SHREE H. N. SHUKLA GROUP OF COLLEGES

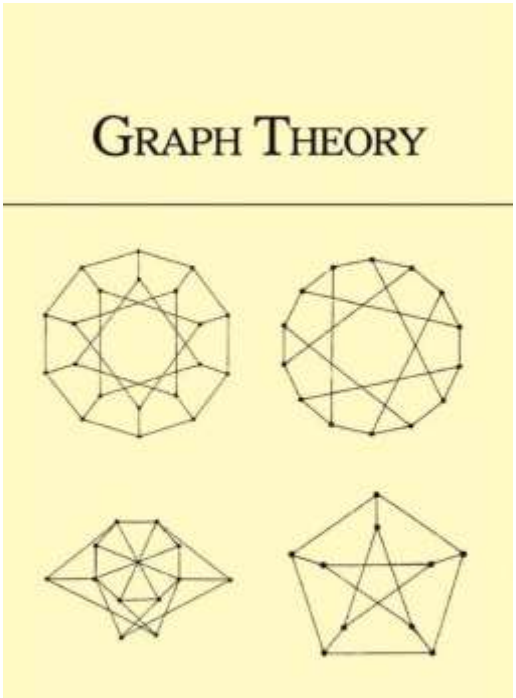
T.Y.B.Sc. Sem-6

Subject: Mathematics

Paper-601

Unit-1

GRAPH THEORY



- Introduction
- Basic Properties
- Types of graphs
- Tree
- Connectivity
- Covering
- Independent set
- Coloring
- Examples

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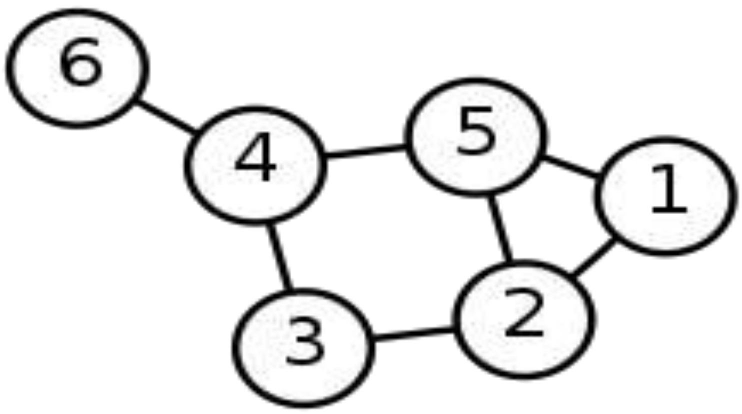
|Maths/Sem-6/P-601/Unit-1|

TITLE EXPLANATION

❖ *GRAPH THEORY*

☑ In mathematics, **graph theory** is the study of graphs, which are mathematical structures, used to model pair wise relations between objects.

☑ A **graph** in this context is made up of vertices (also called nodes or points) which are connected by edges (also called links or lines).



A graph with 6 vertices and 7 edges

LEARNING OUTCOME

☑ Your goal for this class is to develop the mathematical sophistication needed to understand what properties to search for in graphs (simple networks), and prove results about them using the knowledge about graphs' structure.

☑ In doing this, you will

- Use definitions in graph theory to identify and construct examples and to distinguish examples from non-example.
- Apply theories and concepts to test and validate intuition and independent mathematical thinking in problem solving.
- Integrate core theoretical knowledge of graph theory to solve problems.
- Reason from definitions to construct mathematical proofs
- Evaluate and synthesize published research papers.
- Analyze new networks using the main concepts of graph theory.
- Read and write graph theory in a coherent and technically accurate manner.

Table of Contents

1. Graph theory - Introduction

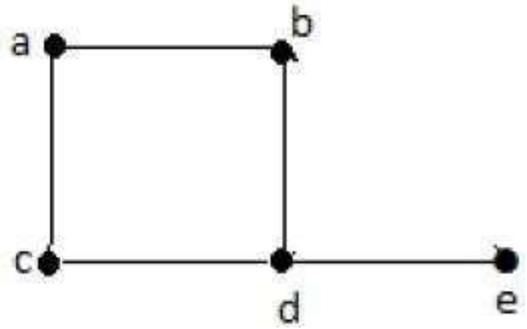
❖ *What is a Graph?*

- A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links.

- The interconnected objects are represented by points termed as **vertices,** and the links that connect the vertices are called **edges**.

- Formally, a graph is a pair of sets **(V, E),** where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices.

- Take a look at the following graph:



In the above graph,
V= {a, b, c, d, e}
E= {ab, ac, bd, cd, de}

Sr. No.	Question	Answer
1	Vertices a and b are _________ relation in above graph.	**Adjacent**
2	Vertices c and d are terminal vertices of edge de in above graph (T/F)?	**False**
3	The interconnected objects are represented by points termed as……	**Vertices**
4	The links that connect the vertices are called…….	**Edges**

2. Graph Theory—Fundamentals

❖Introduction:

- A graph is a diagram of points and lines connected to the points.
- It has at least one line joining a set of two vertices with no vertex connecting itself.
- The concept of graphs in graph theory stands up on some basic terms such as point, line, vertex, edge, degree of vertices, properties of graphs, etc.
- Here, in this chapter, we will cover these fundamentals of graph theory.

❖Point

- **A point** is a particular position in a one-dimensional, two-dimensional, or three-dimensional space.
- For better understanding, a point can be denoted by an alphabet.
- It can be represented with a dot.

Example

Here, the dot is a point named ‘a’.

❖Line

- A **Line** is a connection between two points.
- It can be represented with a solid line.

Example

Here, ‘a’ and ‘b’ are the points. The link between these two points is called a line.

Sr. No.	Question	Answer
1	A particular position in a one-dimensional, two-dimensional, or three-dimensional space is said to be………	**Point**
2	A connection between two points is called………	**Line**

3	A point can be denoted by……	**Alphabet**
4	A point can be represented with……	**Dot**

❖Vertex

- A vertex is a point where multiple lines meet.
- It is also called a **node**.
- Similar to points, a vertex is also denoted by an alphabet.

Example

Here, the vertex is named with an alphabet 'a'.

❖Edge

- An edge is the mathematical term for a line that connects two vertices.
- Many edges can be formed from a single vertex.
- Without a vertex, an edge cannot be formed.
- There must be a starting vertex and an ending vertex for an edge.

Example

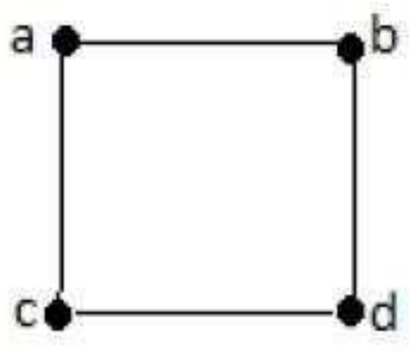
Here, 'a' and 'b' are the two vertices and the link between them is called an edge.

Sr. No.	Question	Answer
1	A line connects two vertices is said to be……	**Edge**
2	Many edges can be formed from a _______ vertex.	**single**
3	Vertex is also known as………	**Node**
4	Vertex is denoted by……	**Alphabet**
5	Without a _________, an edge cannot be formed.	**Vertex**

❖Graph

A graph 'G' is defined as G = (V, E) Where V is a set of all vertices and E is a set of all edges in the graph.

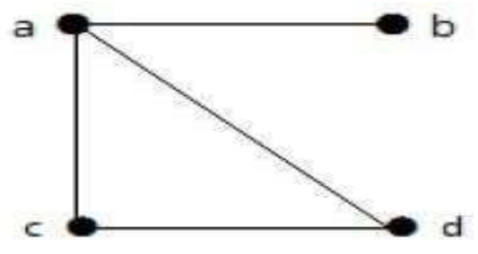
Example-1



In the above example,

- ab, ac, cd, and bd are the edges of the graph.
- Similarly, a, b, c, and d are the vertices of the graph.

Example-2



In this graph, there are four vertices a, b, c, and d, and four edges ab, ac, ad, and cd.

❖Loop

In a graph, if an edge is drawn from vertex to itself, it is called a loop.

Example-1

In the above graph, V is a vertex for which it has an edge (V, V) forming a loop.

Example-2

In this graph, there are two loops which are formed at vertex a, and vertex b.

Sr. No.	Question	Answer
1	In a graph, if an edge is drawn from vertex to itself, it is called……	**Loop**
2	A _______ is defined as G = (V, E) Where V is a set of all vertices and E is a set of all edges in the graph.	**Graph**
3	In a graph, set of edges is denoted as……	**E**
4	In a graph, set of vertices is denoted as……	**V**

❖ Degree of Vertex

- It is the number of vertices adjacent to a vertex V.
- **Notation** –deg (V).
- In a simple graph with n number of vertices, the degree of any vertices is: $deg(v) \le n - 1 \forall v \in G$
- A vertex can form an edge with all other vertices except by itself.
- So the degree of a vertex will be up to the **number of vertices in the graph minus 1**.
- This 1 is for the self-vertex as it cannot form a loop by itself.

- If there is a loop at any of the vertices, then it is not a Simple Graph.

- Degree of vertex can be considered under two cases of Graphs:
  - Undirected Graph
  - Directed Graph

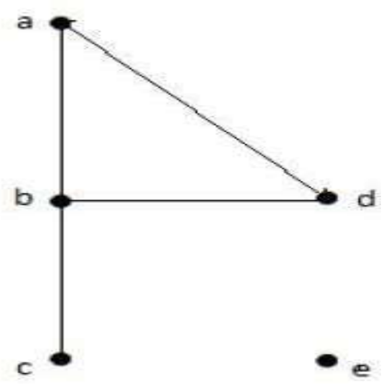
Degree of Vertex in an Undirected Graph

- An undirected graph has no directed edges.

- Consider the following examples.

Example-1

Take a look at the following graph:



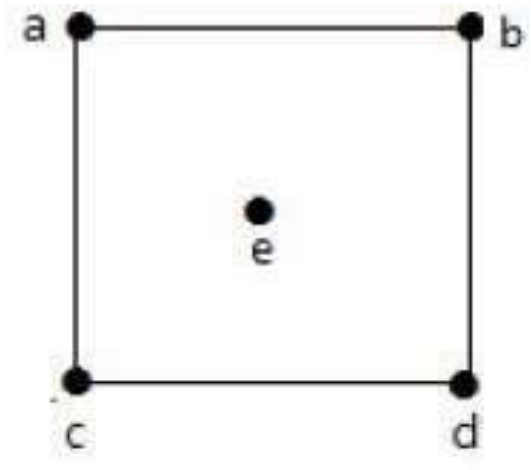
In the above Undirected Graph,

- Deg (a) =2, as there are 2 edges meeting at vertex 'a'.
- Deg (b) =3, as there are 3 edges meeting at vertex 'b'.
- Deg (c) =1,as there is 1 edge formed at vertex 'c'
- So 'c' is a **pendent vertex**.

- Deg (d) =2, as there are 2 edges meeting at vertex'd'.
- Deg (e) =0, as there are 0 edges formed at vertex 'e'.
- So 'e' is an **isolated vertex**.

Example-2

Take a look at the following graph:



In the above graph,

- deg(a)=2, deg(b)= 2,deg(c)=2,deg(d)=2,anddeg(e)=0.
- The vertex 'e' is an isolated vertex.
- The graph does not have any pendent vertex.

❖ Degree of Vertex in a Directed Graph

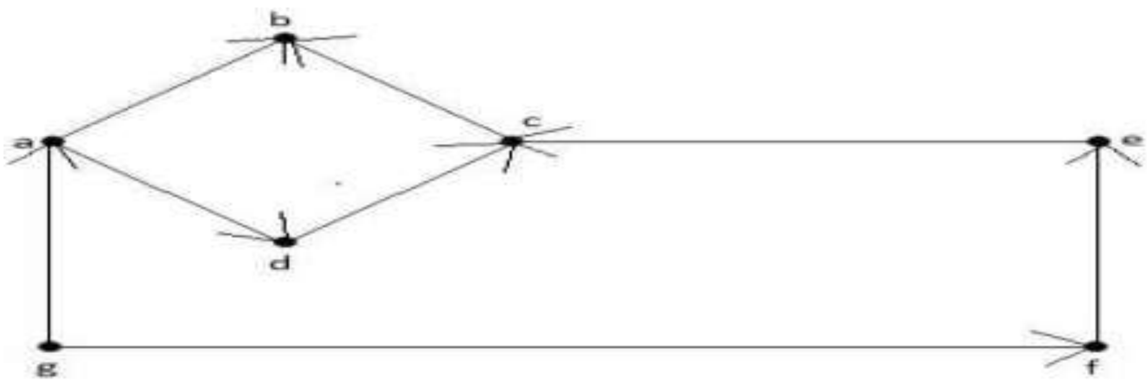
- In a directed graph, each vertex has an in degree and an out degree.

- **In degree of a Graph**
  - In degree of vertex V is the number of edges which are coming into the vertex V.
  - **Notation** $-\deg^{-}(V)$.

- **Out degree of a Graph**
  - OutdegreeofvertexVisthenumberofedgeswhic haregoingoutfromthevertexV.
  - **Notation**$-\deg^{+}(V)$.

Example-1

Take a look at the following directed graph. Vertex 'a' has two edges, 'ad' and 'ab', which are going outwards. Hence its out degree is 2. Similarly, there is an edge 'ga', coming towards vertex 'a'. Hence the in degree of 'a' is 1.

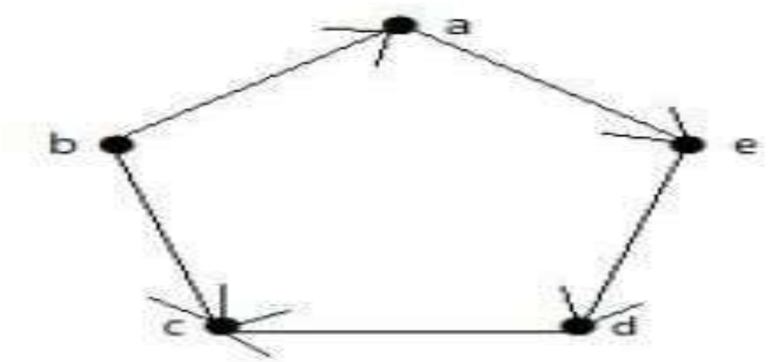


The in degree and out degree of other vertices are shown in the following table:

Vertex	Indegree	Outdegree
a	1	2
b	2	0
c	2	1
d	1	1
e	1	1
f	1	1
g	0	2

Example-2

Take a look at the following directed graph. Vertex 'a' has an edge 'ae' going out wards from vertex 'a'. Hence its outdegree is 1. Similarly, the graph has an edge 'ba' coming towards vertex 'a'. Hence the indegree of'a'is1.



The indegree and outdegree of other vertices are shown in the following table–

Vertex	Indegree	Outdegree
a	1	1
b	0	2
c	2	0
d	1	1
e	1	1

Sr. No.	Question	Answer
1	A ________ can form an edge with all other vertices except by itself.	**Vertex**
2	In a directed graph, each vertex has…….	**an in degree and an out degree**
3	An undirected graph has no _______ edges.	**directed**

❖ Pendent Vertex

By using degree of a vertex, we have two special types of vertices. A vertex with degree one is called a pendent vertex.

Example

- Here, in this example, vertex 'a' and vertex 'b' have a connected edge 'ab'.
- So with respect to the vertex 'a', there is only one edge towards vertex 'b' and similarly with respect to the vertex 'b', there is only one edge towards vertex 'a'.
- Finally, vertex 'a' and vertex 'b' has degree as one which are also called as the pendent vertex.

❖ Isolated Vertex

A vertex with degree zero is called an isolated vertex.

Example

Here, the vertex 'a' and vertex 'b' has no connectivity between each other and also to any other vertices. So the degree of both the vertices 'a' and 'b' are zero. These are also called as isolated vertices.

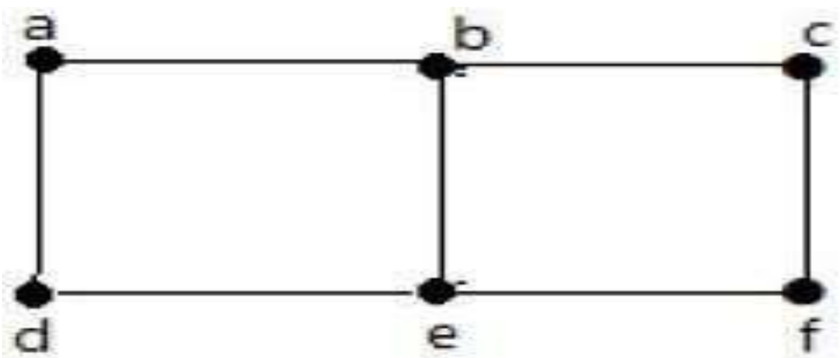
Sr. No.	Question	Answer
1	Degree of vertex is 1, then it is said to be ………	**Pendant vertex**
2	Give the degree of isolated vertex.	**0**
3	No. of edges incidence on vertex is called……	**Degree of vertex**
4	Any edge is loop then the degree of vertex count ________.	**twice**

❖ Adjacency

Here are the norms of adjacency:

- In a graph, two vertices are said to be **adjacent,** if there is an edge between the two vertices. Here, the adjacency of vertices is maintained by the single edge that is connecting those two vertices.
- In a graph, two edges are said to be adjacent, if there is a common vertex between the two edges. Here, the adjacency of edges is maintained by the single vertex that is connecting two edges.

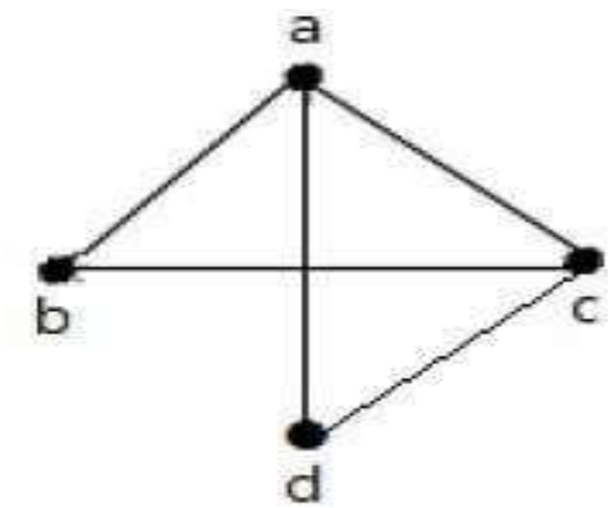
Example-1



In the above graph:

- ‘a’ and ‘b’ are the adjacent vertices, as there is a common edge ‘ab’ between them.
- ‘a’ and‘d’ are the adjacent vertices, as there is a common edge ‘ad’ between them.
- ab’ and ‘be’ are the adjacent edges, as there is a common vertex ‘b’ between them.
- be’ and ‘de’ are the adjacent edges, as there is a common vertex ‘e’ between them.

Example-2

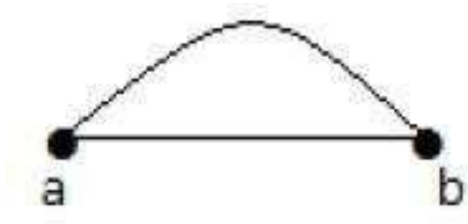


In the above graph:

- a’ and ‘d’ are the adjacent vertices, as there is a common edge ‘ad’ between them.
- ‘c’ and ‘b’ are the adjacent vertices, as there is a common edge ‘cb’ between them.
- ‘ad’ and ‘cd’ are the adjacent edges, as there is a common vertex ‘d’ between them.
- ac’ and ‘cd’ are the adjacent edges, as there is a common vertex ‘c’ between them.

❖ Parallel Edges

In a graph, if a pair of vertices is connected by more than one edge, then those edges are called parallel edges.



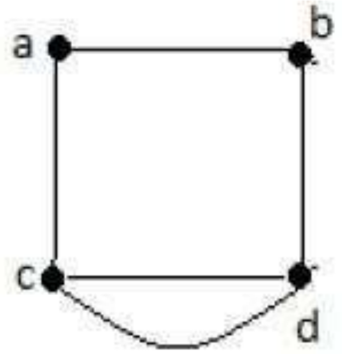
In the above graph, 'a' and 'b' are the two vertices which are connected by two edges 'ab' and 'ab' between them. So it is called as a parallel edge.

Sr. No.	Question	Answer
1	In a graph, if a pair of vertices is connected by more than one edge, then those edges are called………	**Parallel edges**
2	In a graph, two vertices are said to be ________, if there is an edge between the two vertices.	**adjacent**
3	The adjacency of edges is maintained by the single vertex that is connecting _____ edges.	**two**
4	The adjacency of vertices is maintained by the single edge that is connecting _________.	**two vertices**

❖ Multi Graph

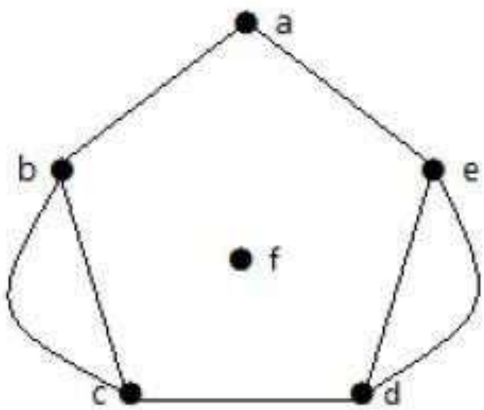
A graph having parallel edges is known as a Multi graph.

Example-1



In the above graph, there are five edges 'ab', 'ac', 'cd', 'cd', and 'bd'. Since 'c' and'd' have two parallel edges between them, it a Multi graph.

Example-2

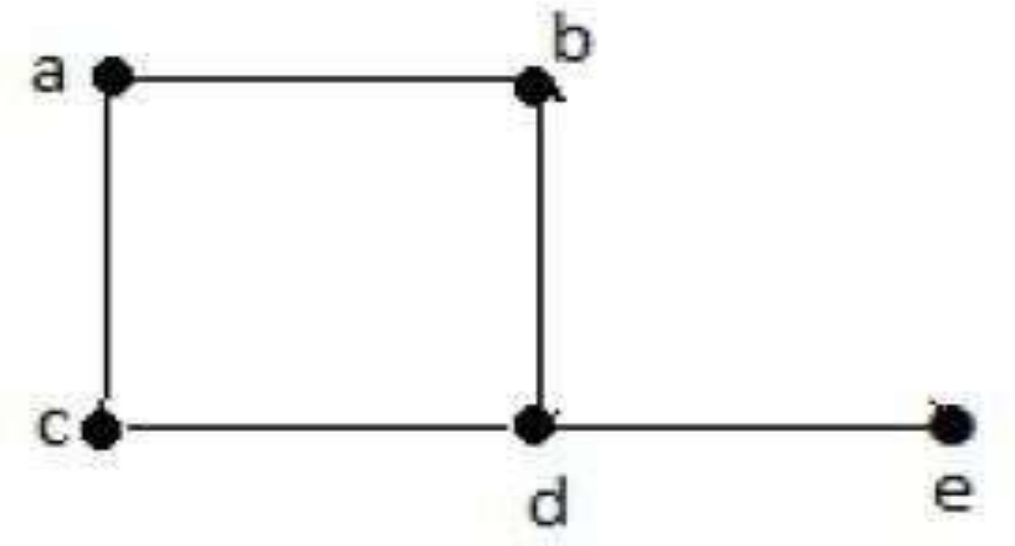


In the above graph, the vertices 'b' and 'c' have two edges. The vertices 'e' and'd' also have two edges between them. Hence it is a Multi graph.

❖ Degree Sequence of a Graph

If the degrees of all vertices in a graph are arranged in descending or ascending order, then the sequence obtained is known as the degree sequence of the graph.

Example-1

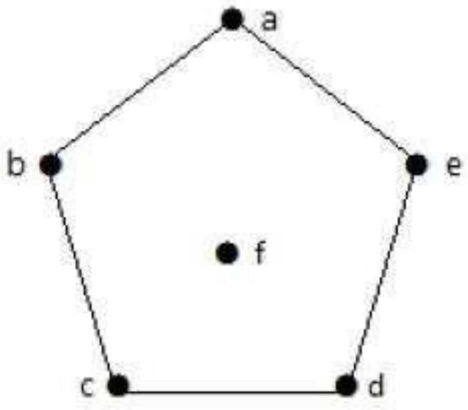


Vertex	A	B	c	d	e
Connecting to	b, c	a, d	a, d	c, b, e	d

Degree	2	2	2	3	1

In the above graph, for the vertices {d, a, b, c, e}, the degree sequence is {3, 2, 2, 2, 1}.

Example-2



Vertex	A	b	c	d	e	F
Connecting to	b, e	a, c	b, d	c, e	a, d	-
Degree	2	2	2	2	2	0

In the above graph, for the vertices {a, b, c, d, e, f}, the degree sequence is {2, 2, 2, 2, 2, 0}.

Sr. No.	Question	Answer
1	A graph having parallel edges is known as……	Multi graph
2	If the degrees of all vertices in a graph are arranged in descending or ascending order, then the sequence obtained is known as……	Degree sequence of a graph

3. Graph Theory – Basic Properties

❖ Introduction

- Graphs come with various properties which are used for characterization of graphs depending on their structures.
- These properties are defined in specific terms pertaining to the domain of graph theory.
- In this chapter, we will discuss a few basic properties that are common in all graphs.

❖ Distance between Two Vertices

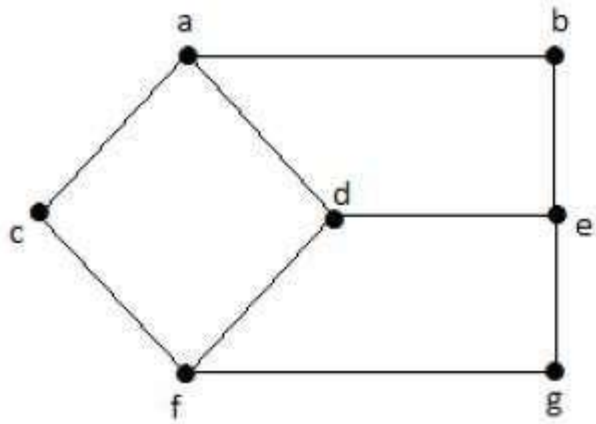
It is number of edges in a shortest path between Vertex U and Vertex V. If there are multiple paths connecting two vertices, the n the shortest path is considered as the distance between the two vertices.

Notation –d (U,V)

There can be any number of paths present from one vertex to other. Among those, you need to choose only the shortest one.

Example

Take a look at the following graph:



Here, the distance from vertex 'd' to vertex 'e' or simply 'de' is 1 as there is one edge between them. There are many paths from vertex'd' to vertex 'e':

- da, ab, be
- df, fg, ge
- de (It is considered for distance between the vertices)
- df ,fc ,ca ,ab ,be
- da, ac, cf, fg, ge

❖ Eccentricity of a Vertex

The maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex.

Notation−e (V)

The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

Example

In the above graph, the eccentricity of 'a' is 3.Thedistancefrom'a'to 'b' is 1 ('ab'),

From 'a' to 'c' is 1('ac'),

From 'a' to'd' is 1 ('ad'),

From 'a' to 'e' is 2 ('ab'- 'be') or ('ad'- 'de'),

From 'a' to 'f' is 2 ('ac'- 'cf') or ('ad'- 'df'),

From 'a' to 'g' is 3 ('ac'- 'cf'- 'fg') or ('ad'- 'df'- 'fg').

So the eccentricity is 3, which is a maximum from vertex 'a' from the distance between 'ag' which is maximum.

In other words, e(b) = 3, e(c) = 3, e(d) = 2, e(e) = 3, e(f) = 3, e(g) = 3

Sr. No.	Question	Answer
1	If there are multiple paths connecting two vertices, the n the shortest path is considered as……	**Distance between the two vertices**
2	The maximum distance between a vertex to all other vertices is considered as……	**Eccentricity of a vertex**

❖ Radius of a Connected Graph

The minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G.

Notation−r(G)

From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.

Example

In the above graph r(G) =2, which is the minimum eccentricity for‘d’.

❖ Diameter of a Graph

The maximum eccentricity from all the vertices is considered as the diameter of the Graph G. The maximum among all the distances between a vertex to all other vertices is considered as the diameter of the Graph G.

Notation−d(G):

From all the eccentricities of the vertices in a graph, the diameter of the connected graph is the maximum of all those eccentricities.

Example

In the above graph, d(G)=3; which is the maximum eccentricity.

Sr. No.	Question	Answer
1	The minimum eccentricity from all the vertices is considered as……	**Radius of a connected graph**
2	The maximum eccentricity from all the vertices is considered as……	**Diameter of a graph**

❖ Central Point

If the eccentricity of a graph is equal to its radius, then it is known as the central point of the graph.

If e(V) = r(V),

Then 'V' is the central point of the Graph 'G'.

Example

In the example graph, 'd' is the central point of the graph.
e(d) = r(d) = 2

❖ Centre

The set of all central points of 'G' is called the centre of the Graph.

Example

In the example graph, {'d'} is the centre of the Graph.

Sr. No.	Question	Answer
1	If the eccentricity of a graph is equal to its radius, then it is known as the ________ of the graph.	**Central point**
2	The set of all central points of 'G' is called……	**Centre of the graph**

4. Graph Theory – Type of graphs

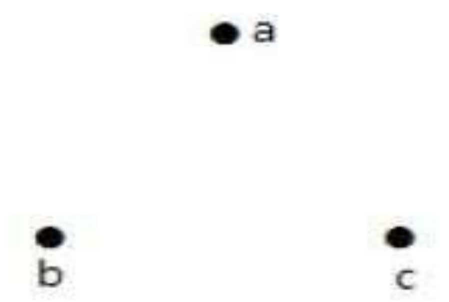
❖ Introduction:

There are various types of graphs depending upon the number of vertices, number of edges, interconnectivity, and their overall structure. We will discuss only a certain few important types of graphs in this chapter.

❖ Null Graph

A **graph having no edges** is called a Null Graph.

Example

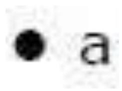


In the above graph, there are three vertices named ‘a’, ‘b’, and ‘c’, but there are no edges among them. Hence it is a Null Graph.

❖ Trivial Graph

A **graph with only one vertex** is called a Trivial Graph.

Example



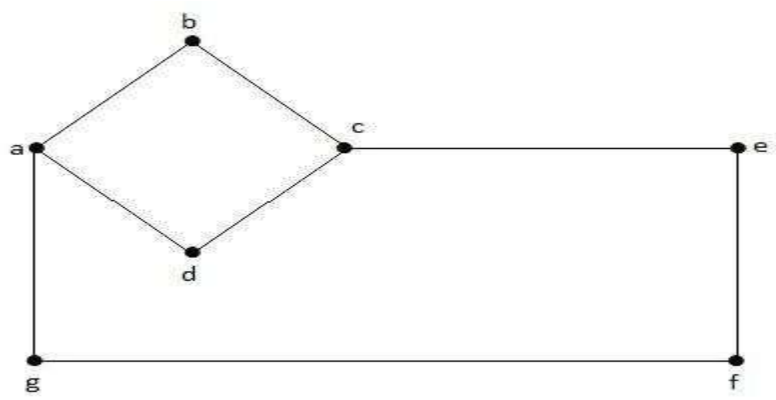
In the above shown graph, there is only one vertex ‘a’ with no other edges. Hence it is a trivial graph.

Sr. No.	Question	Answer
1	A graph having no edges is called….	**Null graph**
2	A Trivial graph has only ________ vertex.	**One**
3	A Null graph having no vertices. (T/F)	**False**

❖ **Non-Directed Graph**

A non-directed graph contains edges but the edges are not directed ones.

Example

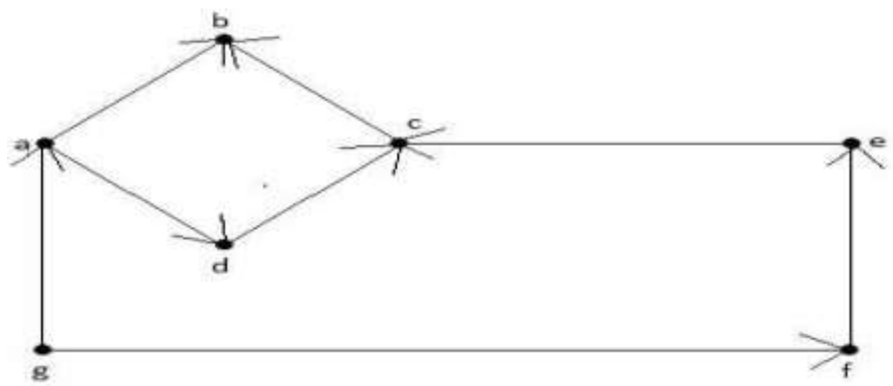


- In this graph, 'a', 'b', 'c', 'd', 'e', 'f', 'g' are the vertices, and 'ab', 'bc', 'cd', 'da', 'ag', 'gf', 'ef' are the edges of the graph. Since it is a non-directed graph, the edges 'ab' and 'ba' are same.
- Similarly other edges also considered in the same way.

❖ Directed Graph

In a directed graph, each edge has a direction.

Example



- Intheabovegraph,wehavesevenvertices'a','b','c','d','e','f',and'g',andeightedges'ab', 'cb', 'dc', 'ad', 'ec', 'fe', 'gf', and 'ga'. As it is a directed graph, each edge bears a narrow mark that shows its direction.
- Note that in a directed graph, 'ab' is different from 'ba'.

Sr. No.	Question	Answer
1	A ________ contains edges but the edges are not directed ones.	**Non-directed graph**
2	In a directed graph, each edge has ________	**direction**

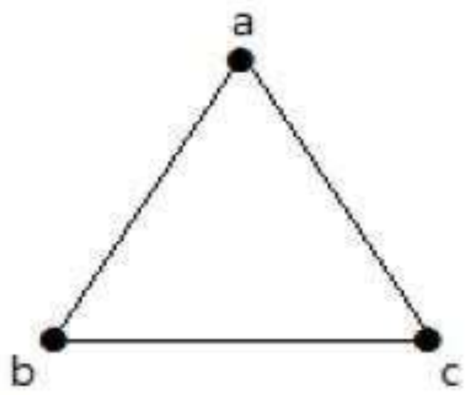
❖ Simple Graph

A graph **with no loops** and **no parallel edges** is called a simple graph.

- The maximum number of edges possible in a single graph with 'n' vertices is ${}^{n}C_2$ where ${}^{n}C_2=n\ (n-1)/2$.
- The number of simple graphs possible with 'n' vertices $=2^{{}^{n}c_2}=2^{n(n-1)/2}$.

Example

In the following graph, there are 3 vertices with 3 edges which is maximum excluding the parallel edges and loops. This can be proved by using the above formulae.



The maximum number of edges with n=3vertices:

$$
\begin{aligned}
{}^{n}C_2 &= n\ (n-1)/2 \\
&= 3(3-1)/2 \\
&= 6/2 \\
&= 3\text{edges}
\end{aligned}
$$

The maximum number of simple graphs with n=3vertices:

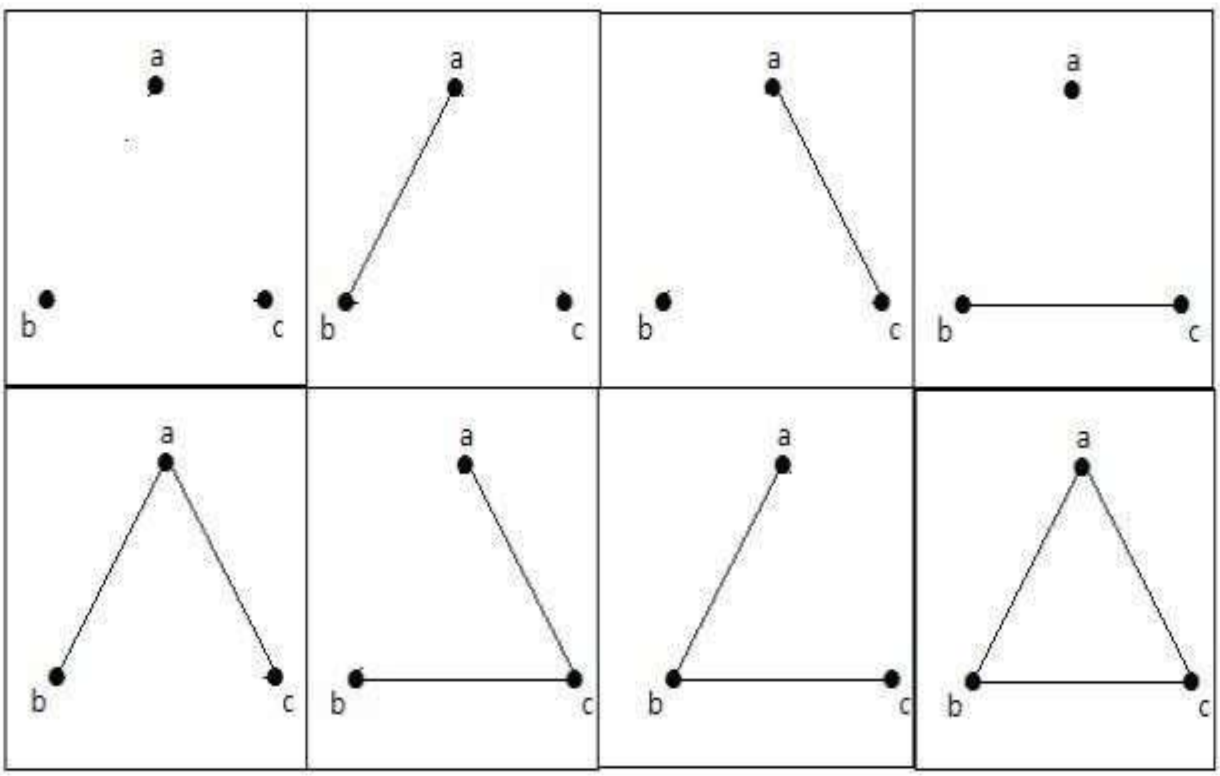
$$2^{nC_2} = 2^{n(n-1)/2}$$

$$= 2^{3(3-1)/2}$$

$$= 2^3$$

$$= 8$$

These 8 graphs are as shown below:



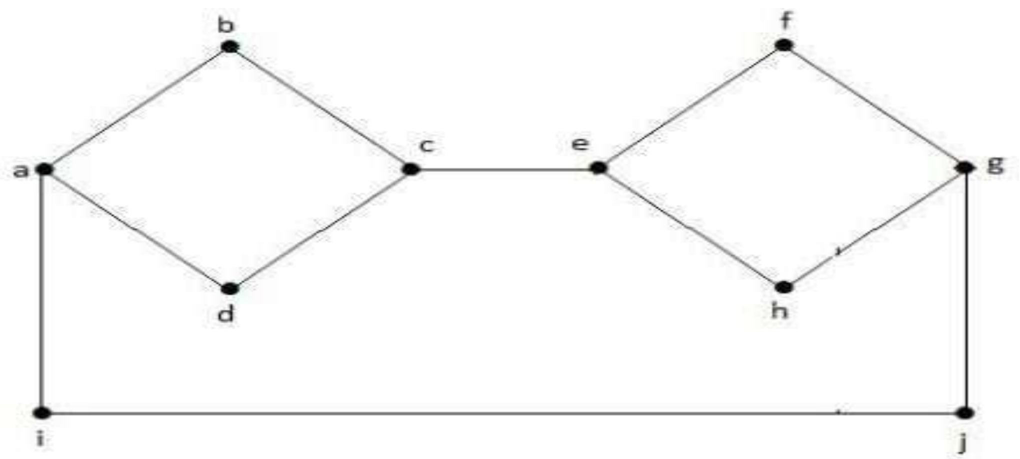
Sr. No.	Question	Answer
1	A graph with no loops and no parallel edges is called…….	**Simple graph**
2	Give the formula to find the number of simple graphs possible with 'n' vertices.	$2^{nc_2} = 2^{n(n-1)/2}$

❖ Connected Graph

- A graph G is said to be connected **if there exists a path between every pair of vertices**.
- There should be at least one edge for every vertex in the graph.
- So that we can say that it is connected to some other vertex at the other side of the edge.

Example

- In the following graph, each vertex has its own edge connected to other edge.
- Hence it is a connected graph.

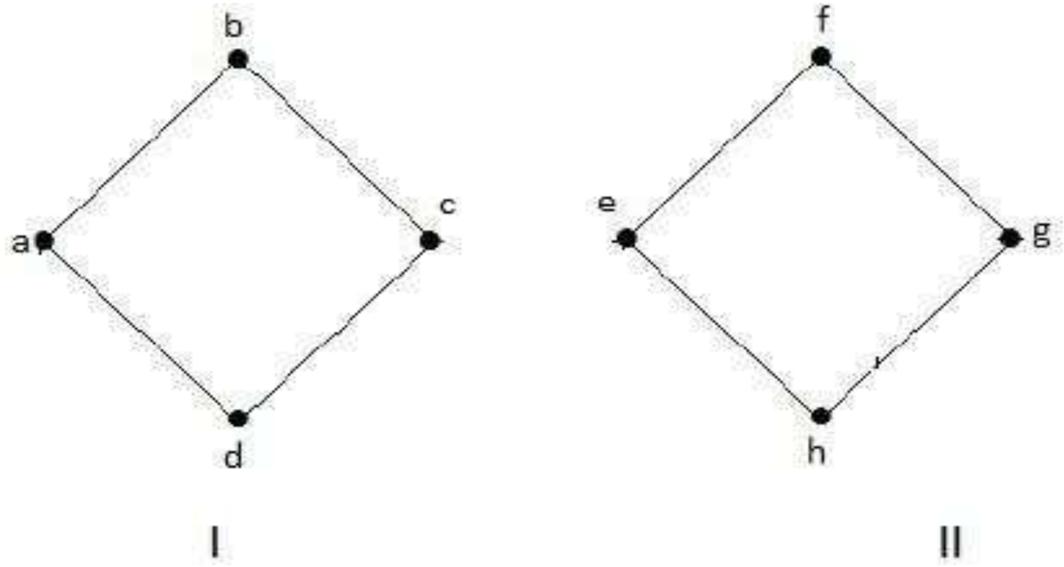


❖ Disconnected Graph

A graph G is disconnected, if it does not contain at least two connected vertices.

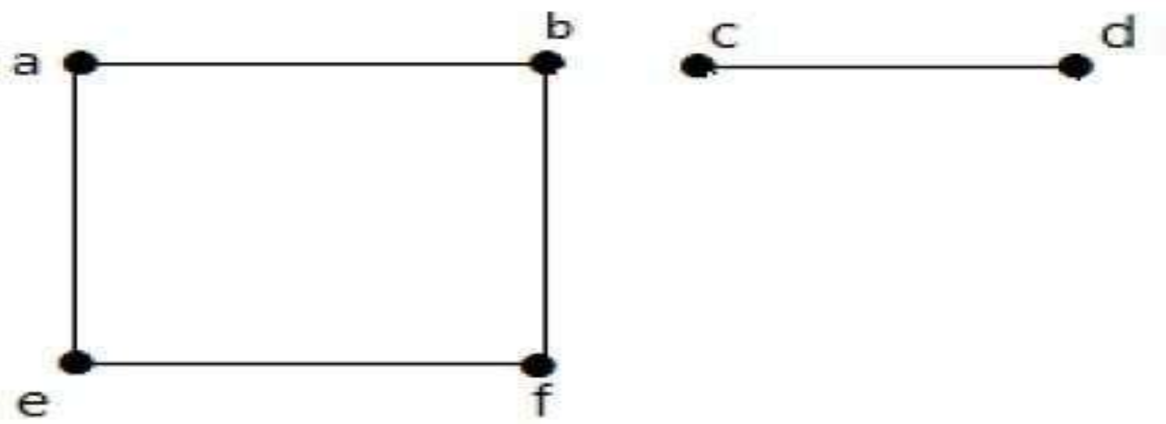
Example1

The following graph is an example of a Disconnected Graph, where there are two components, one with 'a', 'b', 'c', 'd' vertices and another with 'e', 'f', 'g', 'h' vertices.



The two components are independent and not connected to each other. Hence it is called disconnected graph.

Example2



- In this example, there are two independent components, a-b-f-e and c-d, which are not connected to each other.
- Hence this is a disconnected graph.

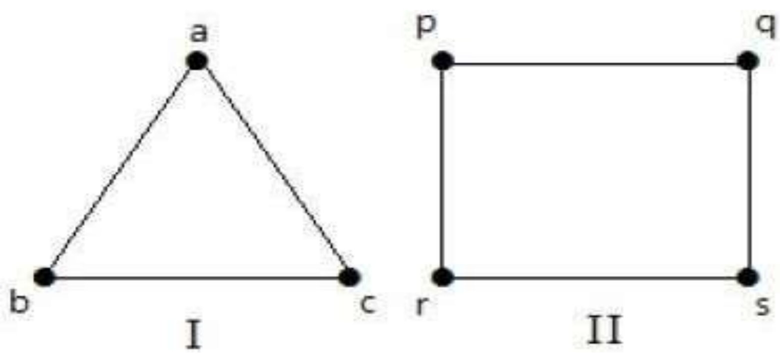
Sr. No.	Question	Answer
1	A graph G is said to be_______ if there exists a path between every pair of vertices.	**Connected**
2	A graph G is disconnected, if it does not contain at least______ connected vertices.	**two**
3	In which graph each vertex has its own edge connected to other edge?	**Connected graph**

❖ Regular Graph

- A graph G is said to be regular, **if all its vertices have the same degree**.
- In a graph, if the degree of each vertex is 'k', then the graph is called a 'k-regular graph'.

Example

- In the following graphs, all the vertices have the same degree.
- So these graphs are called regular graphs.



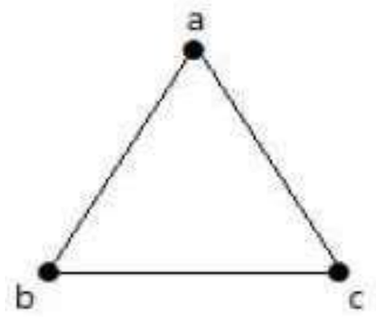
In both the graphs, all the vertices have degree2. They are called 2-Regular Graphs.

❖ Complete Graph

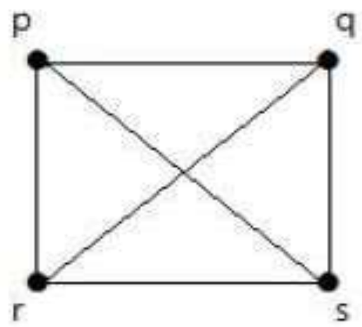
- A simple graph with 'n' mutual vertices is called a complete graph and it is **denoted by'Kn'**. In the graph, **a vertex should have edges with all other vertices,** and then it called a complete graph.
- In other words, if a vertex is connected to all other vertices in a graph, then it is called a complete graph.

Example

In the following graphs, each vertex in the graph is connected with all the remaining vertices in the graph except by itself.



I



II

In graph I,

	a	b	C
a	Not Connected	Connected	Connected
b	Connected	Not Connected	Connected
c	Connected	Connected	Not Connected

In graph II,

	p	q	r	S
p	Not Connected	Connected	Connected	Connected
q	Connected	Not Connected	Connected	Connected
r	Connected	Connected	Not Connected	Connected
s	Connected	Connected	Connected	Not Connected

Sr. No.	Question	Answer
1	In a graph, if the degree of each vertex is 'k', then the graph is called……	**k-regular graph**
2	A graph G is said to be _______, if all its vertices have the same degree.	**Regular graph**
3	If a vertex is connected to all other vertices in a graph, then it is called……..	**Complete graph**
4	Complete graph with n vertices is denoted as……..	**K_n**

❖ Cycle Graph

- A simple graph with 'n' vertices (n>=3) and 'n' edges is called a cycle graph if all its edges form a cycle of length 'n'.
- If the degree of each vertex in the graph is two, then it is called a Cycle Graph.

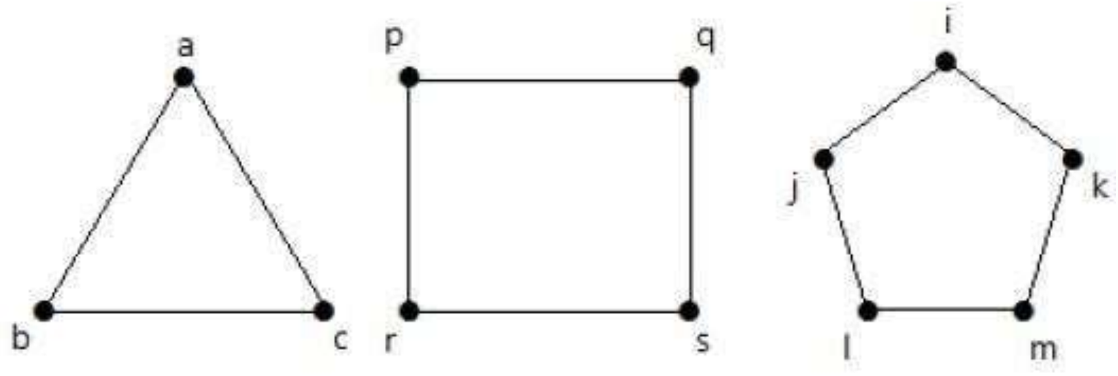
Notation:

Cn

Example

Take a look at the following graphs:

- Graph I has 3 vertices with 3 edges which is forming a cycle 'ab-bc-ca'.
- Graph II has 4 vertices with 4 edges which is forming a cycle 'pq-qs-sr-rp'.
- Graph III has 5 vertices with 5 edges which is forming a cycle 'ik-km-ml-lj-ji'.



I II III

Hence all the given graphs are cycle graphs.

Wheel Graph

A wheel graph is obtained from a cycle graph Cn-1 by adding a new vertex. That new vertex is called a **Hub** which is connected to all the vertices of Cn.

Notation:

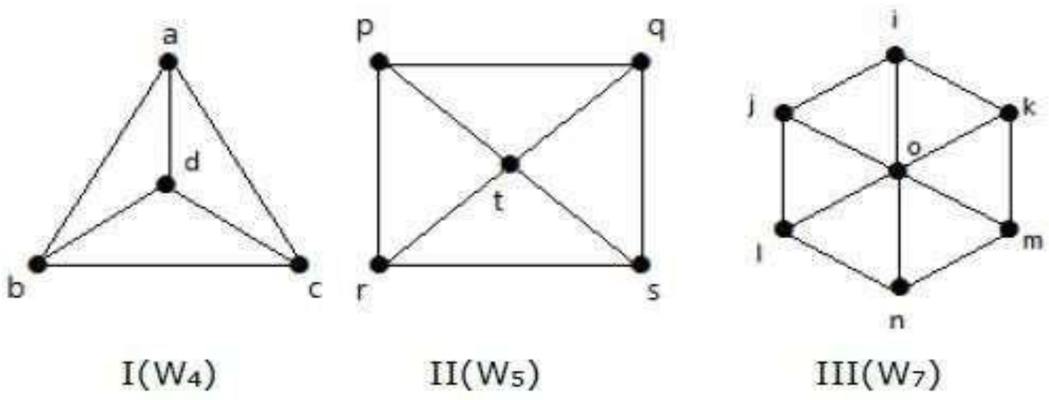
Wn

No. of edges in Wn=No. of edges from hub to all other vertices +

No. of edges from all other node sin cycle graph without a hub. = (n−1)+(n−1)= 2(n−1)

Example

Take a look at the following graphs. They are all wheel graphs.



- In graph I, it is obtained from C3 by adding an vertex at the middle named as'd'. It is denoted as W4.

  Number of edges in W4=2(n-1) =2(3) =6

- In graph II, it is obtained from C4 by adding a vertex at the middle named as't'. It is denoted as W5.

  Number of edges In W5=2(n-1) =2(4) =8

- In graph III, it is obtained from C6 by adding a vertex at the middle named as 'o'. It is denoted as W7.

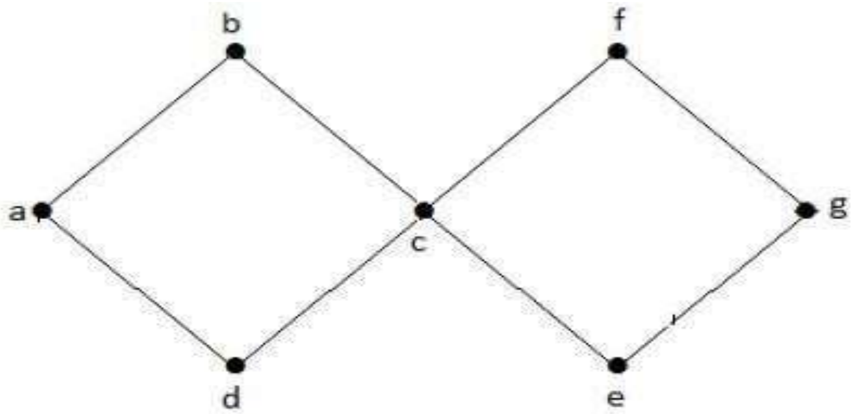
  Number of edges in W4=2(n-1) =2(6) =12

Sr. No.	Question	Answer
1	If the degree of each vertex in the graph is two, then it is called……	**Cycle graph**
2	Which graph is obtained from a Cycle graph?	**Wheel graph**
3	Give the formula to find number of edges in Wheel graph.	**2(n-1)**

❖ Cyclic Graph

A graph **with at least one** cycle is called a cyclic graph.

Example

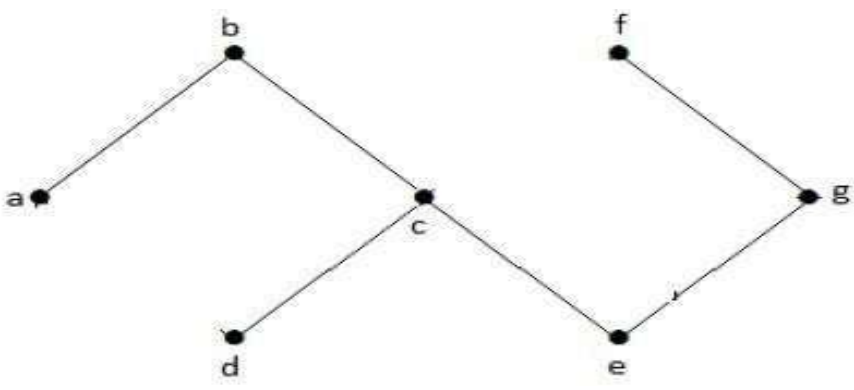


- In the above example graph, we have two cycles a-b-c-d-a and c-f-g-e-c.
- Hence it is called a cyclic graph.

❖ Acyclic Graph

A graph **with no cycles** is called an acyclic graph.

Example



- In the above example graph, we do not have any cycles.
- Hence it is a non-cyclic graph.

Sr. No.	Question	Answer
1	A graph **with at least one** cycle is called………	**Cyclic graph**
2	A graph **with no cycles** is called…..	**Acyclic graph**

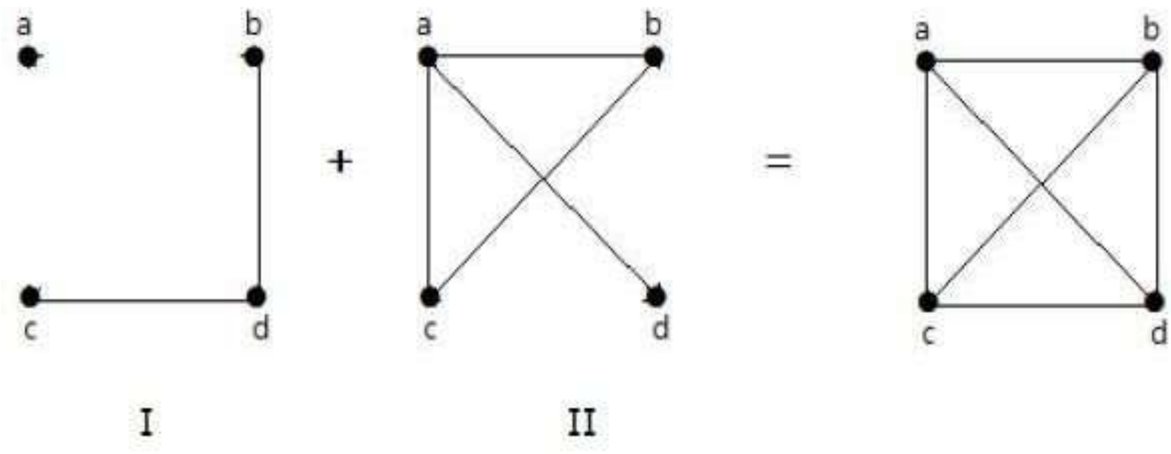
❖ Complement of a Graph

- Let $'G-'$ be a simple graph with some vertices as that of 'G' and an edge {U, V} is present in $'G-'$, if the edge is not present in G. It means, two vertices are adjacent in $'G-'$ if the two vertices are not adjacent in G.

- If the edges that exist in graph I are absent in another graph II, and if both graph I and graph II are combined together to form a complete graph, then graph I and graph II are called complements of each other.

Example

- In the following example, graph-I has two edges 'cd' and 'bd'.
- Its complement graph-II has four edges.



- Note that the edges in graph-I are not present in graph-II and vice versa.
- Hence, the combination of both the graphs gives a complete graph of ‘n’ vertices.

Note:

A combination of two complementary graphs gives a

complete graph. If ‘G’ is any simple graph, then

|E(G)|+|E('G-')|=|E(Kn)|, where n= number of vertices in the graph.

Example

- Let ‘G’ be a simple graph with nine vertices and twelve edges, find the number of edges in *'G-'*.

  You have, |E(G)| + |E('G-')| = |E(Kn)|∴12+|E('G-')| = 9(9-1)/2= 9C2∴ 12+|E('G-')|=36 ∴|E('G-')|=24

- ‘G’ is a simple graph with 40 edges and its complement *'G−'* has 38 edges. Find the number of vertices in the graph G or *'G−'*.

Let the number of vertices in the graph be 'n'. We have,
$|E(G)|+|E(\bar{G})|=|E(K_n)|$ ∴ $40+38=n(n-1)/2$ ∴ $156=n(n-1)$
∴ $13(12)=n(n-1)$ ∴ $n=13$

Sr. No.	Question	Answer
1	In a Simple graph, E^c is complement of E then the graph is said to be………	**Complement of graph**
2	In Complement of graph, E^c=_____	**$E^c=(V*V)-E$**

5. Graph Theory - Trees

❖ Introduction:

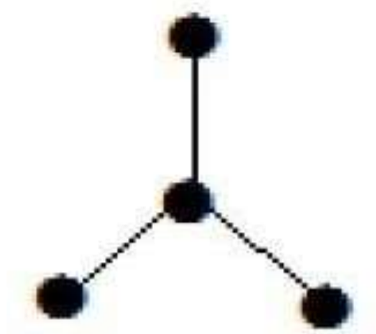
- Trees are graphs that do not contain even a single cycle. They represent hierarchical structure in a graphical form. Trees belong to the simplest class of graphs. Despite their simplicity, they have a rich structure.
- Trees provide a range of useful applications as simple as a family tree to as complex as trees in data structures of computer science.

❖ Tree

- A **connected acyclic graph** is called a tree. In other words, a connected graph with no cycles is called a tree.
- The edges of a tree are known as **branches**. Elements of trees are called their **nodes**. The nodes without child nodes are called **leaf nodes**.
- A tree with 'n' vertices has 'n-1' edges. If it has one more edge extra than 'n-1', then the extra edge should obviously has to pair up with two vertices which leads to form a cycle. Then, it becomes a cyclic graph which is a violation for the tree graph.

Example1

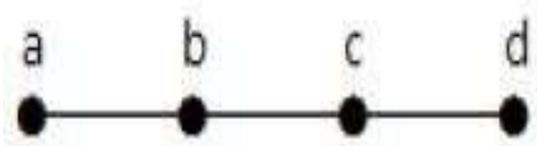
The graph shown here is a tree because it has no cycles and it is connected. It has four vertices and three edges, i.e., for ‘n’ vertices ‘n-1’edgesasmentionedinthedefinition.



Note:

Every tree has at least two vertices of degree one.

Example2



In the above example, the vertices ‘a’ and‘d’ has degree one. And the other two vertices ‘b’ and ‘c’ have degree two. This is possible because for not forming a cycle, there should be at least two single edges anywhere in the graph. It is nothing but two edges with a degree of one.

Sr. No.	Question	Answer
1	A connected acyclic graph is called……	**Tree**
2	Edges of a tree are known as……	**Branches**
3	Elements of trees are called their…..	**Nodes**
4	The nodes without child nodes are called……	**leaf nodes**

❖ Forest

- A **disconnected acyclic graph** is called a forest.
- In other words, a disjoint collection of trees is called a forest.

Example

- The following graph looks like two sub-graphs; but it is a single disconnected graph.
- There are no cycles in this graph.
- Hence, clearly it is a forest.

❖ **Spanning Trees**

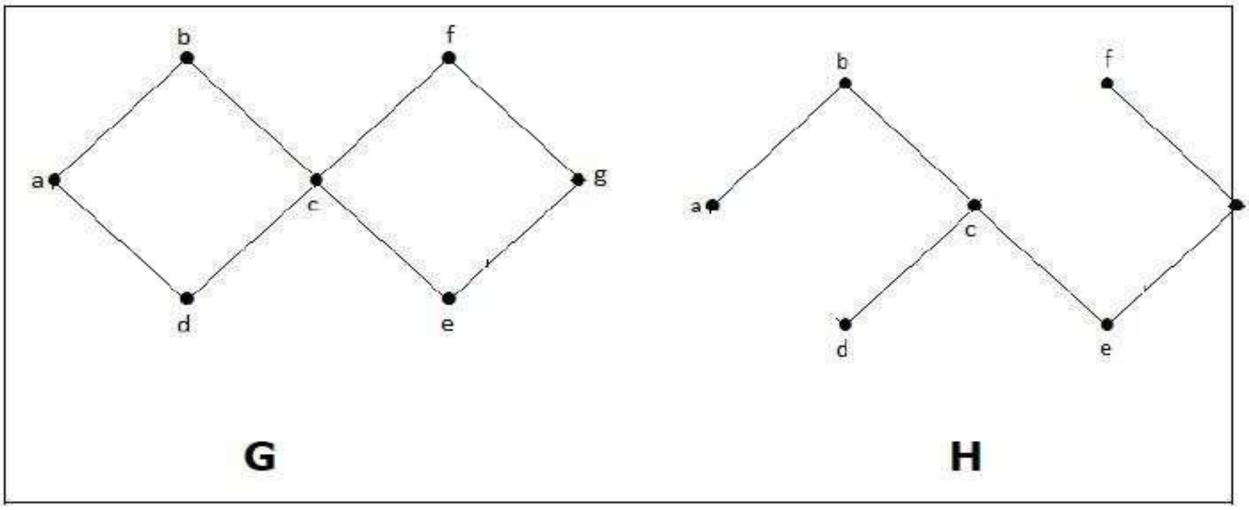
- Let G be a connected graph, then the sub-graph H of G is called a spanning tree of G if:

➢ H is a tree

➢ H contains all vertices of G.

- A spanning tree T of an undirected graph G is a sub graph that includes all of the vertices of G.

Example



- In the above example, G is a connected graph and H is a sub-graph of G.
- Clearly, the graph H has no cycles; it is a tree with six edges which is one less than the total number of vertices.
- Hence H is the Spanning tree of G.

Sr. No.	Question	Answer
1	A __________ is called a forest.	**disconnected acyclic graph**
2	A disjoint collection of trees is called….	**Forest**
3	A ________ of an undirected graph G is a sub graph that includes all of the vertices of G.	**Spanning tree**

6. Graph Theory - Connectivity

❖ Introduction:

- Whether it is possible to traverse a graph from one vertex to another is determined by how a graph is connected.
- Connectivity is a basic concept in Graph Theory.
- Connectivity defines whether a graph is connected or disconnected.
- It has sub topics based on edge and vertex, known as edge connectivity and vertex connectivity.
- Let us discuss them in detail.

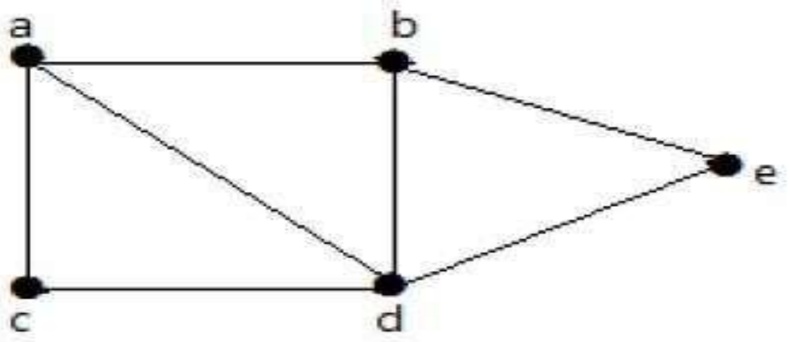
❖ Connectivity

- A graph is said to be **connected if there is a path between every pair of vertex**.
- From every vertex to any other vertex, there should be some path to traverse.
- That is called the connectivity of a graph.
- A graph with multiple disconnected vertices and edges is said to be disconnected.

Example1

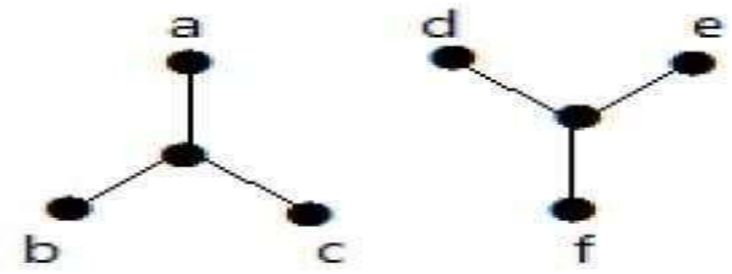
➢ In the following graph, it is possible to travel from one vertex to any other vertex.

- For example, one can traverse from vertex 'a' to vertex 'e' using the path 'a-b-e'.



Example2

- In the following example, traversing from vertex 'a' to vertex 'f' is not possible because there is no path between them directly or indirectly.
- Hence it is a disconnected graph.



Sr. No.	Question	Answer
1	A graph with multiple disconnected vertices and edges is said to be…..	**disconnected**
2	Connectivity is a basic concept in-_________.	**Graph theory**
3	A graph is said to be connected if there is a_____ between every pair of vertex.	**path**

❖ Cut Vertex

- Let 'G' be a connected graph. A vertex V ∈ G is called a cut vertex of 'G', if 'G-V' (Delete 'V' from 'G') results in a disconnected graph.
- Removing a cut vertex from a graph break sit into two or more graphs.

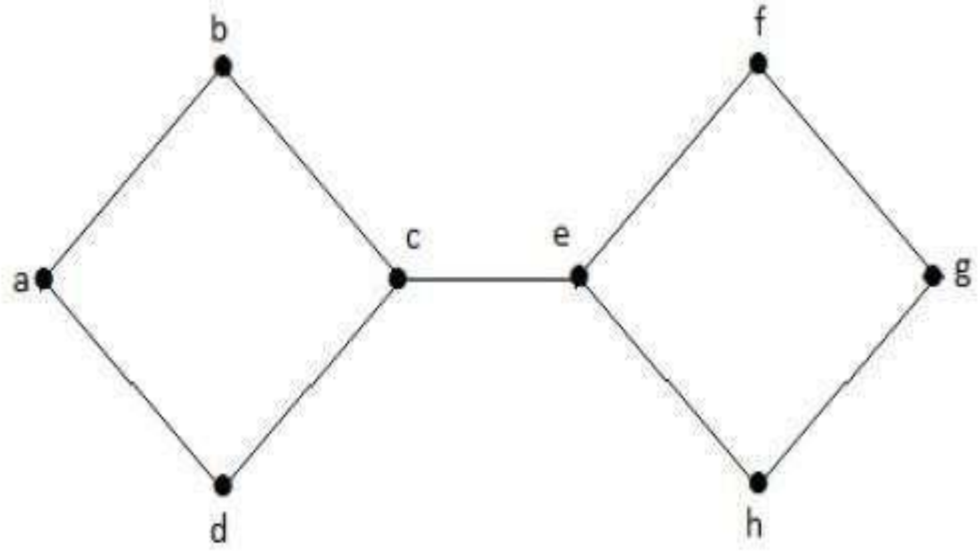
Note:

✓ Removing a cut vertex may rend era graph disconnected.

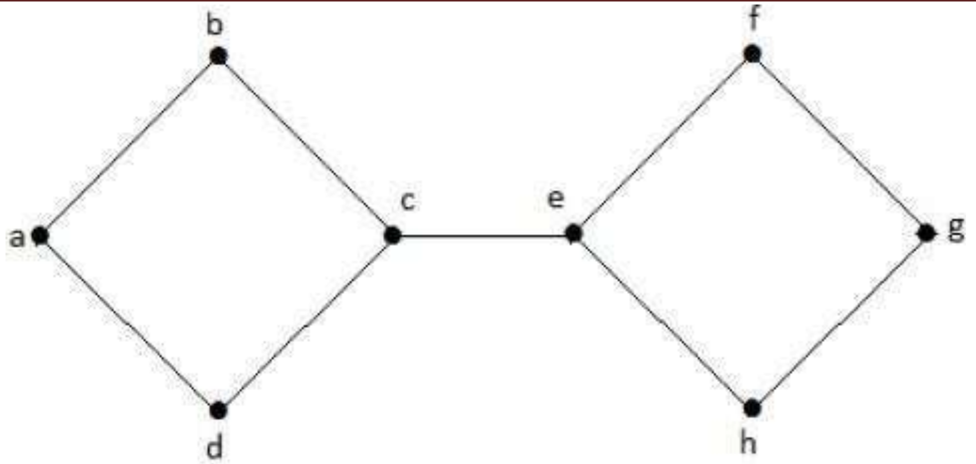
✓ A connected graph 'G' may have atmost (n−2) cut vertices.

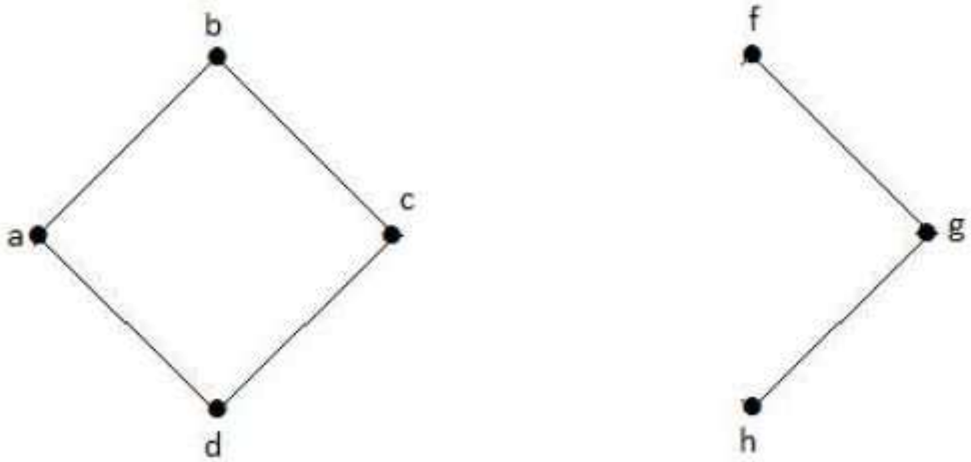
Example

In the following graph, vertices 'e' and 'c' are the cut vertices.



By removing 'e' or 'c', the graph will become a disconnected graph.





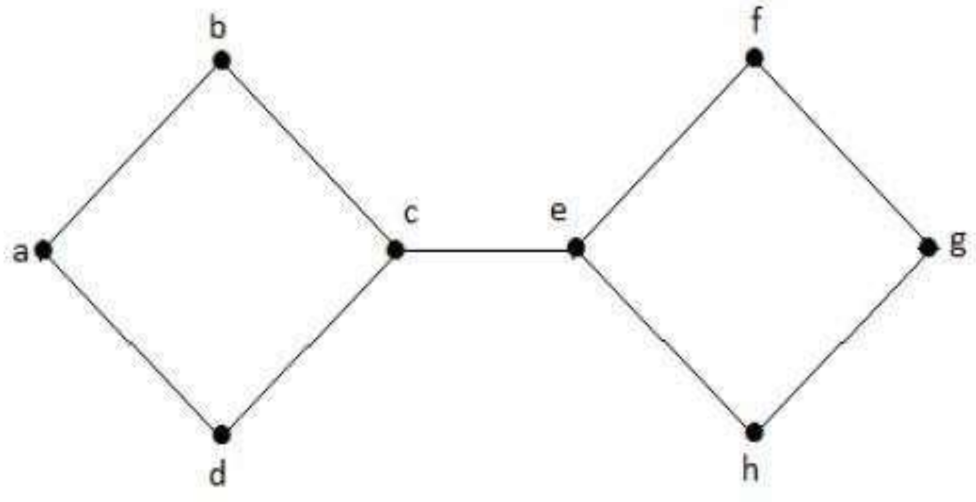
Without ‘g’, there is no path between vertex ‘c’ and vertex ‘h’ and many other. Hence it is a disconnected graph with cut vertex as ‘e’. Similarly, ‘c’ is also a cut vertex for the above graph.

❖ Cut Edge (Bridge)

- Let ‘G’ be a connected graph. An edge ‘e’ ∈G is called a cut edge if ‘G-e’ results in a disconnected graph.
- If removing an edge in a graph results into two or more graphs, then that edge is called a Cut Edge.

Example

In the following graph, the cut edge is [(c, e)].



- By removing the edge (c, e) from the graph, it becomes a disconnected graph.
- In the above graph, removing the edge (c, e) breaks the graph into two which is nothing but a disconnected graph.
- Hence, the edge(c, e) is a cut edge of the graph.

Note:

Let 'G' be a connected graph with 'n' vertices, then;

- A cut edge e∈G if and only if the edge 'e' is not a part of any cycle in G.
- The maximum number of cut edges possibleis'n-1'.
- Whenever cut edges exist, cut vertices also exist because at least one vertex of a cut edge is a cut vertex.
- If a cut vertex exists, then a cut edge may or may not exist.

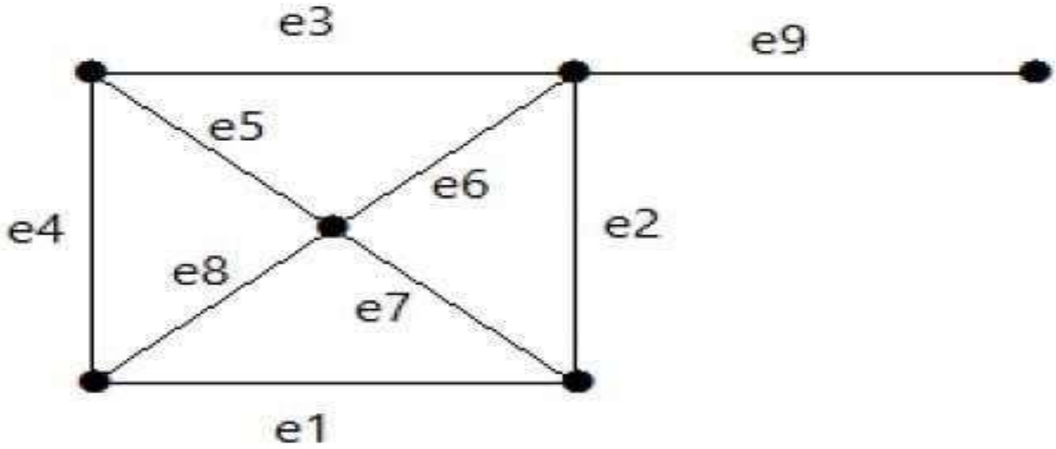
Sr. No.	Question	Answer
1	Let 'G' be a connected graph. A vertex V ∊ G is called a _________, if 'G-V' (Delete 'V' from 'G') results in a disconnected graph.	**Cut vertex**
2	If removing an edge in a graph results into two or more graphs, then that edge is called……..	**Cut edge**
3	A connected graph 'G' may have atmost ______ cut vertices.	**(n-2)**
4	By removing the edge from the graph, it becomes a _________ graph.	**disconnected**

❖ Cut Set of a Graph

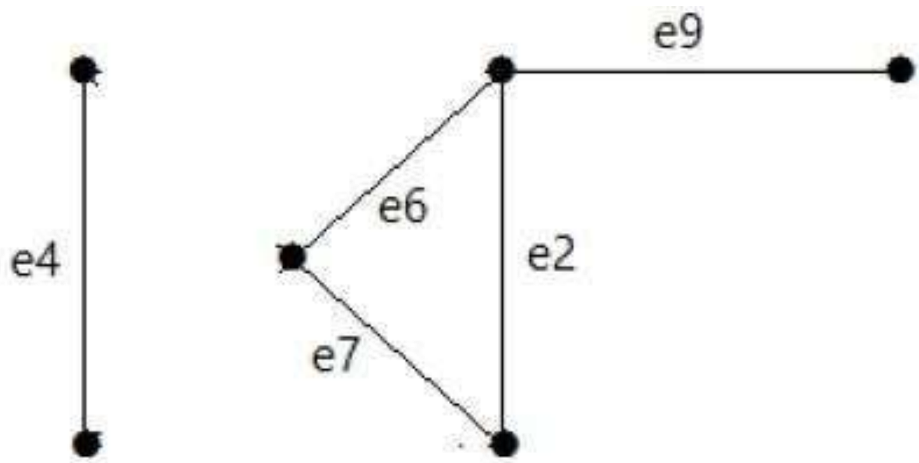
- Let 'G' = (V, E) be a connected graph. A subset E' of E is called a cut set of G if deletion of all the edges of E' from G makes G disconnect.
- If deleting a certain number of edges from a graph makes it disconnected, then those deleted edges are called the cut set of the graph.

Example

- Take a look at the following graph.
- Its cut set is E1 ={e1,e3,e5,e8}.



- After removing the cut set E1 from the graph, it would appear as follows−



- Similarly, there are other cut sets that can disconnect the graph:
- E3={e9}–Smallest cut set of the graph.
- E4={e3, e4,e5}

Sr. No.	Question	Answer
1	Let 'G' = (V, E) be a connected graph, A subset E' of E is called a_______ if deletion of all the edges of E' from G makes G disconnect.	**Cut set of graph G**
2	If deleting a certain number of edges from a graph makes it disconnected, then those deleted edges are called……..	**Cut set**

❖ Edge Connectivity

Let 'G' be a connected graph. The minimum number of edges whose removal makes 'G' disconnected is called edge connectivity of G.

Notation:

- λ(G)
- In other words, the **number of edges in a smallest cut set of G** is called the edge connectivity of G.
- If 'G' has a cut edge, then *λ(G)* is1.(edge connectivity of G.)

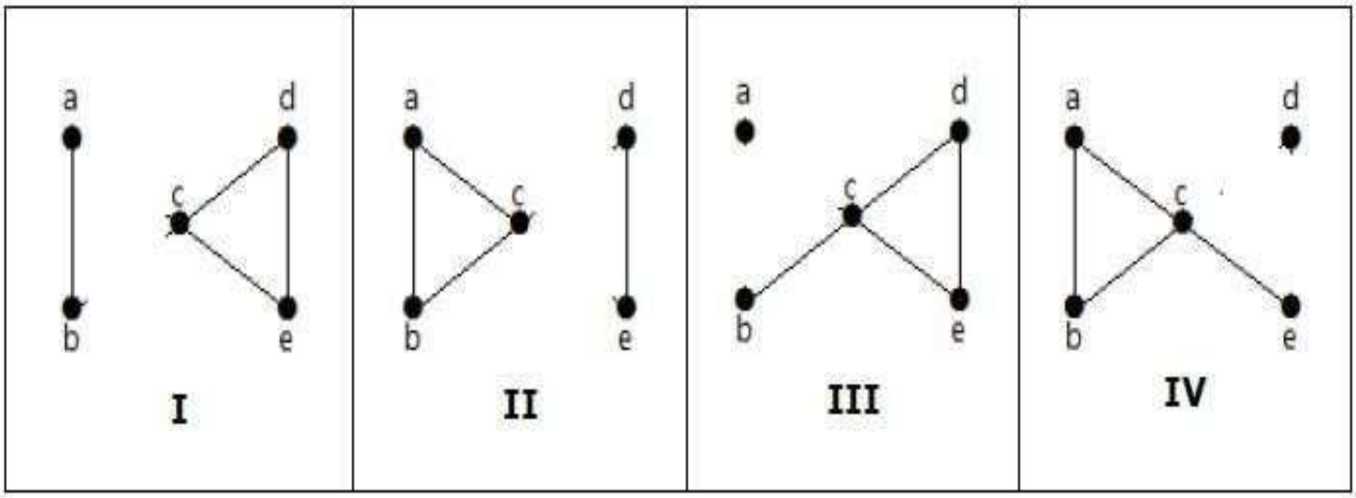
Example

Take a look at the following graph.

By removing two minimum edges, the connected graph becomes disconnected.

Hence, its edge connectivity (λ(G)) is 2.

Here are the four ways to disconnect the graph by removing two edges:



❖ **Vertex Connectivity**

- Let 'G' be a connected graph.

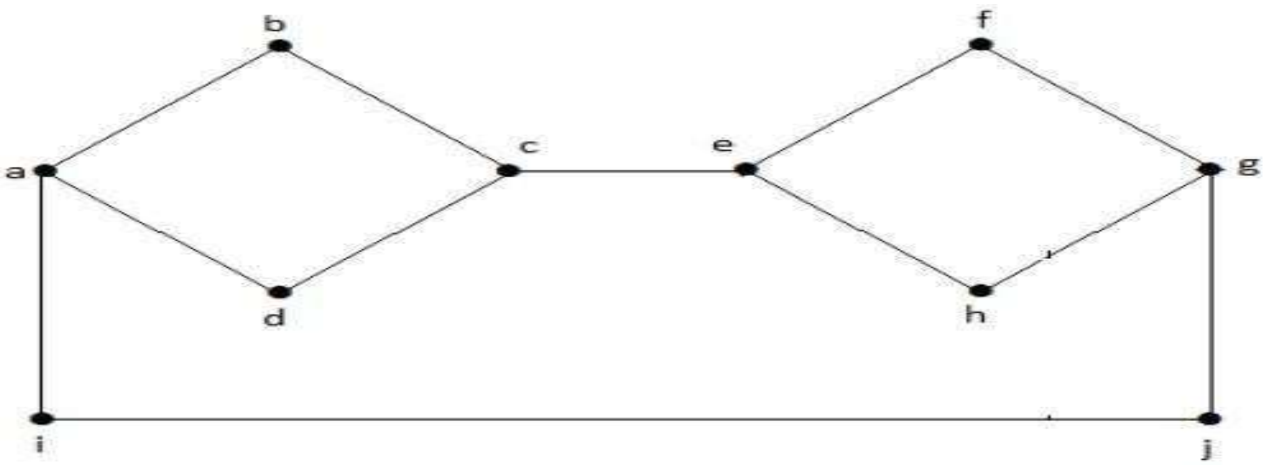
- The minimum number of vertices whose removal makes 'G' either disconnected or reduces 'G' into a trivial graph is called its vertex connectivity.

Notation:

K(G)

Example

In the above graph, removing the vertices 'e' and 'i' makes the graph disconnected.



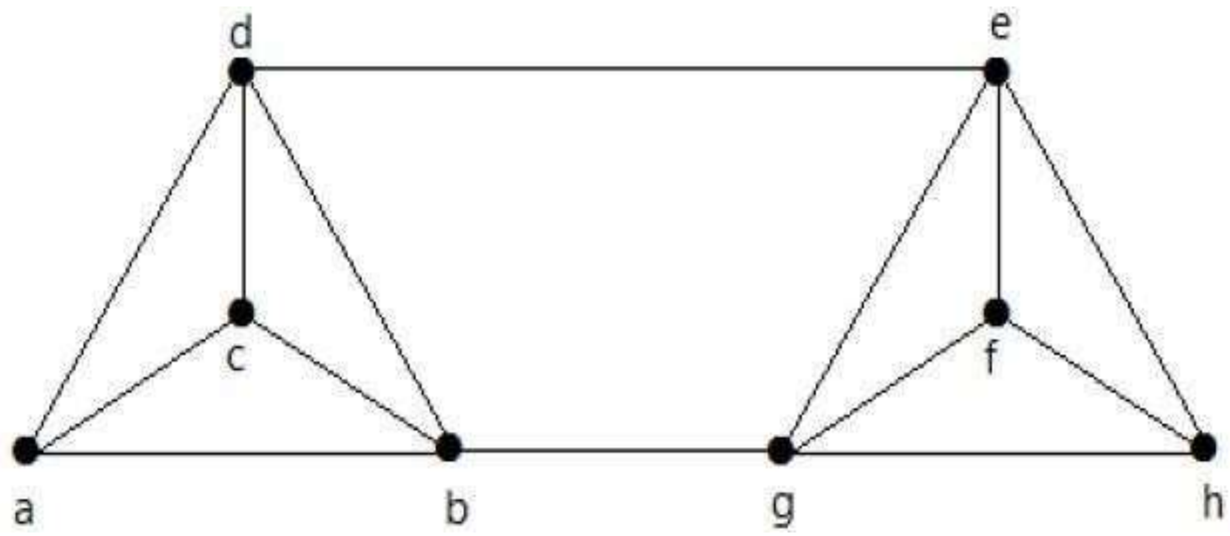
- If G has a cut vertex, then K(G) = 1.

Notation:

For any connected graph G,K(G)≤λ(G) ≤ δ(G); Vertex connectivity (K(G)), edge connectivity (λ(G)), minimum number of degrees of G (δ(G)).

Example

Calculate λ(G) and K(G) for the following graph:



Solution

From the graph, $\delta(G)=3$

$K(G) \leq \lambda(G) \leq \delta(G)=3$(1)
$K(G) \geq 2$(2)

Deleting the edges {d, e} and {b, h}, we can disconnect G.

Therefore,
$\lambda(G)=2$

$2 \leq \lambda(G) \leq \delta(G)=2$(3)

From (2) and (3), vertex connectivity $K(G)=2$

Sr.No.	Question	Answer
1	The minimum number of edges whose removal makes ‘G’ disconnected is called…….	**Edge connectivity**
2	The minimum number of vertices whose removal makes ‘G’ either disconnected or reduces ‘G’ into a trivial graph is called…….	**Vertex connectivity**
3	Give the notation of vertex connectivity.	**K(G)**
4	λ(G) is the notation of Edge connectivity.(T/F)	**True**

7. Graph Theory - Covering

❖ Introduction:

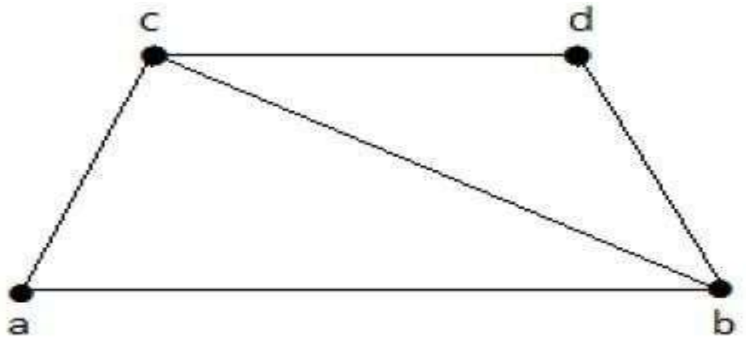
- A covering graph is a sub graph which contains either all the vertices or all the edges corresponding to some other graph.
- A sub graph which contains all the vertices is called a **line/edge covering**.
- A sub graph which contains all the edges is called a **vertex covering**.

❖ Line Covering

- Let G= (V, E) be a graph. A subset C (E) is called a line covering of G if every vertex of G is incident with at least one edge in C,
- i.e., deg (V) $\geq$ 1 $\forall$ V$\in$G; because each vertex is connected with another vertex by an edge.
- Hence it has a minimum degree of 1.

Example

Take a look at the following graph:



- Its sub graphs having line covering are as follows:

  C1= {{a, b}, {c, d}}

  C2= {{a, d}, {b, c}}

  C3= {{a, b}, {b, c}, {b, d}}

  C4= {{a, b}, {b, c}, {c, d}}

- Line covering of 'G' does not exist if and only if 'G' has an isolated vertex.

- Line covering of a graph with 'n' vertices has at least [n/2] edges.

❖ Minimal Line Covering

A line covering C of a graph G is said to be minimal **if no edge can be deleted from C**.

Example

In the above graph, the sub graphs having line covering are as follows:

C1= {{a, b}, {c, d}}

C2= {{a, d}, {b, c}}

C3= {{a, b}, {b, c}, {b, d}}

C4= {{a, b}, {b, c}, {c, d}}

Here, C1, C2, C3 are minimal line coverings, while C4 is not because we can delete {b, c}.

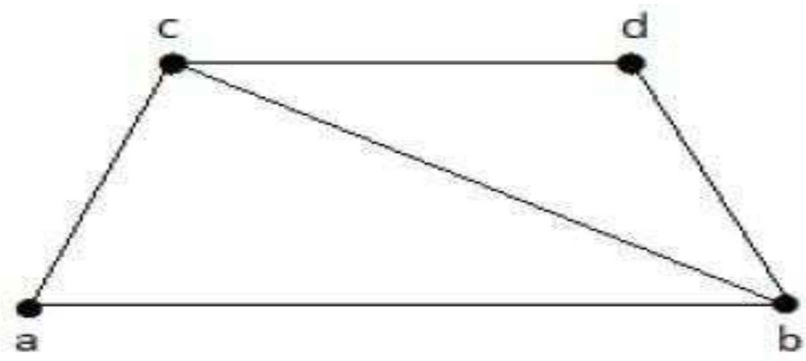
Sr. No.	Question	Answer
1	A sub graph which contains all the vertices is called………	**Line covering**
2	Line covering of ‘G’ does not exist if and only if ‘G’ has an __________.	**isolated vertex**
3	A line covering C of a graph G is said to be _________ **if no edge can be deleted from C**.	**Minimal line covering**

❖ Vertex Covering

Let ‘G’= (V, E) be a graph. A subset K of V is called a vertex covering of ‘G’, if every edge of ‘G’ is incident with or covered by a vertex in ‘K’.

Example

Take a look at the following graph:



The sub graphs that can be derived from the above graph are as follows:

K1= {b, c}

K2= {a, b, c}

K3= {b, c, d}

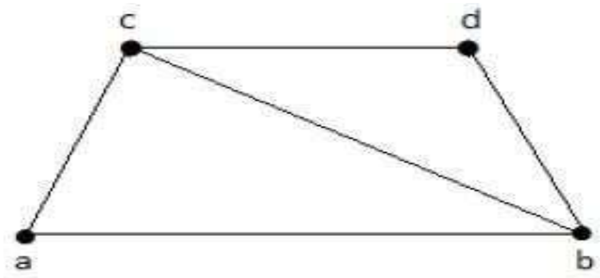
K4= {a, d}

Here, K1, K2, and K3 have vertex covering, whereas K4 does not have any vertex covering as it does not cover the edge {bc}.

- **Minimal Vertex Covering**

  A vertex 'K' of graph 'G' is said to be minimal vertex covering if no vertex can be deleted from 'K'.

Example



In the above graph, the sub graphs having vertex covering are as follows:

K1= {b, c}

K2= {a, b, c}

K3= {b, c, d}

Here, K1 and K2 are minimal vertex coverings, whereas in K3, vertex'd' can be deleted.

Sr. No.	Question	Answer
1	A sub graph which contains all the edges is called………	**Vertex covering**
2	A vertex 'K' of graph 'G' is said to be ________ if no vertex can be deleted from 'K'.	**Minimal vertex covering**

8. Graph Theory – Independent Set

❖ Introduction:

Independent sets are represented in sets, in which

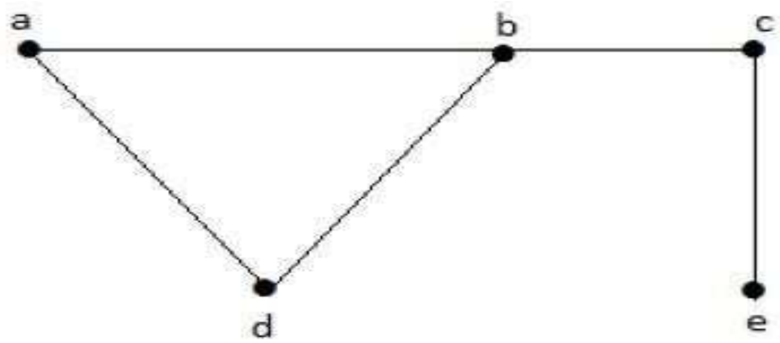
- There should not be **any edges adjacent to each other**. There should not be any common vertex between any two edges.

- There should not be **any vertices adjacent to each other**. There should not be any common edge between any two vertices.

❖ Independent Line Set

Let 'G'= (V, E) be a graph. A subset L of E is called an independent line set of 'G' if not wo edges in Lare adjacent. Such a set is called an independent line set.

Example



Let us consider the following subsets:

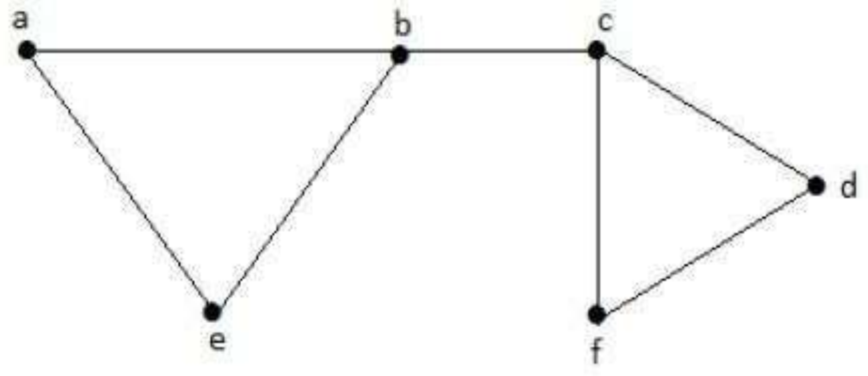
```
L1= {a, b}
L2= {a, b} {c, e}
L3= {a, d} {b, c}
```

In this example, the subsets L2 and L3 are clearly not the adjacent edges in the given graph. They are independent line sets. However L1 is not an independent line set, as for making an independent line set, there should be at least two edges.

❖ Maximal Independent Line Set

An independent line set is said to be the maximal independent line set of a graph 'G' if no other edge of 'G' can be added to 'L'.

Example



Let us consider the following subsets:

L1= {a, b}

L2= {{b, e}, {c, f}}

L3= {{a, e}, {b, c},

{d, f}}

L4 = {{a, b},{c, f}}

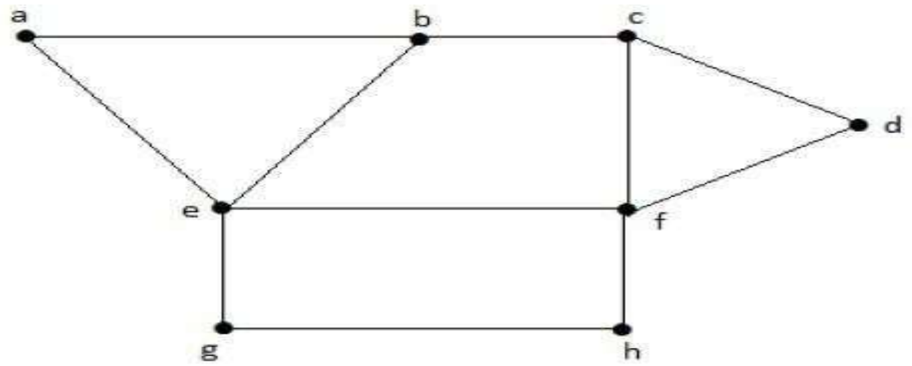
- L2 and L3 are maximal independent line sets/maximal matching. As for only these twosubsets, there is no chance of adding any other edge which is not an adjacent.
- Hence these two sub sets are considered as the maximal independent line sets.

Sr. No.	Question	Answer
1	Let 'G'= (V, E) be a graph. A subset L of E is called an ________ of 'G' if not wo edges in Lare adjacent.	**Independent line set**
2	An independent line set is said to be the _______ of a graph 'G' if no other edge of 'G' can be added to 'L'.	**Maximal Independent line set**

❖ Independent Vertex Set

Let 'G'= (V, E) be a graph. A sub set of 'V' is called an independent set of 'G' if no two vertices in 'S' are adjacent.

Example



Consider the following sub sets from the above graphs:

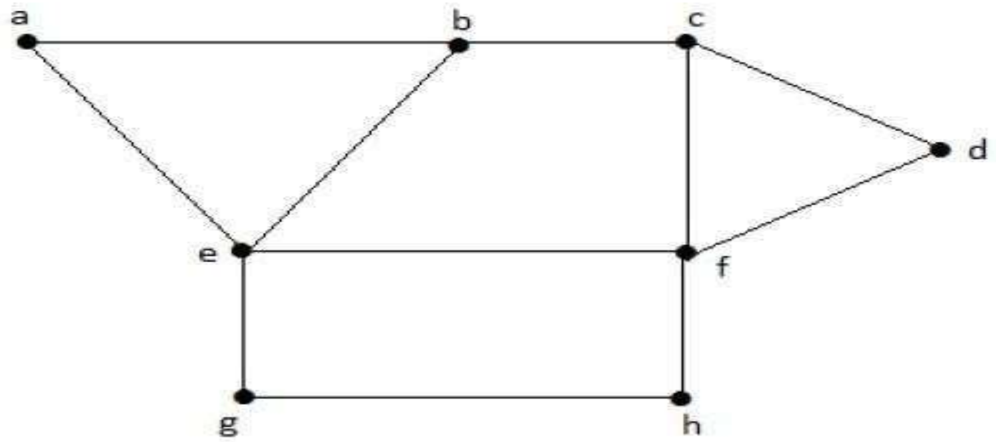
```
S1= {e}
S2= {e, f}
S3= {a, g, c}
S4= {e, d}
```

Clearly S1 is not an independent vertex set, because for getting an independent vertex set, there should be at least two vertices in the from a graph. But here it is not that case. The subsets S2, S3, and S4 are the independent vertex sets because there is no vertex that is adjacent to any one vertex from the subsets.

❖ Maximal Independent Vertex Set

Let 'G' be a graph, then an independent vertex set of 'G' is said to be maximal if no other vertex of 'G' can be added to 'S'.

Example



Consider the following subsets from the above graphs.

```
S1= {e}
S2= {e, f}
S3= {a, g, c}
S4= {e, d}
```

S2 and S3 are maximal independent vertex sets of 'G'. In S1 and S4, we can add other vertices; but in S2and S3, we cannot add any other vertex.

Sr. No.	Question	Answer
1	Let 'G'= (V, E) be a graph. A sub set of 'V' is called an _________ of 'G' if no two vertices in 'S' are adjacent.	**Independent vertex set**
2	Let 'G' be a graph, then an independent vertex set of 'G' is said to be _______ if no other vertex of 'G' can be added to 'S'.	**Maximal independent vertex set**

9. Graph Theory - Coloring

❖ Introduction:

- Graph coloring is nothing but a simple way of labeling graph components such as vertices, edges, and regions under some constraints.
- In a graph, no two adjacent vertices, adjacent edges, or adjacent regions are colored with minimum number of colors.
- This number is called the **chromatic number** and the graph is called a **properly colored graph**.
- While graph coloring, the constraints that are set on the graph are colors, order of coloring, the way of assigning color, etc.
- A coloring is given to a vertex or a particular region.
- Thus, the vertices or regions having same colors form independent sets.

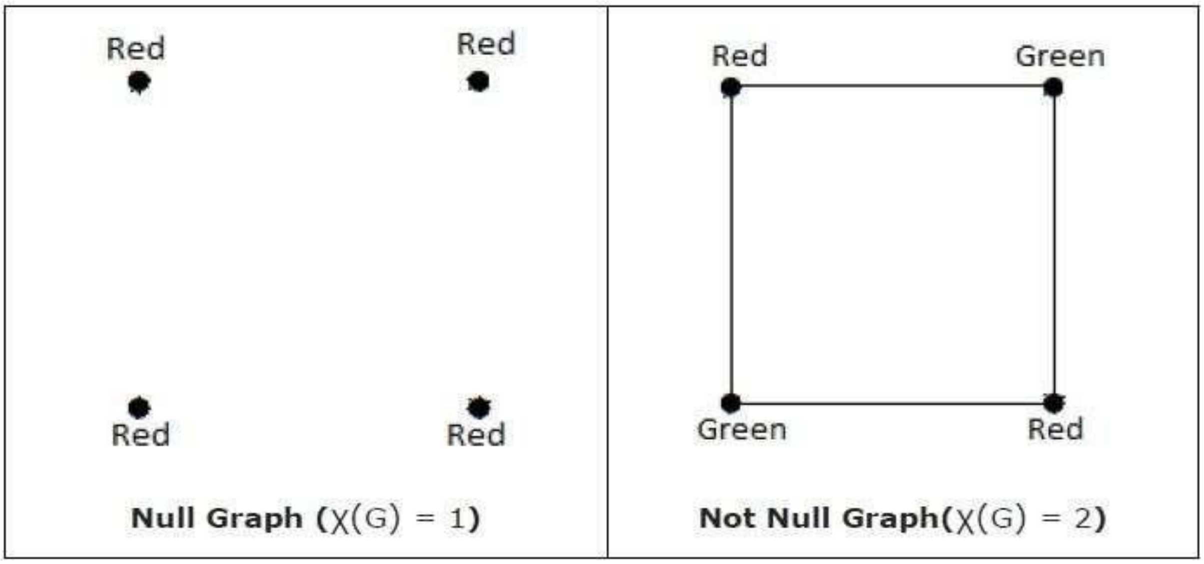
❖ Vertex Coloring

Vertex coloring is an assignment of colors to the vertices of a graph 'G' such that no two adjacent vertices have the same color. Simply put, no two vertices of an edge should be of the same color.

❖ Chromatic Number

- The minimum number of colors required for vertex coloring of graph 'G' is called as the chromatic number of G, denoted by X(G).
- X (G)=1 if and only if 'G' is a null graph.
- If 'G' is not a null graph, then χ (G) ≥ 2.

Example



Note:

A graph 'G' is said to be n-coverable if there is a vertex coloring that uses at most n colors, i.e., X(G)≤n.

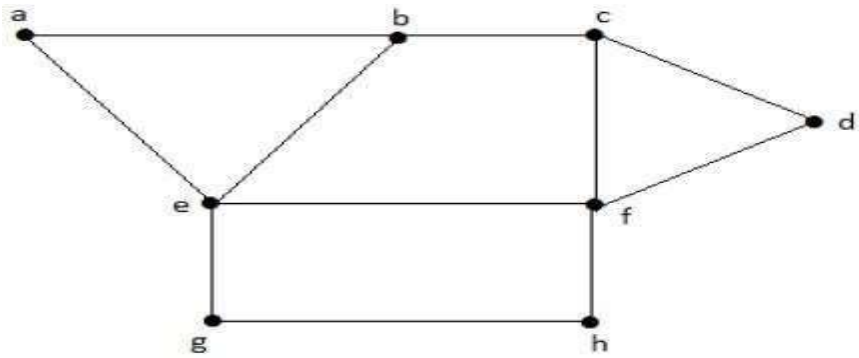
Sr. No.	Question	Answer
1	In a graph, no two adjacent vertices, adjacent edges, or adjacent regions are colored with ____________ of colors.	**minimum number**
2	X (G)=1 if and only if 'G' is _______	**Null graph**
3	The minimum number of colors required for vertex coloring of graph 'G' is called….	**Chromatic number**
4	Chromatic number is denoted by……	**X(G)**
5	If 'G' is not a null graph, then χ (G) ≥ 2.(T/F)	**True**

❖ Region Coloring

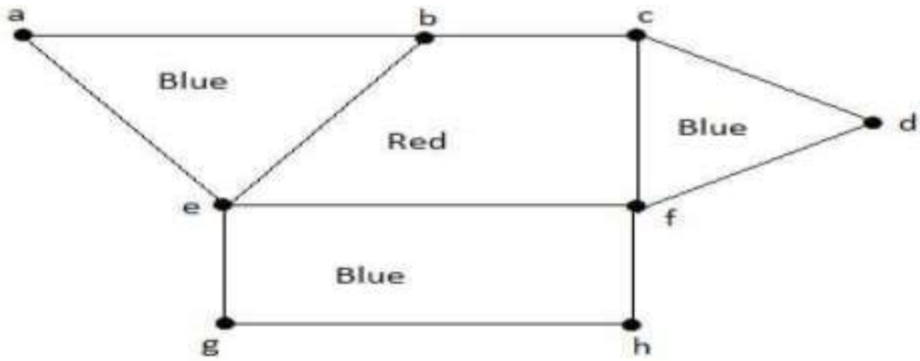
Region coloring is an assignment of colors to the regions of a planar graph such that no two adjacent regions have the same color. Two regions are said to be adjacent if they have a common edge.

Example

- Take a look at the following graph.
- The regions 'aeb' and 'befc' are adjacent, as there is a common edge 'be' between those two regions.



Similarly, the other regions are also colored based on the adjacency. This graph is colored as follows:



Sr. No.	Question	Answer
1	Two regions are said to be adjacent if they have a _____.	**Common edge**
2	Region coloring is an assignment of colors to the regions of a planar graph such that no two adjacent regions have the same color.(T/F)	**True**

10. Graph Theory - Isomorphism

❖ Introduction:

- A graph can exist in different forms having the same number of vertices, edges, and also the same edge connectivity.
- Such graphs are called isomorphic graphs.
- Note that we label the graphs in this chapter mainly for the purpose of referring to them and recognizing them from one another.

❖ Isomorphic Graphs

Two graphs G1and G2 are said to be isomorphic if:

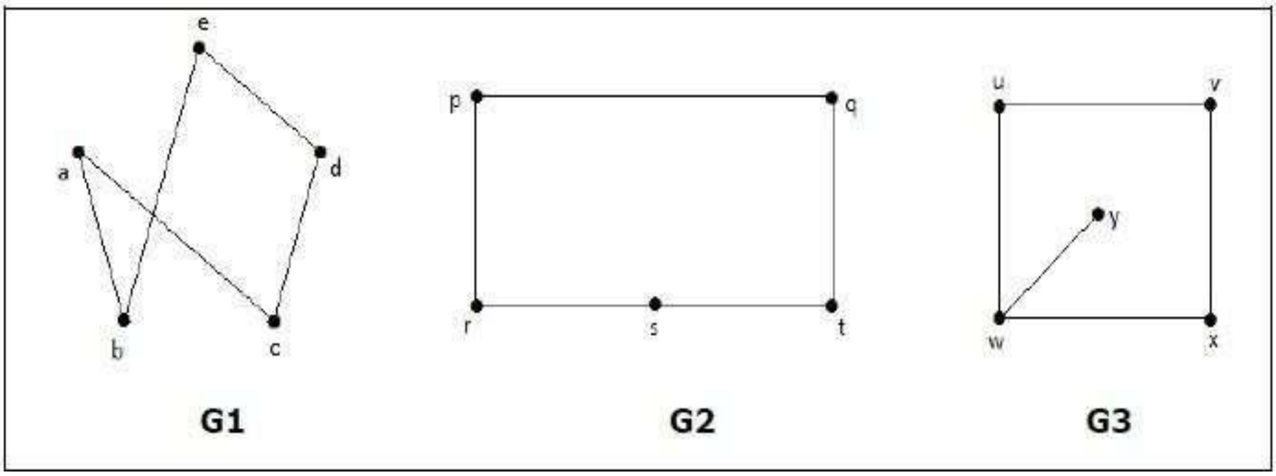
- Their number of components (vertices and edges) is same.
- Their edge connectivity is retained.

Note–

- In short, out of the two isomorphic graphs, one is at weaked version of the other.
- An unlabelled graph also can be thought of as an isomorphic graph.

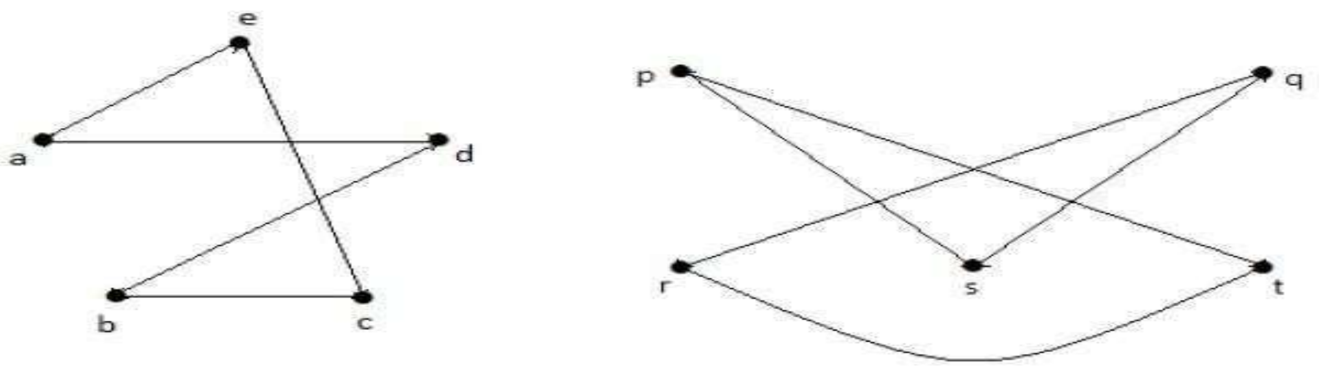
Example

Which of the following graphs are isomorphic?



- In the graph G3, vertex ‘w’ has only degree 3, whereas all the other graph vertices has degree 2.
- Hence G3 not isomorphic to G1or G2.

Taking complements of G1and G2, you have:



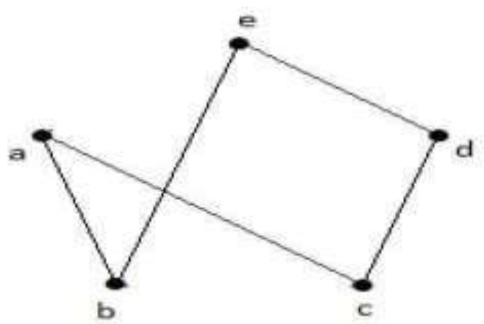
Here, ($G1^{-} \equiv G2^{-}$), hence (G1≡ G2).

Sr. No.	Question	Answer
1	A graph can exist in different forms having the same number of vertices, edges, and also the same edge connectivity. Such graphs are called……	**Isomorphic graphs**
2	An_________also can be thought of as an isomorphic graph.	**Unlabelled graph**

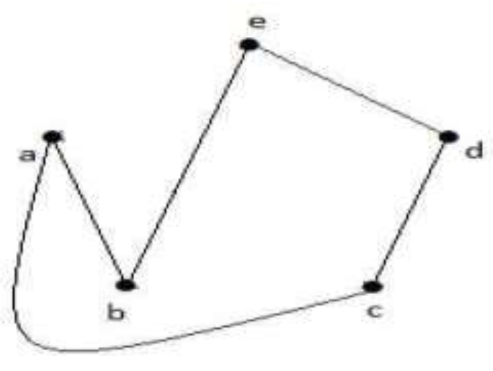
❖ **Planar Graphs**

A graph 'G' is said to be planar if it can be drawn on a plane or a sphere so that no two edges cross each other at a non-vertex point.

Example



NON - PLANAR GRAPH



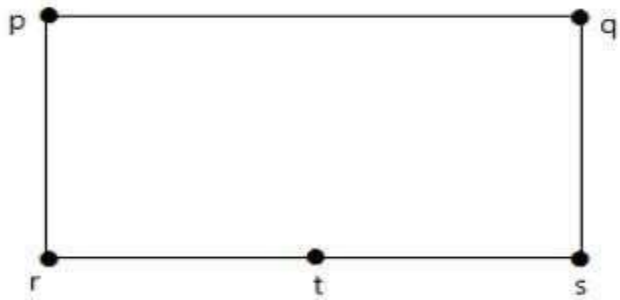
PLANAR GRAPH

❖ Homomorphism

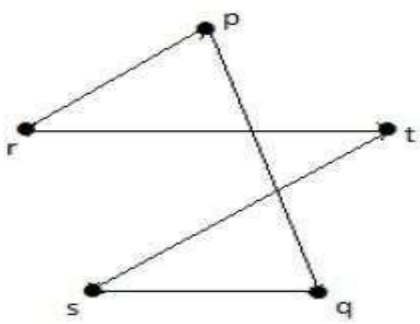
Two graphs G1 and G2 are said to be homomorphic, if each of these graphs can be obtained from the same graph ‘G’ by dividing some edges of G with more vertices. Take a look at the following example:

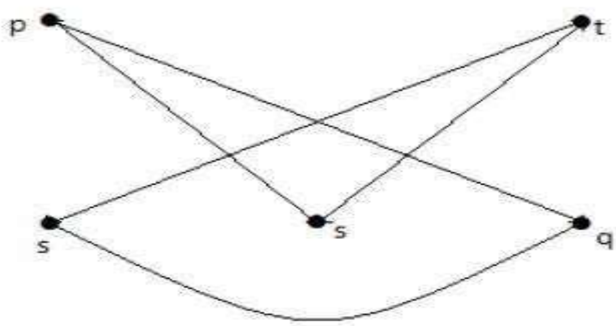
G

Divide the edge ‘rs’ into two edges by adding one vertex.



The graphs shown below are homomorphic to the first graph.





If G1 is isomorphic to G2, then G is homomorphic to G2 but the converse need not be true.

- Any graph with 4 or less vertices is planar.
- Any graph with 8 or less edges is planar.
- A complete graph Kn is planar if and only if n ≤4.
- A simple non-planar graph with minimum number of vertices is the complete graph K5.
- The simple non-planar graph with minimum number of edges is K3, 3.

Sr. No.	Question	Answer
1	A graph 'G' is said to be _______ if it can be drawn on a plane or a sphere so that no two edges cross each other at a non-vertex point.	**Planner graph**
2	A complete graph Kn is planar if and only if n ≤2. (T/F)	**False**
3	Two graphs G1 and G2 are said to be ______ , if each of these graphs can be obtained from the same graph 'G' by dividing some edges of G with more vertices.	**Homorphic**

11. Graph Theory - Traversability

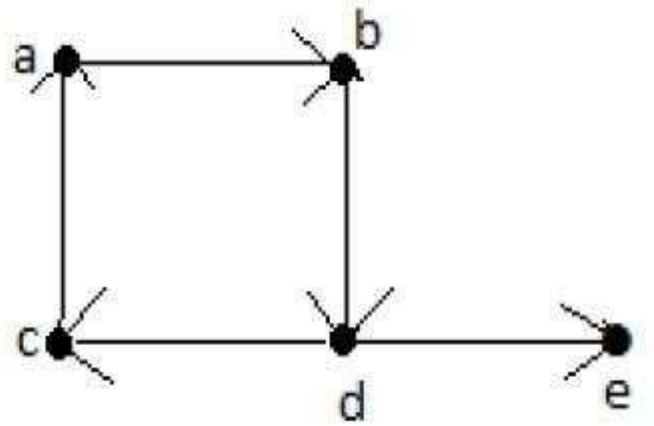
❖Introduction:

A graph is traversable if you can draw a path between all the vertices without retracing the same path. Based on this path, there are some categories like Euler's path and Euler's circuit which are described in this chapter.

❖ Euler's Path

An Euler's path contains each edge of 'G' exactly once and each vertex of 'G' at least once. A connected graph G is said to be traversable if it contains an Euler's path.

Example

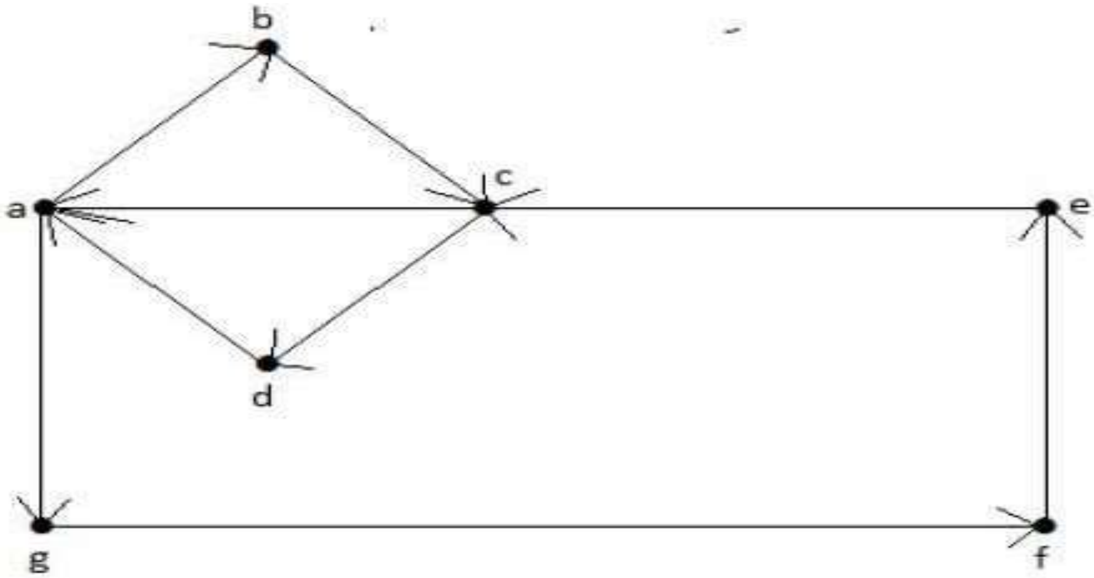


Euler's Path=d-c-a-b-d-e.

❖ Euler's Circuit

In a Euler's path, if the starting vertex is same as its ending vertex, then it is called an Euler's circuit.

Example



Euler's Circuit=a-b-c-d-a-g-f-e-c-a.

Note:

In a connected graph G, if the number of vertices with odd degree = 0, then Euler's circuit exists.

Sr.No.	Question	Answer
1	In a Euler's path, if the starting vertex is same as its ending vertex, then it is called……….	**Euler's circuit**
2	In a connected graph G, if the number of vertices with odd degree = 0, then Euler's circuit exists. (T/F)	**True**
3	An ________ contains each edge of 'G' exactly once and each vertex of 'G' at least once.	**Euler's path**

❖ Hamiltonian Graph

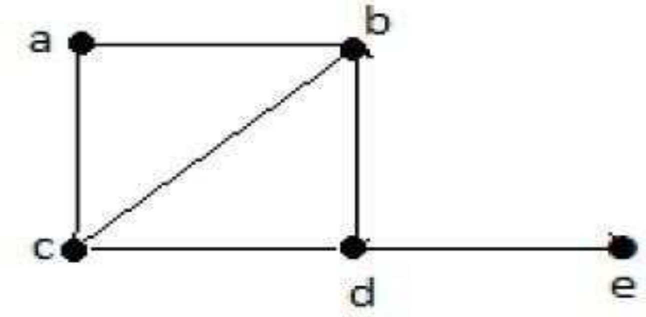
- A connected graph G is said to be a Hamiltonian graph, if there exists a cycle which contains all the vertices of G.

- Every cycle is a circuit but a circuit may contain multiple cycles. Such a cycle is called a **Hamiltonian cycle** of G.

❖ Hamiltonian Path

A connected graph is said to be Hamiltonian if it contains each vertex of G exactly once. Such a path is called a **Hamiltonian path**.

Example



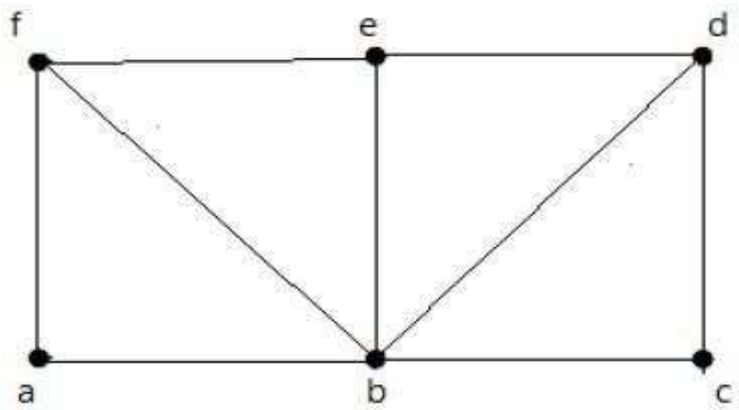
Hamiltonian Path−e-d-b-a-c.

Note

- Euler's circuit contains each edge of the graph exactly once.
- In a Hamiltonian cycle, some edges of the graph can be skipped.

Example

Take a look at the following graph:



For the graph shown above:

- Euler path exists– false
- Euler circuit exists–false
- Hamiltonian cycle exists–true
- Hamiltonian path exists–true

G has four vertices with odd degree, hence it is not traversable. By skipping the internal edges, the graph has a Hamiltonian cycle passing through all the vertices.

Sr. No.	Question	Answer
1	A connected graph G is said to be a_______, if there exists a cycle which contains all the vertices of G.	**Hamiltonian graph**
2	A _________ graph is said to be Hamiltonian if it contains each vertex of G exactly once.	**Connected**
3	Which circuit contains each edge of the graph exactly once?	**Euler's circuit**

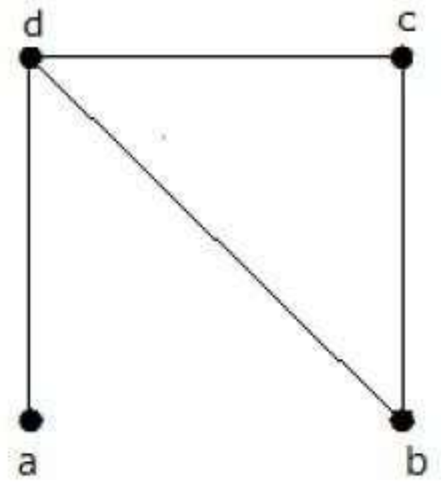
12. Graph Theory - Examples

❖ Introduction:

In this chapter, we will cover a few standard examples to demonstrate the concepts we already discussed in the earlier chapters.

Example1

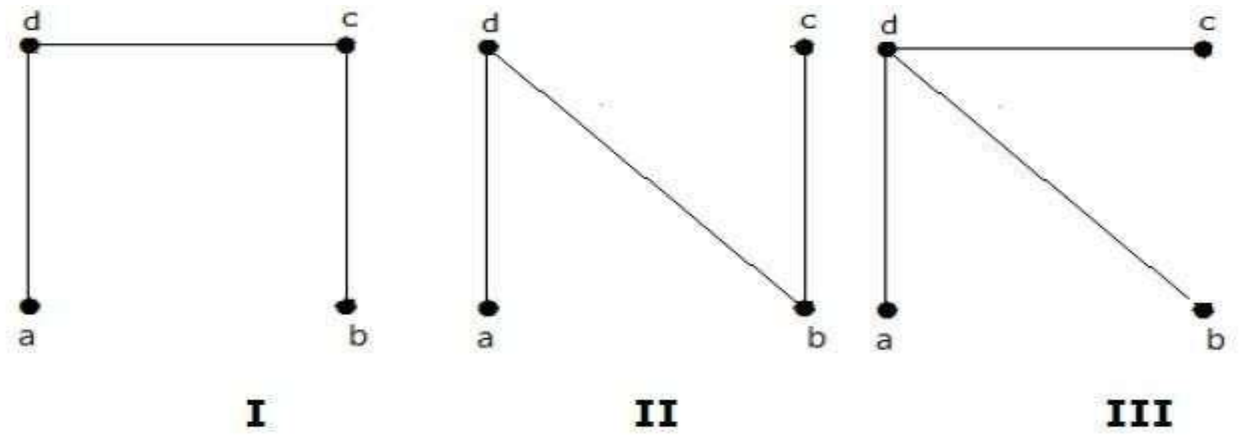
Find the number of spanning trees in the following graph.



Solution

The number of spanning trees obtained from the above graph is 3.

They are as follows:



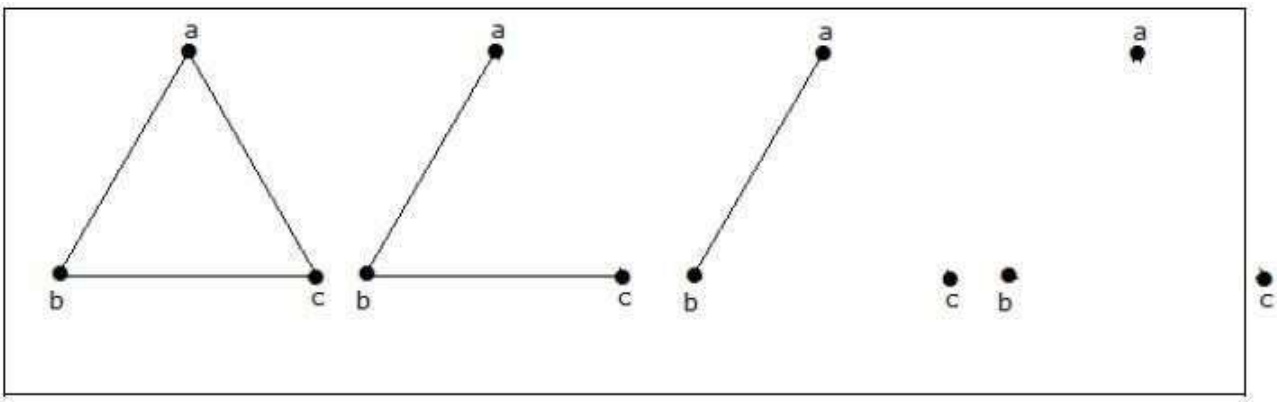
- These three are the spanning trees for the given graphs.
- Here the graphs I and II are isomorphic to each other.
- Clearly, the number of non-isomorphic spanning trees is two.

Example2

How many simple non-isomorphic graphs are possible with 3 vertices?

Solution

There are 4 non-isomorphic graphs possible with 3 vertices.
They are shown below.



Example3

Let 'G' be a connected planar graph with 20 vertices and the degree of each vertex is 3.Find the number of regions in the graph.

Solution

By the sum of degrees theorem,20∑i=1deg(Vi) =2|E|
20(3) =2|E|
|E|=30

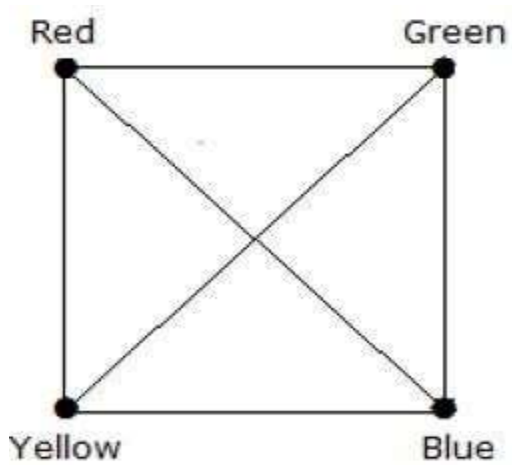
By Euler's formula,
|V| + |R| = |E| + 220+|R|=30+ 2
|R|=12
Hence, the number of regions is 12.

Example4

What is the chromatic number of

complete graph Kn?

Solution



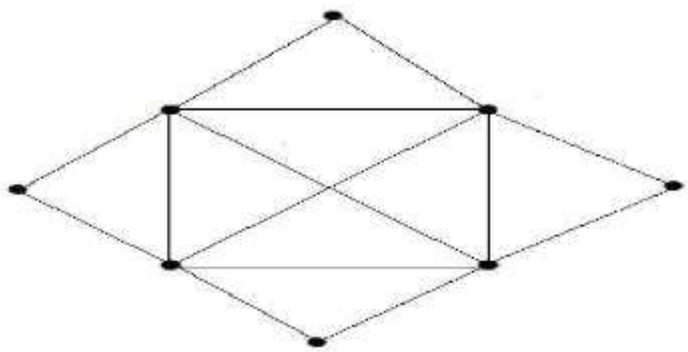
- In a complete graph, each vertex is adjacent to is remaining (n−1) vertices.
- Hence, each vertex requires a new color. Hence the chromatic number $Kn=n$.

Example5

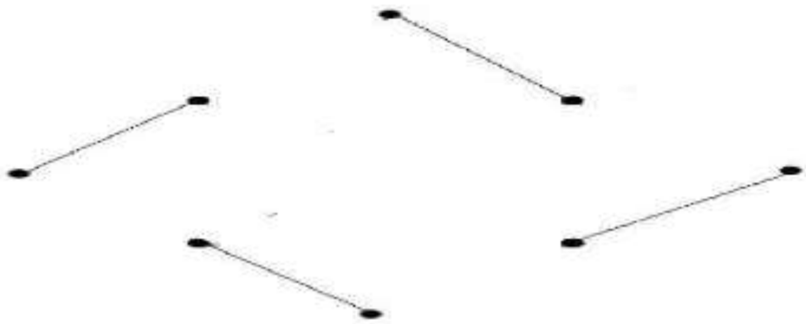
What is the matching number for the

following graph?

Solution



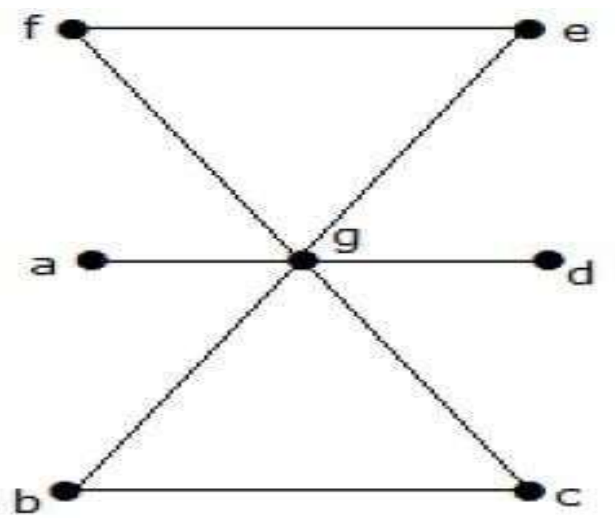
Number of vertices=9
We can match only 8 vertices.
Matching number is 4.



Example6

What is the line covering number of for the following graph?

Solution



Number of vertices =|V|=n=7
Line covering number = (α1) ≥ [n/2] = 3α1≥3
By using 3 edges, we can cover all the vertices.

Hence, the line covering number is 3