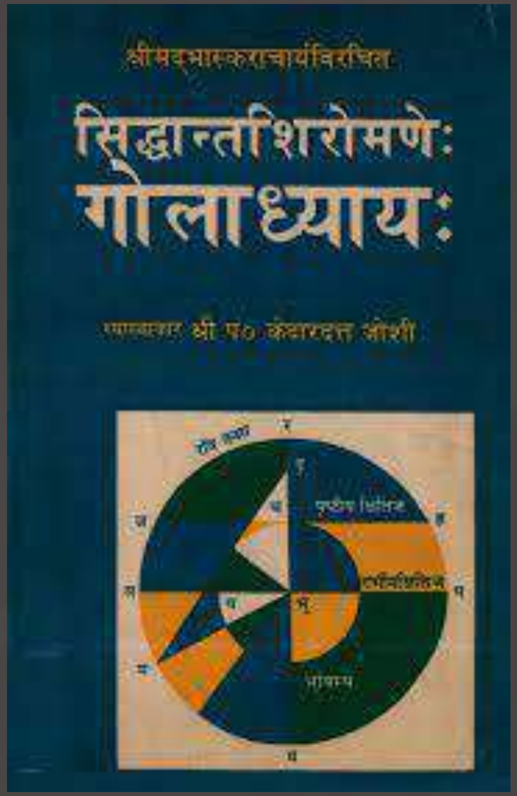








Contributions to Mathematics

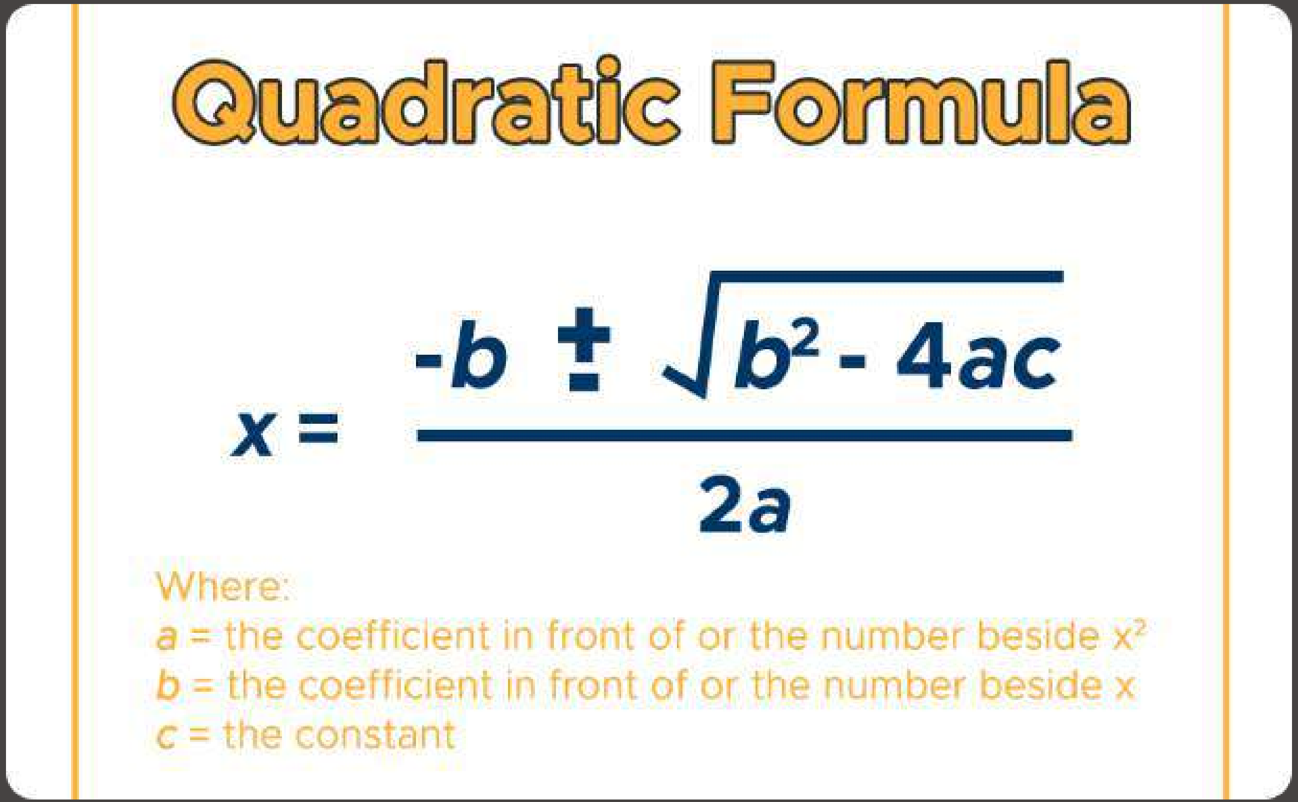
1 Lilavati

- To console his daughter, who remained a widow the rest of her life, Bhaskara promised to nbame a book after her and named it Lilavati (The Beautiful), one that would remain till the end of time as a good name is akin to a second life. It covers calculations, progression, measurement, permutations and other topics.

2 Bijaganita

- He also authored "Bijaganita," a treatise on algebra that introduced innovative solutions for equations. It discusses zero, infinity, positive and negative numbers, and indeterminate equations including Pell’s equation, solving it using a Kuttaka method.

Quadratic formula



- This is because most of the world simply refers to Bhaskara's discovery as the quadratic formula for the solutions of quadratic equations although it was pre discovered by Sridhar Acharya (870- 930 CE India).

Spherical Trigonometry

- Bhaskara developed many of the trigonometric identities and formulas used throught mathematics, navigation, geodesy and astronomy.
- For instance, the Goladhyaya is the first publication to report the now-famous formulas :

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

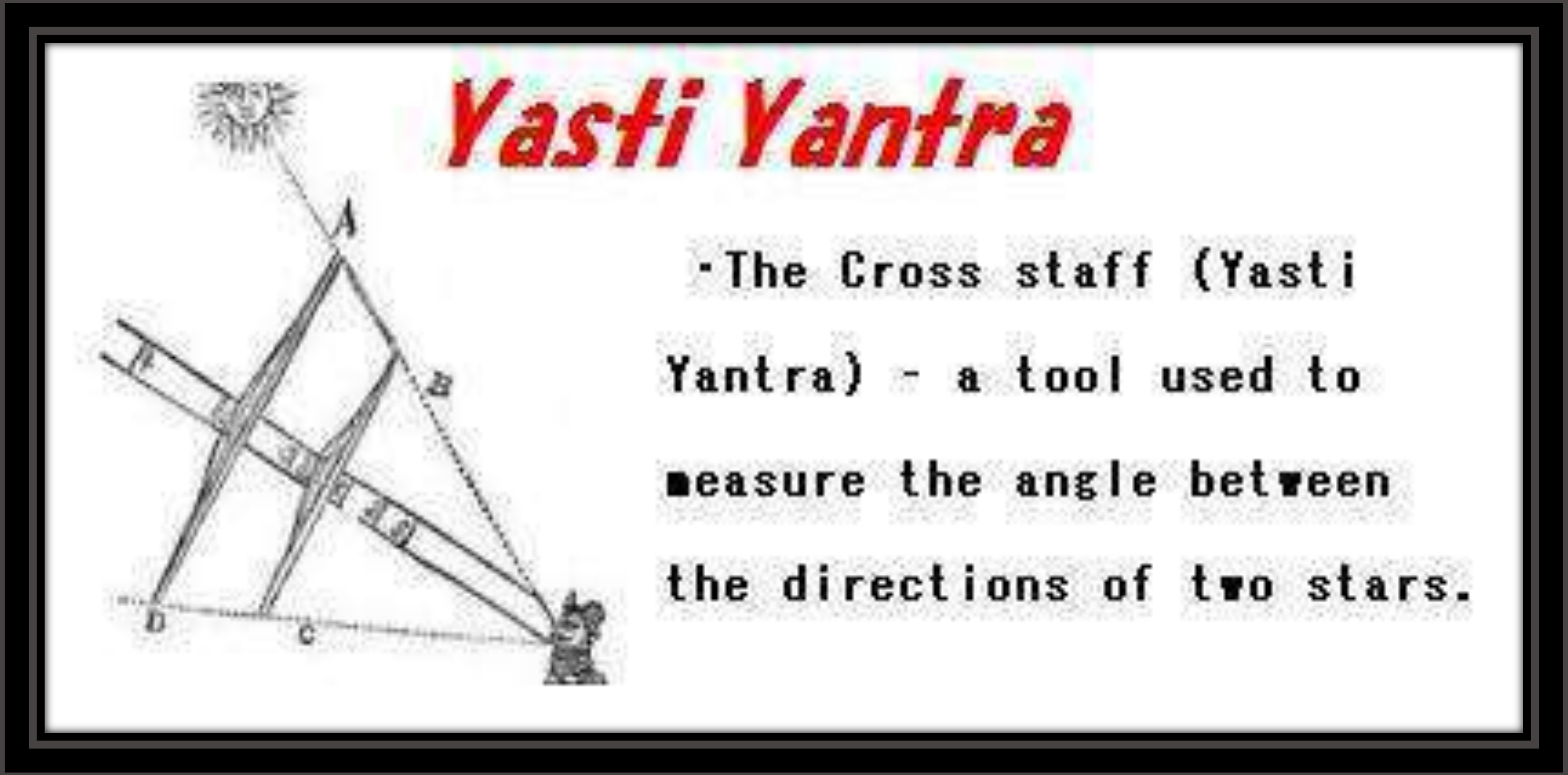
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

The Bhaskara Wheel

- The earliest reference to a perpetual motion machine date back to 1150, when Bhaskara II described a wheel that he claimed would run forever.

Yasti- Yantra

- Bhaskara II used a measuring device known as Yasti-Yantra. This device could vary from a simple stick to V- shaped staffs designed specifically for determining angles with the help of a calibrated scale



Work on Indeterminate Equations

1

Exploration

Bhaskara II investigated indeterminate equations, which involve finding integer solutions.

2

Solutions

He developed techniques to find specific solutions that satisfy certain conditions.

3

Algebraic Manipulation

Bhaskara II explored advanced algebraic manipulation to solve these intricate equations.

Legacy and Impact

Modern Mathematics

Bhaskara II's work continues to influence modern mathematics and algebraic techniques.

Cultural Importance

His contributions are celebrated as a significant part of India's rich scientific heritage.

Inspiration

Bhaskara II's dedication to education and exploration serves as an inspiration to future generations of mathematicians.

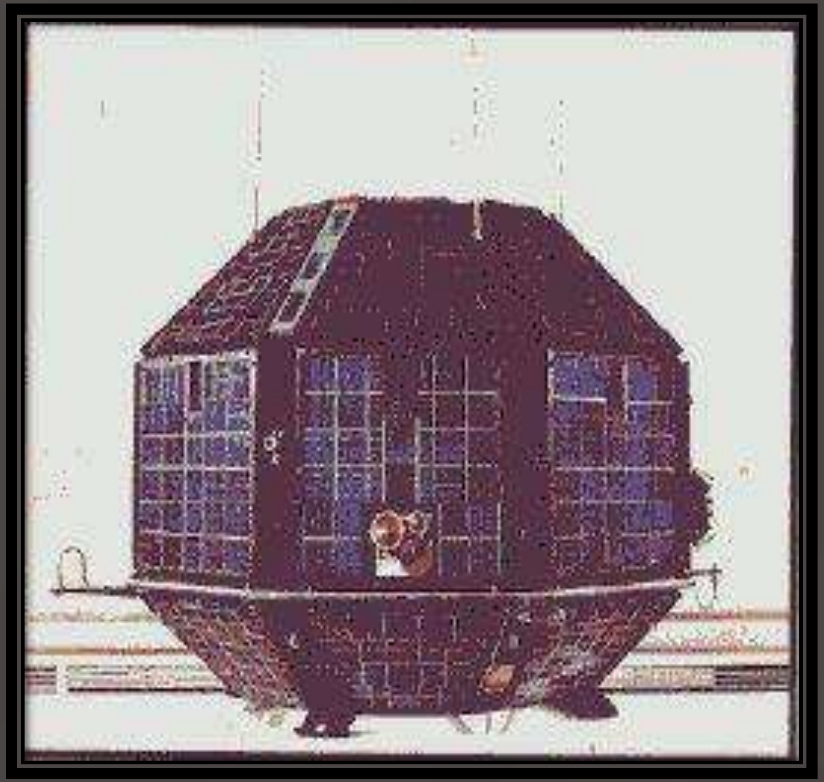
Death

- Died in 1185.

- A number of institutes and colleges in India are named after him, including Bhaskaracharya Pratishthana in Pune, Bhaskaracharya College of Applied sciences in Delhi, Bhaskaracharya Institute for Space Applications and Geo-Informatics in Gandhinagar.

In his Honour

- On 20 November 1981 the Indian Space Research Organization (ISRO) launched the Bhaskara II satellite honoring the mathematician and astronomer.



Bhaskara II satellite

Conclusion

- Bhaskara II's pioneering efforts in mathematics laid the foundation for many mathematical principles still used today. His intriguing methods and ingenious solutions continue to captivate mathematicians around the world.

- His works show the influence of Brahmagupta, Sridhara, Mahavira, Padmanabha and other predecessors.

THANK YOU

Shree H. N. Shukla Group of Colleges, Rajkot

Bachelor of Computer Applications

Sem-1

CS-07 Mathematics in Ancient India: Exploring the rich Heritage of Vedic Mathematics

UNIT-2

Biography of Remarkable Indian Mathematicians

In this unit we will study the biography of following great Indian mathematicians

- Srinivasa Ramanujan
- C. R. Rao
- P. C. Mahalanobis
- D. R. Kaprekar
- Satyendranath Bose
- Shakuntala Devi

SRINIVASA RAMANUJAN

- Srinivasa Ramanujan was a great Indian mathematician. He is counted among the greatest mathematicians of modern times.
- He was the second Indian to become a member of the Royal Society and the first to become a member of Trinity College in Cambridge.
- Based on his talent and passion, he made wonderful inventions in mathematics and simultaneously illuminated the name of India in the whole world.
- He contributed Mathematical Analysis, Number Theory, Infinite Serie and continued Fractions.

Early Life

- Ramanujan was born on December 22, 1887, in the village of Kumbakonam, Tamil Nadu, to a Brahmin family. His father worked as a clerk in sari store, while his mother was a homemaker who sang at a neighboring temple.

- They lived in a small traditional home on Sarangapani Sannidhi Street in the town of Kumbakonam. The family home is now a museum.

- Ramanujan's intellectual growth as a youngster differed from that of other children. Ramanujan did not learn to talk until he was three years old. As a result, his parents wondered if he was mentally ill.

- In nature, he was peaceful, kind, and emotional. He would take a close look at everything and begin to consider it. Ramanujan was an insatiable questioner. His professors found his queries to be a little odd at times. For example, who was the first guy in the world? How far is the earth from the clouds?

- On 1 October 1892, Ramanujan was enrolled at the local school and everyone was astounded by his brilliance. He topped the entire district in the primary exams at 10 and went to Town High School for additional education. He succeeded in all subjects, especially mathematics. After that, he enrolled in Town High School and spent six years there.

- A child prodigy by age 11, he had exhausted the mathematical knowledge of two college students who were lodgers at his home.

- He was later lent a book written by S.L. Loney on advanced trigonometry. He mastered this by the age of 13 while discovering sophisticated theorems on his own.

- By 14, he received merit certificates and academic awards that continued throughout his school career, and he assisted the school in the logistics of assigning its 1,200 students (each with differing needs) to its approximately 35 teachers.

- Ramanujan was shown how to solve cubic equations in 1902. He would later develop his own method to solve the quartic.

- In 1903, when he was 16, Ramanujan obtained from a friend a library copy of A Synopsis of Elementary Results in Pure and Applied Mathematics, G. S. Carr's collection of 5,000 theorems. Ramanujan reportedly studied the contents of the book in detail.

- The next year, Ramanujan independently developed and investigated the Bernoulli numbers and calculated the Euler–Mascheroni constant up to 15 decimal places.

- When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. Ranganatha Rao prize for mathematics by the school's headmaster, Krishnaswami Iyer.

- Iyer introduced Ramanujan as an outstanding student who deserved scores higher than the maximum. He received a scholarship to study at Government Arts College, Kumbakonam, but was so intent on mathematics that he could not focus on any

- In August 1905, Ramanujan ran away from home, heading towards Visakhapatnam, and stayed in Rajahmundry for about a month. He later enrolled at Pachaiyappa's College in Madras. There, he passed in mathematics, choosing only to attempt questions that appealed to him and leaving the rest unanswered, but performed poorly in other subjects, such as English, physiology, and Sanskrit.

- Ramanujan failed his Fellow of Arts exam in December 1906 and again a year later. Without an FA degree, he left college and continued to pursue independent research in mathematics, living in extreme poverty and often on the brink of starvation.

- In 1910, after a meeting between the 23-year-old Ramanujan and the founder of the Indian Mathematical Society, V. Ramaswamy Aiyer, Ramanujan began to get recognition in Madras's mathematical circles, leading to his inclusion as a researcher at the University of Madras.

Adulthood in India

- On 14 July 1909, Ramanujan married Janaki (Janakiammal; 21 March 1899 – 13 April 1994), a girl his mother had selected for him a year earlier and who was ten years old when they married.

- After the marriage, Ramanujan developed a hydrocele testis. The condition could be treated with a routine surgical operation that would release the blocked fluid in the scrotal sac, but his family could not afford the operation. In January 1910, a doctor volunteered to do the surgery at no cost.

- After his successful surgery, Ramanujan searched for a job. He stayed at a friend's house while he went from door to door around Madras looking for a clerical position. To make money, he tutored students at Presidency College who were preparing for their Fellow of Arts exam.

- In late 1910, Ramanujan was sick again. He feared for his health, and told his friend R. Radakrishna Iyer to "hand [his notebooks] over to Professor Singaravelu Mudaliar [the mathematics professor at Pachaiyappa's College] or to the British professor Edward B. Ross, of the Madras Christian College.

- After Ramanujan recovered and retrieved his notebooks from Iyer, he took a train from Kumbakonam to Villupuram, a city under French control. In 1912, Ramanujan moved with his wife and mother to a house in Saiva Muthaiah Mudali street, George Town, Madras, where they lived for a few months. In May 1913, upon securing a research position at Madras University, Ramanujan moved with his family to Triplicane.

Attention from Mathematicians

- In 1910, Ramanujan met deputy collector V. Ramaswamy Aiyer, who founded the Indian Mathematical Society. Wishing for a job at the revenue department where Aiyer worked, Ramanujan showed him his mathematics notebooks. As Aiyer later recalled:

  I was struck by the extraordinary mathematical results contained in [the notebooks]. I had no mind to smother his genius by an appointment in the lowest rungs of the revenue department.

- Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras. Some of them looked at his work and gave him letters of introduction to R. Ramachandra Rao, the district collector for Nellore and the secretary of the Indian Mathematical Society. Rao was impressed by Ramanujan's research but doubted that it was his own work. Ramanujan mentioned a correspondence he had with Professor Saldhana, a notable Bombay mathematician, in which Saldhana expressed a lack of understanding of his work but concluded that he was not a fraud. Ramanujan's friend C. V. Rajagopalachari tried to quell Rao's

- Rao agreed to give him another chance, and listened as Ramanujan discussed elliptic integrals, hypergeometric series, and his theory of divergent series, which Rao said ultimately convinced him of Ramanujan's brilliance. When Rao asked him what he wanted, Ramanujan replied that he needed work and financial support. Rao consented and sent him to Madras. He continued his research with Rao's financial aid. With Aiyer's help, Ramanujan had his work published in the Journal of the Indian Mathematical Society.

- One of the first problems he posed in the journal was to find the value of:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+\cdots}}}$$

- He wrote his first formal paper for the journal on the properties of Bernoulli Numbers.
- In early 1912 he got job in the Madras Accountant Generals office with a salary of Rs. 20 per month.
- Later he applied for a position under the Chief Accountant of the Madras Port Trust.
- Accepted as a Class III, Grade IV Accounting clerk making Rs 30 per month and

Contacting English Mathematicians

- In the spring of 1913, Narayana Iyer, Ramachandra Rao and E. W. Middlemast tried to present Ramanujan's work to British mathematicians. M. J. M. Hill of University College London commented that Ramanujan's papers were riddled with holes. He said that although Ramanujan had "a taste for mathematics, and some ability", he lacked the necessary educational background and foundation to be accepted by mathematicians.

- Although Hill did not offer to take Ramanujan on as a student, he gave thorough and serious professional advice on his work. With the help of friends, Ramanujan drafted letters to leading mathematicians at Cambridge University.

- Later on Ramanujan wrote to G.H. Hardy, he was an academician at Cambridge University, he was a prominent English Mathematician, known for his achievements in Number Theory and Mathematical Analysis.

- Hardy recognized some of his formula but other seemed scarcely possible to believe.

- Hardy believed that Ramanujan's theorems must be true otherwise no one could have imagined to invent them.

- Hardy considered him to be "a mathematician of the highest quality, a man of altogether exceptional originality and power".

- Hardy's one colleague, E. H. Neville, later remarked that "not one [theorem] could have been set in the most advanced mathematical examination in the world".

- Hardy invited Ramanujan to Cambridge but Ramanujan refused to leave his country to "go to a foreign land".

- Hardy then enlisted E.H. Neville to bring Ramanujan to England.

- With his parents supporting him he agreed to the proposal this time.

Life in England

- Ramanujan departed from Madras aboarded the S.S. *Nevasa* on 17 March, 1914 and arrived in London on 14 April.
- Ramanujan began his work with Littlewood and Hardy.
- Hardy had already received 120 theorems from Ramanujan in the first two letters, but there were many more results and theorems in the notebooks.
- Ramanujan left a deep impression on Hardy and Littlewood. Littlewood commented, "I can believe that he's at least a Jacobi" while Hardy said he "can compare him only with Euler or Jacobi."
- Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood.

Ramanujan’s Honours

- Ramanujan was awarded a Bachelor of Arts by Research degree(later calledPhD degree) in March 1916 for his work on highly composite numbers.
- On 6 December 1917, Ramanujan was elected to the London Mathematical Society.
- On 2 May 1918, he was elected a Fellow of the Royal Society, the second Indian admitted, after Ardaseer Cursetjee in 1841. At age 31, Ramanujan was one of the youngest Fellows in the Royal Society's history.
- On 13 October 1918, he was the first Indian to be elected a Fellow of Trinity College, Cambridge.

Illness & Death

- ❖Ramanujan had numerous health problems throughout his life.
- ❖He was diagnosed with tuberculosis and a severe vitamin deficiency, and confined to a sanatorium.
- ❖In 1919, he returned to Kumbakonam, Madras Presidency, and in 1920 he died at the age of 32.
- ❖After his death, in 1994 Dr. D.A.B. Young analysed his records and concluded he had hepatic amoebiasis.

Personality & Spiritual life

- A person with a somewhat shy and quiet disposition, a dignified man with pleasant manners.
- He credited his acumen to his family goddess, Namagiri Thayar (Goddess Mahalakshmi) of Namakkal.
- Ramanujan often said, "An equation for me has no meaning unless it expresses a thought of God."

Ramanujan’s Notebook

- ❖ Ramanujan recorded the bulk of his results in four notebooks of looseleaf paper.
- ❖ Result were mostly written up without any derivations.
- ❖ Mathematician Bruce C. Berndt, in his review of these notebooks and Ramanujan's work, says that Ramanujan most certainly was able to prove most of his results, but chose not to record the proofs in his notes.
- ❖ This may have been for any number of reasons. Since paper was very expensive, Ramanujan did most of his work and perhaps his proofs on slate, after which he transferred the final results to paper.
- ❖ He was also quite likely to have been influenced by the style of G. S. Carr's book, which stated results without proofs.

- Hardy himself wrote papers exploring material from Ramanujan's work, as did G. N. Watson, B. M. Wilson, and Bruce Berndt.

- In 1976, George Andrews rediscovered a fourth notebook with 87 unorganised pages, the so-called "lost notebook".

Ramanujan-Hardy Number

- The number 1729 is known as the Hardy–Ramanujan number.

- Hardy arrived in a cab numbered 1729 to see Ramanujan at a hospital. In hardy's words:

I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No", he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Other Mathematician’s views of Ramanujan

- J.H Hardy was highly impressed by Ramanujan.
- Hardy said that the solutions were ”arrived at by a process of mingled argument, intuition, and induction, of which he was entirely unable to give any coherent account” He also said that he had "never met his equal, and can compare him only with Euler or Jacobi".
- On the basis of pure talent on a scale from 0 to 100. Hardy gave himself a score of 25, J. E. Littlewood 30, David Hilbert 80 and Ramanujan 100.

Recognition

- Ramanujan's home state of Tamil Nadu celebrates 22 December (Ramanujan's birthday) as 'State IT Day’.

- 22 December annually celebrated as Ramanujan Day by the Government Arts College, Kumbakonam, where he studied, and at the IIT Madras in Chennai.

- In 2011, on the 125th anniversary of his birth, the Indian government declared that 22 December will be celebrated every year as National Mathematics Day. Then Indian Prime Minister Manmohan Singh also declared that 2012 would be celebrated as National Mathematics Year and 22 December as National Mathematics Day of India.

- SASTRA University, a private university based in Tamil Nadu, has instituted the SASTRA Ramanujan Prize of US$10,000 to be given annually to a mathematician not exceeding age 32 for outstanding contributions in an area of mathematics influenced by Ramanujan.

- Stamps released by the government in 1962.

In Popular Culture

- The Man Who Loved Numbers is a 1988 PBS NOVA documentary about Ramanujan.

- The Man Who Knew Infinity is a 2015 film based on Kanigel's book of the same name. British actor Dev Patel portrays Ramanujan.

- A Disappearing Number is a British stage production by the company Complicite that explores the relationship between Hardy and Ramanujan.

- In the famous film ”Good Will Hunting” the main character is compared to Ramanujan.

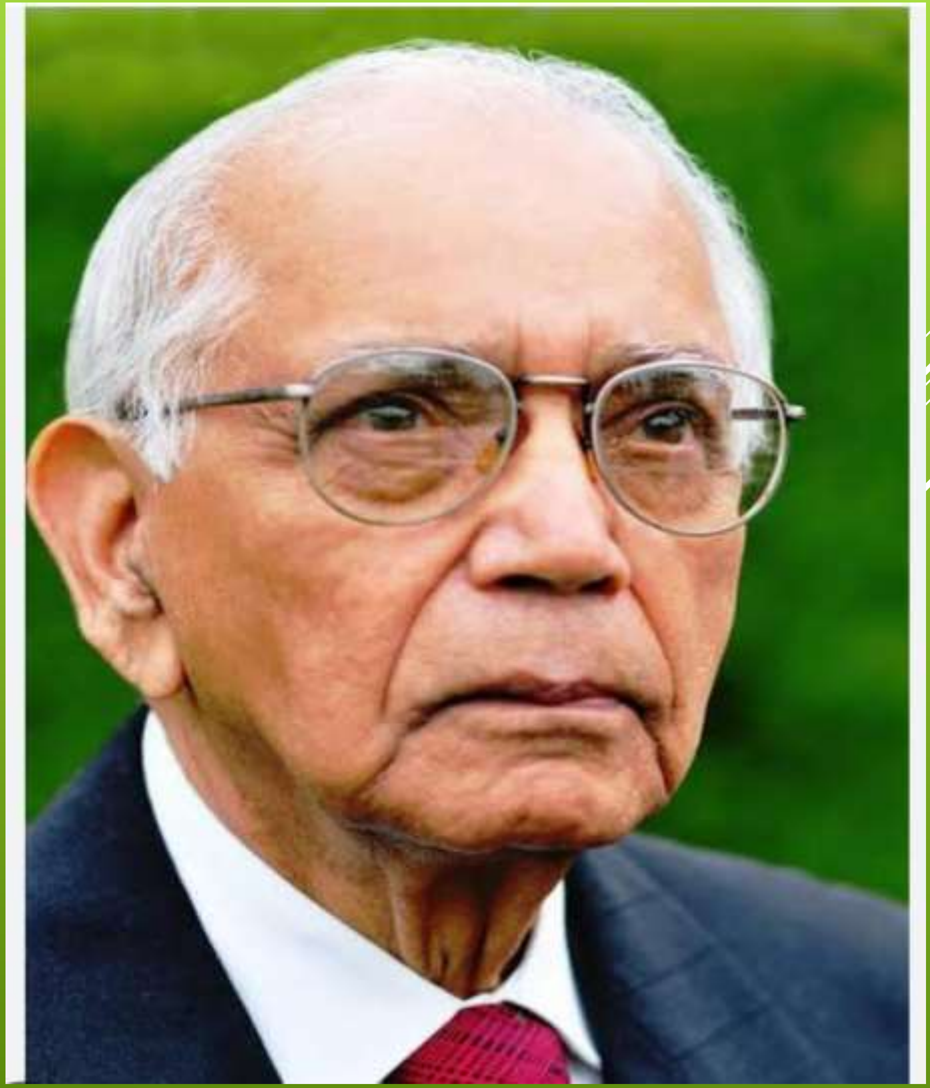
- Google honored Ramanujan on his 125th birth anniversary by replacing its logo with a doodle on its home page.

CALYAMPUDI RADHAKRISHNA RAO

- **Calyampudi Radhakrishna Rao** was an Indian-American mathematician and statistician.

- He was professor emeritus at Pennsylvania State University and Research Professor at the University at Buffalo.

- Rao was honored by numerous colloquia, honorary degrees, and festschrifts and was awarded the US National Medal of Science in 2002.



- The American Statistical Association has described him as "a living legend whose work has influenced not just statistics, but has had far reaching implications for fields as varied as economics, genetics, anthropology, geology, national planning, demography, biometry, and medicine.“

- *The Times of India* listed Rao as one of the top 10 Indian scientists of all time.
- In 2023, Rao was awarded the International Prize in Statistics, an award often touted as the "statistics’ equivalent of the Nobel Prize".

- Rao was also a Senior Policy and Statistics advisor for the Indian Heart Association non-profit focused on raising South Asian cardiovascular disease awareness.

Early Life

- Calyampudi Radhakrishna Rao was born on 10 sep, 1920 at hadagali in Karnataka, India.
- His schooling was completed in Gudur, Nuzvid, Nandigama, and Visakhapatnam, all in the present state of Andhra Pradesh.
- He received an MSc in mathematics from Andhra University and an MA in statistics from Calcutta University in 1943.
- He obtained a PhD degree at King's College, Cambridge, under R. A. Fisher in 1948, to which he added a DSc degree, also from Cambridge, in 1965.

Career

- Rao first worked at the Indian Statistical Institute and the Anthropological Museum in Cambridge.

- Later he held several important positions, as the Director of the Indian Statistical Institute, Jawaharlal Nehru Professor and National Professor in India, University Professor at the University of Pittsburgh and Eberly. Professor and Chair of Statistics and Director of the Center for Multivariate Analysis at Pennsylvania State University.

- As Head and later Director of the Research and Training School at the Indian Statistical Institute for a period of over 40 years, Rao developed research and training programs and produced several leaders in the field of Mathematics. On the basis of Rao's recommendation, the Asian Statistical Institute (ASI), now known as the Statistical Institute for Asia and Pacific, was established in Tokyo to provide training to statisticians working in government and industrial organizations.

- Among his best-known discoveries are the Cramér–Rao bound and the Rao–Blackwell theorem both related to the quality of estimators. Other areas he worked in include multivariate analysis, estimation theory, and differential geometry. His other contributions include the Fisher–Rao theorem, Rao distance, and orthogonal arrays. He was the author of 14 books and published over 400 journal publications.

- Rao received 38 honorary doctoral degrees from universities in 19 countries around the world and numerous awards and medals for his contributions to statistics and science. He is a member of eight National Academies in India, the United Kingdom, the United States, and Italy.

Areas of Research Contributions

- Estimation theory
- Statistical inference & linear models
- Multivariate Analysis
- Combined Designs
- Orthogonal Arrays
- Biometry
- Statistical Genetics
- Generalized Matrix Inverses
- Functional equations

Awards

- Rao has received 38 honorary doctoral degrees from universities in 19 countries around the world and numerous awards and medals for his contributions to statistics and science.

- Rao was awarded with the United States ***National Medal of Science***, which is the nation's highest award for lifetime achievement in fields of scientific research, in June 2002.

- He was given the ***India Science Award*** in 2010, the highest honor conferred by the government of India in a scientific domain. In 2013, he was nominated for the Nobel Peace Prize, along with Miodrag Lovric and ***Shlomo Sawilowsky***, for their contribution to the ***International Encyclopedia of Statistical Science***.

- He was most recently honored with his 38^{th} honorary doctorate by the Indian Institute of Technology, Kharagpur, on 26 July 2014 for his contributions to the foundations of modern statistics.

❖ He has received many recognitions including the ***Padma Bhushan*** and fellow of the Royal Society UK.

❖ He was awarded with the ***Padma Vibhushan.***

❖ The ***American Statisticalc Association*** has described him as "a ***living legend*** whose work has influenced not just statistics, but has had far reaching implications for fields as varied as economics, genetics, anthropology, geology, national planning, demography, biometry, and medicine.

❖ The C. R. Roy Award for statistics was instituted in his honor, to be given in 2 years.

Recognition & Death

- He died in Buffalo, New York, U.S., on 22 August 2023, at the age of 102.
- The Pennsylvania State University has established C. R. and Bhargavi Rao Prize in Statistics.
- CR Rao Advanced Institute of Mathematics, Statistics and Computer Science.
- National Award in Statistics established by Ministry of Statistics and Programme Implementation (MoSPI), Government of India.
- The road from IIIT Hyderabad passing along Central University of Hyderabad crossroads to Alind Factory, Lingampally is named as "Prof. C.R. Rao Road".

PRASANTA CHANDRA MAHALANOBIS

- **Prasanta Chandra Mahalanobis** was an Indian scientist and statistician.

- He is best remembered for the Mahalanobis distance, a statistical measure, and for being one of the members of the first Planning Commission of free India.

- He made pioneering studies in anthropometry in India.
- He founded the Indian Statistical Institute, and contributed to the design of large-scale sample surveys.

- For his contributions, Mahalanobis has been considered the Father of statistics in India.

Early Life

- Prasanta Chandra Mahalanobis was born on 29th June,1893 at Calcutta, Bengal.

- Mahalanobis received his early schooling at the Brahmo Boys School in Calcutta, graduating in 1908.

- He joined the Presidency College, then affiliated with the University of Calcutta, where he was taught by teachers who included Jagadish Chandra Bose, and Prafulla Chandra Ray. Others attending were Meghnad Saha, a year junior, and Subhas Chandra Bose, two years his junior at college.

- Mahalanobis received a Bachelor of Science degree with honours in physics in 1912. He left for England in 1913 to join the University of London.

- After missing a train, he stayed with a friend at King's College, Cambridge. He was impressed by King's College Chapel and his hostel's friend M. A. Candeth suggested that he could try joining there, which he did.

- He interacted with the mathematical genius Srinivasa Ramanujan during the latter's time at Cambridge.

- After his Tripos in physics, Mahalanobis worked with C. T. R. Wilson at the Cavendish Laboratory.

- He took a short break and went to India, where he was introduced to the Principal of Presidency College and was invited to take classes in physics.

- After returning to England, Mahalanobis was introduced to the journal *Biometrika*. This interested him so much that he bought a complete set and took them to India.

- He discovered the utility of statistics to problems in meteorology and anthropology, beginning to work on problems on his journey back to India.

- Mahalanobis married to Nirmalkumari (Rani), daughter of Heramba Chandra Maitra, a leading educationist and member of the Brahmo Samaj, on 27 February 1923, although her father did not completely approve of the marriage.

Indian Statistical Institute

- Many colleagues of Mahalanobis took an interest in statistics. An informal group developed in the Statistical Laboratory, which was located in his room at the Presidency College, Calcutta.

- On 17 December 1931 Mahalanobis called a meeting with Pramatha Nath Banerji (Minto Professor of Economics), Nikhil Ranjan Sen (Khaira Professor of Applied Mathematics) and Sir R. N. Mukherji. Together they established the Indian Statistical Institute (ISI) in Baranagar, and formally registered on 28 April 1932 as a non-profit distributing learned society under the Societies Registration Act XXI of 1860.

- In 1933, the Institute founded the journal *Sankhya*, along the lines of Karl Pearson's *Biometrika*.
- In 1959, the institute was declared as an institute of national importance and a Deemed university.
- The methods pioneered at the institute are now used by the World Bank and the United Nations.
- As Nobel Prize-winning economist Angus Deaton and co-author Valerie Kozel wrote in 2005: "Where Mahalanobis and India led, the rest of the world has followed, so that today, most countries have a recent household income or expenditure survey. Most countries, can only envy India in its statistical capacity".

- Economists TN Srinivasan, Rohini Somanathan, Pranab Bardhan and another Nobel-winner Abhijit Banerjee have since argued that there is "no other instance of an entirely homegrown institution in a developing country becoming a world leader in a large field of general interest".

Contributions to statistics

1. Mahalanobis distance:

Mahalanobis distance is one of the most widely used metrics to find how much a point diverges from a distribution, based on measurements in multiple dimensions. It is widely used in the field of cluster analysis and classification. It was first proposed by Mahalanobis in 1930 in context of his study on racial likeness.

2. Sample survey:

His most important contributions are related to large-scale sample surveys. He introduced the concept of pilot surveys and advocated the usefulness of sampling methods.

Later Life

- In later life, Mahalanobis was a member of the planning commission contributed prominently to newly independent India's five-year plans starting from the second. In the second five-year plan he emphasized industrialization on the basis of a two-sector model.

- In the 1950s, Mahalanobis played a critical role in the campaign to bring India its first digital computers.

- Mahalanobis also had an abiding interest in cultural pursuits and served as secretary to Rabindranath Tagore (about whom he would write in the *Journal of the Oriental Society of Australia*), particularly during the latter's foreign travels, and also worked at his Visva-Bharati University, for some time.

- He received India's second highest civilian award, the Padma Vibhushan from the Government of India for his contribution to science and services to the country.

- Mahalanobis died on 28 June 1972, a day before his seventy-ninth birthday.

- ❖ Even at this age, he was still active doing research work and discharging his duties as the secretary and director of the Indian Statistical Institute and as the honorary statistical advisor to the Cabinet of the Government of India

Honours

- Fellow of the Indian Academy of Sciences (FASc, 1935)
- Fellow of the Indian National Science Academy (FNA, 1935)
- Officer of the Order of the British Empire (Civil Division), 1942 New Year Honours list.
- Weldon Memorial Prize from the University of Oxford (1944)
- Fellow of the Royal Society, London (1945)
- President of Indian Science Congress (1950)
- Fellow of the Econometric Society, US (1951)
- Fellow of the Pakistan Statistical Association (1952)
- Honorary Fellow of the Royal Statistical Society, UK (1954)

- Sir Deviprasad Sarvadhikari Gold Medal (1957)
- Foreign member of the Academy of Sciences of the USSR (1958)
- Honorary Fellow of King's College, Cambridge (1959)
- Fellow of the American Statistical Association (1961)
- Durgaprasad Khaitan Gold Medal (1961)
- Desikottam by Visva Bharati University (1961)
- Padma Vibhushan (1968)
- Srinivasa Ramanujan Gold Medal (1968)
- The government of India decided in 2006 to celebrate Prasanta Chandra Mahalanobis's birthday, 29 June, every year as "National Statistics Day" of India.

- On the occasion of his 125th birth anniversary on 29 June 2018, Indian Vice-President M Venkaiah Naidu released a commemorative coin at a programme at ISI, Kolkata.

In Popular Culture

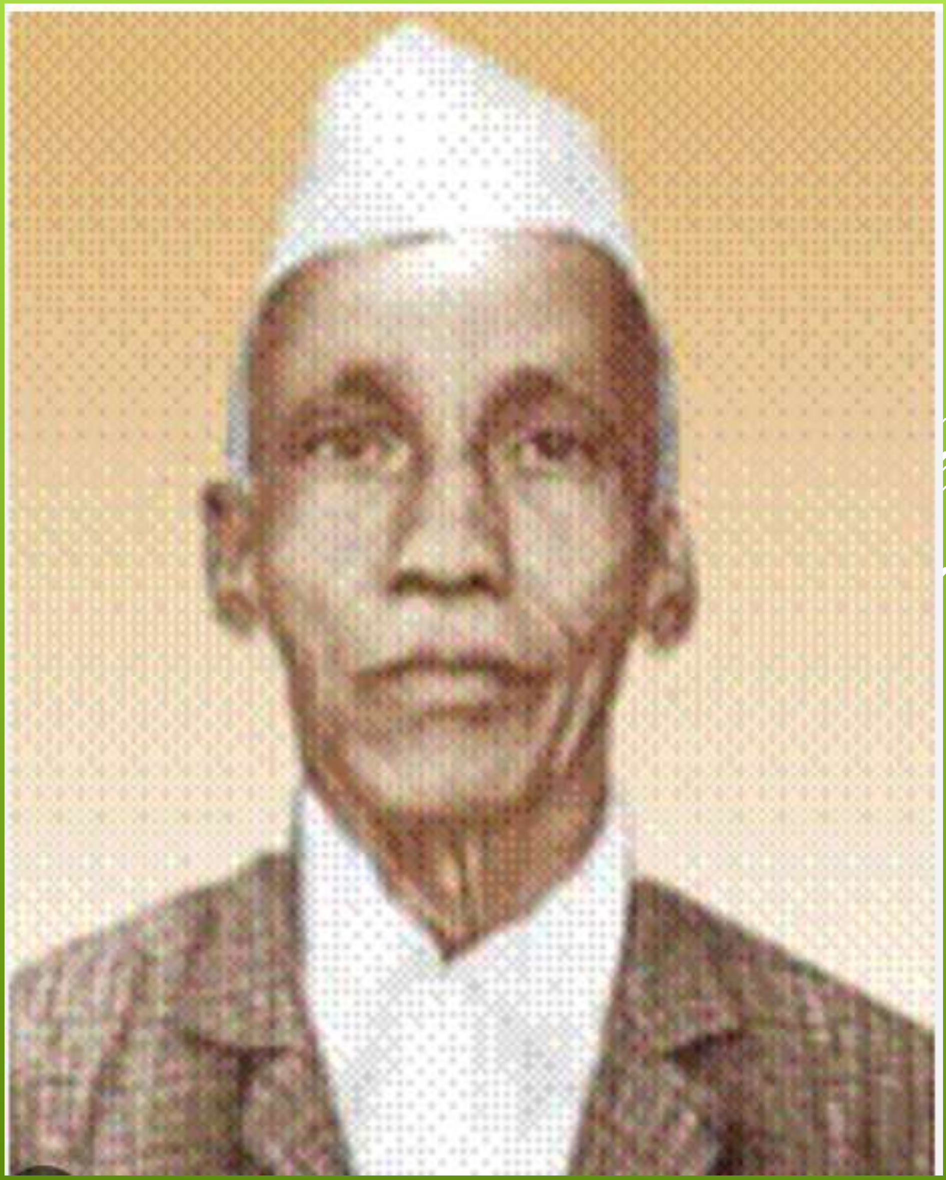
- Mahalanobis appears in the 2015 movie *The Man Who Knew Infinity* about the life of Srinivasa Ramanujan. He is portrayed by Shazad Latif.

- Mahalanobis was honoured with a Google doodle on his 125th birthday on 29 June 2018.

DATTATREYA RAMCHANDRA KAPREKAR

- Dattatreya Ramchandra Kaprekar was an Indian recreational mathematician who described several classes of natural numbers including the Kaprekar, harshad and self numbers and discovered the Kaprekar's constant, named after him.

- Despite having no formal postgraduate training and working as a school teacher, he published extensively and became well known in recreational mathematics circles.



Early Life

- Dattatreya Ramchandra Kaprekar was born on 17th January,1905 at Dahanu, Bombay.
- Ha received his secondary school education in Thane and studied at Cotton College in Guwahati.
- In 1927, he won the Wrangler R. P. Paranjpye Mathematical Prize for an original piece of work in mathematics.
- He obtained his bachelor's degree from University of Mumbai in 1929. He never received any formal postgraduate training.

Career

- For his entire career (1930–1962) he was a schoolteacher at the government junior school in Devlali Maharashtra, India.
- Cycling from place to place he also tutored private students with unconventional methods, cheerfully sitting by a river and "thinking of theorems".
- He published extensively, writing about such topics as recurring decimals, magic squares, and integers with special properties.
- He is also known as "Ganitanand". He died at the age of 81 in 1986.

Discoveries

- Working largely alone, Kaprekar discovered a number of results in number theory and described various properties of numbers.
- In addition he discovered Kaprekar Number, Kaprekar constant, Self number or Devlali number the Harshad numbers and Demlo numbers.
- He also constructed certain types of magic squares related to the Copernicus magic square.

- Initially his ideas were not taken seriously by Indian mathematicians, and his results were published largely in low-level mathematics journals or privately published, but international fame arrived when Martin Gardner wrote about Kaprekar in his March 1975 column of *Mathematical Games* for *Scientific American*.

- Today his name is well-known and many other mathematicians have pursued the study of the properties he discovered.

Kaprekar’s Constant

- The number 6174 is known as Kaprekar's constant after the Indian mathematician D. R. Kaprekar.
- This number is renowned for the following rule: Take any four-digit number, using at least two different digits (leading zeros are allowed).
- Arrange the digits in descending and then in ascending order to get two four-digit numbers, adding leading zeros if necessary.
- Subtract the smaller number from the bigger number.
- Go back to step 2 and repeat.

- The above process, known as Kaprekar's routine, will always reach its fixed point, 6174, in at most 7 iterations.
- Once 6174 is reached, the process will continue yielding 7641 – 1467 = 6174.
- For example choose 6789

9876 – 6789 = 3087

8730 – 0378 = 8352

8532 – 2358 = 6174

7641 – 1467 = 6174

Kaprekar Number

- A Kaprekar number is a positive integer with the property that if it is squared, then its representation can be partitioned into two positive integer parts whose sum is equal to the original number.

- Kaprekar numbers are 45, 703, 2728, 9, 99, 999,….

- For example: $45^2 = 2045$ and 20 + 25 = 45.

  $9^2 = 81$ and 8 + 1 = 9.

  $99^2 = 9801$ and 88 + 01 = 99.

Devlali of Self Number

- A self number or Devlali number is an integer that cannot be written as the sum of any other integer n and the individual digits of n.
- For example: 21 is not devlali number because, 15+1+5=21 while 20 is devlali number.

SATYENDRA NATH BOSE

- **Satyendra Nath Bose** was an Indian mathematician and physicist specializing in theoretical physics.
- He is best known for his work on quantum mechanics in the early 1920s, in developing the foundation for Bose–Einstein statistics and the theory of the Bose–Einstein condensate.
- He was a Fellow of the Royal Society.
- He was awarded India's second highest civilian award, the Padma Vibhushan, in 1954 by the Government of India.
- He had a wide range of interests in varied fields, including physics, mathematics, chemistry, biology, mineralogy, philosophy, arts, literature, and music.
- He served on many research and development committees in India after independence.

Early Life

- Satyendra Nath Bose was born on 1st January,1894 at Calcutta, Bengal.

- He was eldest and only male among seven children.

- His schooling began at the age of five, near his home.

- He passed his entrance examination to the Hindu School in 1909 and stood fifth in the order of merit.

- He then joined the intermediate science course at the Presidency College, Calcutta and studied with renowned Scientist Jagadish Chandra Bose, Sarada Prasanna Das, and Prafulla Chandra Ray.

- Bose received a Bachelor of Science in mixed mathematics from Presidency College and Master o Science in the same subject in 1915 Calcatta University.

- He received such high score on the exams for each degrees that only was he in first standing but, for the later, he even created a new record in the annals of the University of Calcutta, which is yet to be surpassed.

- Between his two degrees, Bose married Ushabati Ghosh at the age of 20. They had nine children, two of whom died in early childhood.

- After completing his MSc, Bose joined the Science College, Calcutta University as a research scholar in 1916 and started his studies in the theory of relativity.

- As a polyglot, Bose was well versed in several languages such as Bengali, English, French, German and Sanskrit as well as the poetry of Lord Tennyson, Rabindranath Tagore and Kalidasa.

- He could play the *esraj*, an Indian instrument similar to a violin. He was actively involved in running night schools that came to be known as the Working Men's Institute.

Research Career

- From 1916 to 1921, he was a lecturer in the physics department of the Rajabazar Science College under University of Calcutta. Along with Saha, Bose prepared the first book in English based on German and French translations of original papers on Einstein's special and general relativity in 1919.
- In 1921, Bose joined as Reader in the Department of Physics of the recently founded University of Dhaka (in present-day Bangladesh).
- Bose set up whole new departments, including laboratories, to teach advanced courses for MSc and BSc honours and taught thermodynamics as well as James Clerk Maxwell's theory of electromagnetism.
- Bose, along Meghnad Saha, presented several papers in theoretical physics and pure mathematics from 1918 onwards.

- In 1924, while working as a reader(Professor without a chair) at the Physics Department of the University of Dhaka, Bose wrote a paper deriving Planck's quantum radiation law without any reference to classical physics.

- This paper was seminal in creating the important field of quantum statistics. Though not accepted at once for publication, he sent the article directly to Albert Einstein in Germany.

- Einstein, recognizing the importance of the paper, translated it into German himself and submitted it on Bose's behalf to the *Zeitschrift für Physik*. As a result of this recognition, Bose was able to work for two years in European X-ray and crystallography laboratories, during which he worked with Louis de Broglie, Marie Curie, and Einstein.

- After his stay in Europe, Bose returned to the University of Dhaka in 1926. Although he did not doctorate, Einstein had recommended he be made a Professor, and so Bose was made head of the Physics department. But upon his return.

- He published another physics paper in 1937, and in the early 1950s work on unified field theories.

- After 25 years in Dhaka, Bose moved back to Calcutta in 1945 and continued to research and teach there until his death in 1974.

- Apart from physics, he did research in biotechnology and literature (Bengali and English). He made studies in chemistry, geology, zoology, anthropology, engineering and other sciences.

- Being Bengali, he devoted significant time to promoting Bengali as a teaching language, translating scientific papers into it, and promoting the development of the region.

Bose- Einstein Statistics

- While presenting a lecture at the University of Dhaka on the theory of radiation and the ultraviolet catastrophe, Bose intended to show his students that the contemporary theory was inadequate, because it predicted results not in accordance with experimental results.

- In the process of describing this discrepancy, Bose for the first time took the position that the Maxwell–Boltzmann distribution would not be true for microscopic particles, where fluctuations due to Heisenberg's uncertainty principle will be significant.

- Thus he stressed the probability of finding particles in the phase space, each state having volume h^3, and discarding the distinct position and momentum of the particles.

- Bose adapted this lecture into a short article called "Planck's Law and the Hypothesis of Light Quanta" and sent it to Albert Einstein.

- Bose's interpretation is now called Bose–Einstein statistics. This result derived by Bose laid the foundation of quantum statistics.

Honours

- In 1937, Rabindranath Tagore dedicated his only book on science, *Visva–Parichay*, to Satyendra Nath Bose.
- Bose was honoured with the title Padma Vibhushan by the Indian Government in 1954.
- In 1959, he was appointed as the National Professor, the highest honour in the country for a scholar, a position he held for 15 years.
- In 1986, the S.N. Bose National Centre for Basic Sciences was established by an act of Parliament, Government of India, in Salt Lake, Calcutta.
- Bose became an adviser to the then newly formed Council of Scientific and Industrial Research.

- He was the president of the Indian Physical Society and the National Institute of Science.
- He was elected general president of the Indian Science Congress.
- He was the vice president and then the president of Indian Statistical Institute.
- In 1958, he became a Fellow of the Royal Society.
- He was nominated as member of Rajya Sabha.
- Partha Ghose has stated that

  Bose's work stood at the transition between the 'old quantum theory' of Planck, Bohr and Einstein and the new quantum mechanics of Schrödinger, Heisenberg, Born, Dirac and others.

Nobel Price Nomination

- Bose was nominated by K. Banerjee (1956), D.S. Kothari (1959), S.N. Bagchi (1962), and A.K. Dutta (1962) for the Nobel Prize in Physics, for his contribution to Bose–Einstein statistics and the unified field theory.

Legacy

- Bosons, a class of elementary subatomic particles in particle physics were named by Dirac after Satyendra Nath Bose to commemorate his contributions to science.

- Although seven Nobel Prizes were awarded for research related to S N Bose's concepts of the boson, Bose–Einstein statistics and Bose–Einstein condensate, Bose himself was not awarded a Nobel Prize.

- When Bose himself was once asked that question, he replied, "I have got all the recognition I deserve" - probably because in the realms of science to which he belonged, what is important is not a Nobel, but whether one's name will live on in scientific discussions in the decades to come.

- One of the main academic buildings of University of Rajshahi, the No 1 science building has been named after him.

- On 4 June 2022, Google honored Bose by featuring him on a Google Doodle marking the 98th anniversary of Bose sending his quantum formulations to the German scientist Albert Einstein who recognized it as a significant discovery in quantum mechanics.

SHAKUNTALA DEVI

- **Shakuntala Devi** was an Indian mental calculator and writer, popularly known as the "Human Computer".

- Her talent earned her a place in the 1982 edition of *The Guinness Book of World Records*.

- Devi was a precocious child and she demonstrated her arithmetic abilities at the University of Mysore without any formal education.

Early Life

- Shakuntala Devi was born on 4 November 1929 at Bangalore, Karnataka to a Kannada Brahmin family. Her father, C V Sundararajan Rao, worked as a trapeze artist, lion tamer, tightrope walker and magician in a circus

- Her Father discovered his daughter's ability to memorise numbers while teaching her a card trick when she was about three years old.

- Her father left the circus and took her on road shows that displayed her ability at calculation. She did this without any formal education. At the age of six she demonstrated her arithmetic abilities at the University of Mysore.

- In 1944, Devi moved to London, United Kingdom with her father.

Mental Calculator

- Devi travelled to several countries around the world demonstrating her arithmetic talents. She was on a tour of Europe throughout 1950 and was in New York City in 1976.

- In 1980, at the Imperial College London she correctly multiplied two 13-digit numbers in 28 seconds. The feat was even more remarkable because it included the time to recite the 26 digit solution.

- It earned her a place in the 1982 edition of the *Guinness Book of World Records*. additionally, she also came to be known as “the human computer”.

- Shakuntala Devi explained many of the methods she used to do mental calculations in her 1977 book *Figuring: The Joy of Numbers*.

Personal Life

- In the mid 1960s, when Shakuntala returned home after displaying her mathematical skill to professors and researchers in London, she married Paritosh Bannerji, an IAS offices from Kolkata.

- They divorced in 1979 due to personal problems.

- In 1980, she contested the Lok Sabha elections as an independent candidate for Mumbai South and for Medak in Andhra Pradesh (now in Telangana).

- In Medak she stood against the former Prime Minister Indira Gandhi, saying she wanted to "defend the people of Medak from being fooled by Mrs. Gandhi“. she came ninth, with 6,514 votes. She returned Bangalore in the early 1980s.

- In addition to her work as a mental calculator, Devi was a notable astrologer and an author of several books, including cookbooks and novels.
- She started with writing short stories and murder mysteries, and had a keen interest in music.

Book on Homosexuality

- In 1977, she wrote *The World of Homosexuals*, the first published academic study of homosexuality in India, for which she was criticised.

- In the documentary *For Straights Only*, she said that her interest in the topic was because of her marriage to a homosexual man and her desire to look at homosexuality more closely to understand it.

Death & Legacy

- In April 2013, Shakuntala Devi was admitted to a hospital in Bangalore with severe respiratory problems. Over the following two weeks she had heart and kidney complications.
- She died in the hospital on 21 April 2013. She was 83 years old. She is survived by her daughter Anupama Banerjee.
- On 4 November 2013, Devi was honoured with a Google Doodle on what would have been her 84th birthday.
- film on her life titled *Shakuntala Devi* was announced in May 2019. The film stars Vidya Balan in the lead title role and features Sanya Malhotra, Amit Sadh, and Jisshu Sengupta in the supporting roles.

Thank You

Shree H. N. Shukla Group of Colleges, Rajkot

BCA Sem-01 (CS-07: Mathematics in Ancient India: Exploring the Rich Heritage of Vedic Mathematics)

Vedic Mathematics

Vedic Mathematics– Overview Contents

Vedic Mathematics– Overview Contents

Vedic Mathematics– Overview Contents

- 5. Squaring

    5.1 Two Digit Number

    5.2 Number Ends With 5

History
Why a talk about Vedic Mathematics?

- Pragmatism – how to make my sabbatical project “A search for appropriate functions for approximate linear programming solutions to queuing network problems” interesting to a general audience?
- Curiosity – I'd received email solicitations for books and curriculum.
- Religion – “Vedic” means from the Vedas

Disclaimers and an Invitation

- I've only just begun to study this subject.
- I have not formally studied the history of mathematics or non-European mathematics.
- Other than some Asian history classes while in college, I've not studied Indian history nor the Hindu religion
- Please speak up if you have anything to add about any of these topics!

What is Vedic Mathematics?

- Vedic period begin around 1500 BC and ended after 500 BC
- Vedas (Books of Knowledge) are the most sacred Hindu Scriptures
- Atharvaveda – supposedly contains a set of sixteen sutras that describe all of mathematics

What is Vedic Mathematics?

- Sutra is often translated word formula and is short and easily memorized and recited
- Vedic Mathematics is a system of mathematics based on these sixteen sutras

A Little History

- Several important mathematical concepts came out of the subcontinent
  - Decimal place value system
  - Arabic numerals based on symbols used here
  - Zero (also discovered independently elsewhere)
- Mathematical astronomy in use by third millennium B.C.

A Little History

- Mathematics was used during Vedic period for the construction of alters
- Jainism followed the Vedic period and found mathematicians working with
  - cubic and quartic equations,
  - permutations and combinations,
  - a rather developed notion of infinity, including multiple “levels” or “sizes” of infinity

Jagadguru Swami
Shri Bharati Krishna Tirthaji Maharaja

- Born in 1884 to an educated and pious family
- Received top marks in school
- Sat for the M.A. exam of the American College of Sciences (Rochester NY) in Sanskrit, Philosophy, English, Mathematics, History and Science.

Jagadguru Swami Shri Bharati Krishna Tirthaji Maharaja

- Became Shankaracharya (major religious leader) of Govardhana Matha (akin to a monastery) in Puri, a city in the east Indian state of Orissa
- Wrote sixteen volumes based on sixteen Sutras written 1911-1918
- Volumes were unaccountably lost without a trace
- Rewrote manuscript from memory in 1956-7 before touring the USA; published posthumously in 1965 as "Vedic Mathematics"

The Sixteen Sutras

Sutra	Translation
1) एकाधिकेन पूर्वेण Ekādhikena Pūrveṇa	By one more than the one before
2) निखिलं नवतश्चरमं दशतः Nikhilaṃ Navataścaramaṃ Daśataḥ	All from 9 and the last from 10
3) ऊर्ध्वतिर्यग्भ्यामं Ūrdhva Tiryagbhyāṃ	Vertically and Crosswise
4) परावर्त्य योजयेत् Parāvartya Yojayet	Transpose and Apply

The Sixteen Sutras

	Sutra	Translation
5)	शून्यं साम्यसमुच्चये Śūnyaṃ Sāmyasamuccaye	If the Samuccaya is the Same it is Zero
6)	आनुरूप्ये शून्यं अन्यत् Ānurūpye Śūnayamanyat	If One is in Ratio the Other is Zero
7)	संकलन व्यवकलनाभ्यां Saṅkalana Vyavakalanābhyāṃ	By Addition and by Subtraction
8)	पूरणापूरणाभ्यां Pūraṇāpūraṇābhyāṃ	By the Completion or Non-Completion

The Sixteen Sutras

Sutra	Translation
09) चलनकलनाभ्याम् Calana Kalanābhyāṃ	Differential Calculus
10) यावदूनं Yāvadūnaṃ	By the Deficiency
11) व्यष्टिसमष्टिः Vyaṣṭisamaṣṭiḥ	Specific and General
12) शेषाण्यङ्केन चरमेण Śeṣāṇyaṅkena Carameṇa	The Remainders by the Last Digit

The Sixteen Sutras

	Sutra	Translation
13)	सोपान्त्यद्वयमन्त्यं Sopāntyadvayamantyaṃ	The Ultimate and Twice the Penultimate
14)	एकन्यूनेन पूर्वेण Ekanyūnena Pūrvena	By One Less than the One Before
15)	गुणितसमुच्चयः Guṇitasamuccayaḥ	The Product of the Sum
16)	गुणकसमुच्चयः Guṇakasamuccayaḥ	All the Multipliers

1. Breaking the number:
1.1 Addition in mind

- Place Value

$33=3\times10+3\times1$

$562=5\times100+6\times10+2\times1$

$5149=5\times1000+1\times100+4\times10+9\times1$

$10120=1\times10000+0\times1000+1\times100+2\times10+0\times1$

1.2 Double/Triple in Mind

- Example of double/Triple in Mind

$$\begin{array}{r} 66 \\ +\ \underline{55} \\ 121 \end{array} \qquad \begin{array}{r} 29 \\ +\ \underline{74} \\ 103 \end{array} \qquad \begin{array}{r} 88 \\ +\ \underline{44} \\ 132 \end{array}$$

1.2 Double/Triple in Mind

- Example of double/Triple in Mind

$$\begin{array}{rcl} 66 & = & 6 \times 10 + 6 \times 1 \\ +\underline{55} & = & \underline{5 \times 10} + \underline{5 \times 1} \\ 121 & & 110 \quad + \quad 11 \end{array}$$

$$1+1$$

$$1 \qquad 2 \qquad 1$$

1.2 Double/Triple in Mind

- Example of double/Triple in Mind

$$
\begin{array}{rcl}
366 & = & 3 \times 100 + 6 \times 10 + 6 \times 1 \\
+\ 4\underline{55} & = & \underline{4 \times 100} + \underline{5 \times 10} + \underline{5 \times 1} \\
821 & & 700 \quad + \quad 110 \quad + \quad 11
\end{array}
$$

$$7+1 \qquad 1+1$$

$$8 \qquad 2 \quad 1$$

3. Multiplication with a Series of 1's:
3.1 Multiplication by 11

- Example: 23×11

- Step 1 : Write the digit on L.H.S. of the number first. Here the number is 23 so, 2 is written first.

- Step 2 : Add the two digits of the given number and write it in between. Here $2 + 3 = 5$

- Step 3 : Now write the second digit on extreme right. Here the digit is 3. So, $23 \times 11 = 253$

3.1 Multiplication by 11

OR

$$23 \times 11 = 2 \;/\; 2+3 \;/\; 3 = 253$$

(Here base is 10 so only 2 digits can be added at a time)

3.1 Multiplication by 11

- Example 2: 243×11

- Step 1: Mark the first, second and last digit of given number

  First digit = 2, second digit = 4, last digit = 3

  Now first and last digits of the number 243 form the first and last digits of the answer.

- Step 2: For second digit (from left) add first two digits of the number i.e. $2 + 4 = 6$

3.1 Multiplication by 11

- Step 3: For third digit add second and last digits of the number i.e. $3 + 4 = 7$

$$\text{So, } 243 \times 11 = 2673$$

OR

$$243 \times 11 = 2 \,/\, 2 + 4 \,/\, 4 + 3 \,/\, 3 = 2673$$

Similarly we can multiply any bigger number by 11 easily.

3.1 Multiplication by 11

- Example 3: 42431×11

$$42431 \times 11 = 4 \;/\; 4+2 \;/\; 2+4 \;/\; 4+3 \;/\; 3+1 \;/\; 1$$

$$= 466741$$

3.2 Multiplication by 111

- Example: 189×111

- Step 1: Mark the first, second and last digit of given number

  First digit = 1, second digit = 8, last digit = 9

  Now first and last digits of the number 189 may form the first and last digits of the answer

- Step 2: For second digit (from left) add first two digits of the number i.e. $1 + 8 = 9$

3.2 Multiplication by 111

- Step 3: For third digit add first, second and last digits of the number to get $1 + 8 + 9 = 18$

(multiplying by 111, so three digits are added at a time)

- Step 4: For fourth digit from left add second and last digit to get, $8 + 9 = 1$

As we cannot have two digits at one place so 1 is shifted and added to the next digit so as to get

$$189 \times 111 = 20979$$

3.2 Multiplication by 111

OR

- Example: 189×111

1	$1 + 8 = 9$	$1 + 8 + 9$	$8 + 9$	9
	$9 + 1 =$	$= 18$	$= 1\ 7$	
$1 + 1 = 2$	$= 1\ 0$	$= 18 + 1$		
		$= 1\ 9$		

$\therefore 189 \times 111 = 20979$

3.2 Multiplication by 111

- Example : 2891×111

2	$2 + 8$	$2 + 8 + 9$	$8 + 9 + 1$	$9 + 1$
$10 +$	$2 \quad = 1$	$9 + 1 \quad 18$	$+ 1 \quad = 1$	$0 \; 1$
	$= 1 \; 2$	$= 2 \; 0$	$= 1 \; 9$	

$2891 \times 111 = 320901$

4. Multiplication by Criss-Cross Method

- **Sutra: Vertically and cross-wise.**

Till now we have learned various methods of multiplication but these are all special cases, where numbers should satisfy certain conditions like near base, or sub base, complimentary to each other etc. Now we are going to learn about a general method of multiplication, by which we can multiply any two numbers in a line. Vertically and cross-wise sutra can be used for multiplying any number.

4.1 Two digit – multiplication

- Example: Multiply 21 and 23

- Step1: Vertical (one at a time)

$$\begin{array}{r} 2\ [1] \\ 2\ [3] \\ \hline \end{array} \quad \downarrow \ 1 \times 3 = 3 \qquad \begin{array}{c|c} & 3 \\ \hline & \end{array}$$

- Step2: Cross –wise (two at a time)

$$\begin{array}{cc} 2 & 1 \\ 2 & 3 \end{array} \qquad (2 \times 3 + 2 \times 1) = 8$$

$$/8\ /\ 3$$

4.1 Two digit – multiplication

- Step3: Vertical (one at a time)

$$\begin{array}{cc} [2] & 1 \\ [2] & 3 \\ \hline \end{array} \qquad 2 \times 2 = 4 \qquad \begin{array}{c|c|c} 4 & 8 & 3 \\ \hline \end{array}$$

Therefore, $21 \times 23 = 483$

4.1 Two digit – multiplication

- Example: Multiply 42 and 26

- Step1: Vertical (one at a time)

$$\begin{array}{r} 42 \\ \underline{26} \end{array} \downarrow \qquad 2 \times 6 = 12 \qquad /\,/12$$

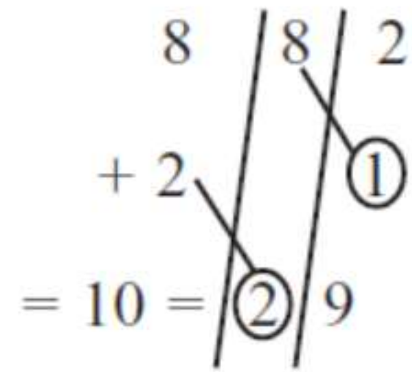
- Step2: Cross –wise (two at a time)

$$\begin{array}{ccc} 4 & & 2 \\ & \times & \\ 2 & & 6 \end{array} \qquad \begin{array}{l} 4 \times 6 + 2 \times 2 \\ 24 + 4 = 28 \end{array} \qquad /\,2_8/\,_12$$

4.1 Two digit – multiplication

- Step3: Vertical (one at a time)

$$\begin{array}{r} 42 \\ \underline{26} \end{array} \qquad 4 \times 2 = 8$$

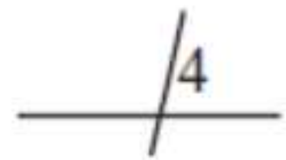


Therefore, $42 \times 26 = 1092$

4.2 Three digit – multiplication

- Example: Multiply 212 and 112
- Step1: Vertical (one at a time)

$$\begin{array}{r} 212 \\ \underline{112} \end{array} \downarrow \qquad 2 \times 2 = 4$$



- Step2: Cross-wise (two at a time)

$$\begin{array}{ccc} 2 & 1 & 2 \\ \hline 1 & 1 & 2 \end{array} \qquad 2 \times 1 + 2 \times 1 = 2 + 2 = 4$$

4.2 Three digit – multiplication

- Step3: Vertical and cross-wise (three at a time)

$$\begin{array}{ccc} 2 & 1 & 2 \\ 1 & 1 & 2 \end{array}$$

$2 \times 2 + 2 \times 1 + 1 \times 1 = 4 + 2 + 1 = 7$

/ 7 / 4 / 4

- Step4: cross wise (Two at a time)

$$\begin{array}{ccc} 2 & 1 & 2 \\ 1 & 1 & 2 \end{array}$$

$2 \times 1 + 1 \times 1$

$= 2 + 1 = 3$

/ 3 / 7 / 4 / 4

4.2 Three digit – multiplication

- Step5: vertical (one at a time)

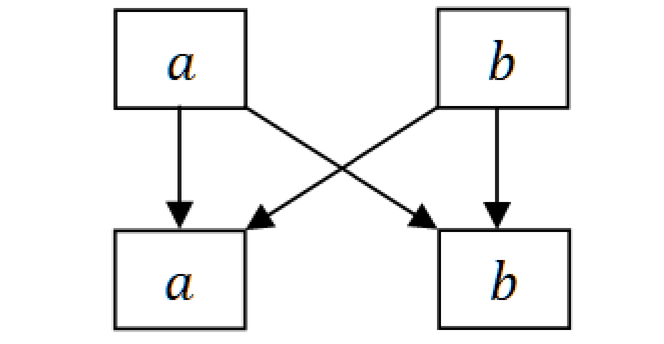
$$\begin{array}{r} 2\,1\,2 \\ \underline{1\,1\,2} \end{array} \qquad 2 \times 1 = 2 \qquad\qquad 2\,/\,3\,/\,7\,/\,4\,/\,4$$

Therefore, $212 \times 112 = 23744$

5. Squaring
5.1 Square of Two Digit Number

- Example: $(ab)^2$

For $(ab)^2$, you write the number below each other:



$$\boxed{a \times a / a \times b + a \times b \ / \ b \times b}$$

VERTICAL / CROSS / VERTICAL

The number are written left to right.

5.1 Square of Two Digit Number

- Multiply:

1) Vertically $\longrightarrow (a \times a)$
2) Crosswise in both directions and add $\longrightarrow (a/a \times b + a \times b)$
3) Vertically $\longrightarrow (b \times b)$

The answer of the form:

$$a \times a/a \times b + a \times b \;/\; b \times b$$

5.1 Square of Two Digit Number

- Example: $(13)^2$

$$
\begin{array}{ccc}
1 & & 3 \\
1 & & 3 \\
\hline
1/ & 3+3 & /\,9 \\
1/ & 6 & /\,9
\end{array}
$$

Therefore, $(13)^2 = 169$

5.1 Square of Two Digit Number

- Example: $(63)^2$

$$\begin{array}{ccc} 6 & & 3 \\ 6 & & 3 \\ \hline 36/ & 6\times 3+6\times 3 & /\,9 \end{array}$$

$$36\,/ \quad 36 \quad /\,9$$

$$36\,/ \quad {}_{3}6 \quad /\,9$$

Carry over the 3:

$$36+3\,/ \quad 6 \quad /\,9$$

$$39\,/ \quad 6 \quad /\,9$$

Therefore $\mathbf{(63)^2 = 3969}$

5.2 Number Ends With 5

- Example: 25^2

  Here the number is 25. We have to find out the square of the number. For the number 25, the last digit is 5 and the 'previous' digit is 2. Hence, 'one more than the previous one', that is, 2+1=3. The sutra, in this context, gives the procedure **'to multiply the previous digit 2 by one more than it self, that is by 3'**. It becomes L.H.S. (left hand side) of the result, that is,

$$2 \times 3 = 6$$

The R.H.S. (right hand side) of the result is 5^2, that is, 25.

5.2 Square of Number Ends With 5

Thus $25^2 = 2 \times 3/25 = 625$

In this way,

$35^2 = 3 \times 4/25 = 1225$

$65^2 = 6 \times 7/25 = 4225$

$105^2 = 10 \times 11/25 = 11025$

$135^2 = 13 \times 14/25 = 18225$

$1225 = 122 \times 123/25 = 1500625$

THANK YOU