Shree H.N. Shukla Group of Colleges

M.Sc. Mathematics

Sub. Code: EMT-2001

Elec. Sub. 1 : Abstract Algebra 02

PRILIMS PAPER

[Time : 2.5 Hr] **[Total marks:70]**

Instructions : (1) Each question carries **equal** marks.

(2) All questions are **compulsory**.

1 Answer following short questions : **7×2=14**

(i) For a ring R, define R-module M and R-submodule of M.

(ii) Let M be an R-module. In Standard notation prove that $(-a)\,m = -(am) = a(-m), \forall a \in R$ and $\forall m \in M$.

(iii) Prove that $x^3 + 3x + 2 \in \mathbb{Z}_7[x]$ is an irreducible polynomial over $\mathbb{Z}_7$.

(iv) Write down the Eisenstein Criterian. Using it deduce that $f(x) = x^3 + 5x^2 + 5x + 10 \in Q[x]$. is irreducible polynomial over Q.

(v) For the field extension $\mathbb{R} | Q$, write down two elements of $\mathbb{R}$ which are algebraic over Q and also write down two elements of $\mathbb{R}$ which are transcendental elements over Q (They are not algebraic over Q).

(vi) Write down minimal polynomial of $2^{1/4} \cdot i$ over Q.

(vii) Prove or disprove $Q\left(2^{1/4}\cdot i\right)/Q$ is a finite, normal and separable extension.

2 Attempt any two : **2×7=14**

(a) Define a field extension. Let *E*/*F*, *K*/*E* both are finite field extensions. Prove that K/F is also a finite field extension with $[K : F] = [K : E]\ [E : F]$.

(b) Let *F* be a finite field. Prove that $F^{*} = F - \{0\}$ is a cyclic group under multiplication.

(c) Prove that $Q\left(\sqrt{2}, \sqrt{3}, \ldots, \sqrt{p}, \ldots\right)|Q$ is an algebraic extension, but it is not a finite field extension.

(d) Let *E*/*F*, *K*/*E* both are algebraic field extensions. Prove that *K*/*F* is also an algebraic field extension.

3 Attempt any one : **1×14=14**

(a) Let *F* be a finite field and $|F| = p^{n}$, for some prime p and $n \in \mathbb{N}$. Prove that Aut (*F*) is a cyclic group of order n.

(b) State and prove Primitive Element Theorem.

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(c) Let E/F be a finite field extension. Prove that E/F is a simple extension iff there are only finite number of subfields of E that containing F.

4 Attempt any two : **2×7=14**

(a) Let $(N_i)i \in \wedge$ be a family of R-submodules of an R-module M. Prove that $\bigcap_{i \in \wedge} N_i$ is also an R-submodule of M.

(b) Let $f : M \to N$ be an R-homomorphism of R-modules. Prove that $Ker\ f$ and $f(M)$ are R-submodules of M and N respectively.

(c) Let R be a ring with unity and M be an R-module. Prove that M is cyclic R-module iff $M \simeq R/I$ for some left ideal I of R.

(d) Let $f : M \to N$ be an onto R-homomorphism of R-modules. In standard notation deduce that $M/Ker\ f \simeq N$.

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5 Fill in the blanks appropriately : **7×2=14**

(i) Let L/K, K/E be both are finite extensions. Let $[L : K] = p$, $[K : E] = n$. What is value of $[L : E] = ...?$ $\{p + n, p^n,$ infinite, none of these$\}$

(ii) Let F be a finite field. What is number of elements of F from followings is a possible case i.e. $|F| = ...? \{49, 50, 51, 100\}$.

(iii) Which of followings is a prime field ?

$\{Q, \mathbb{R}, \overline{Q}, \mathbb{C}\}$.

(iv) Let $\omega = (-1, + \sqrt{3}\, i)/2$. Which of following is the minimal polynomial of ω over Q............ ?

$\{x^2 + x + 1,\ x^3 - 2,\ x^4 - 2,\ x^2 - 3\}$.

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(v) Which of followings is a normal extension ?

$$\left\{Q\left(2^{1/4}\right)|Q,\ Q\left(2^{1/3}\right)|\ Q,\ \bar{Q}\ |\ Q,\ \mathbb{C}|\ \mathbb{R}\right\}.$$

(vi) Which of followings is an irreducible polynomial in $\mathbb{Z}_2[x]$........ ?

$$\left\{x^2+x,\ x^2+1,\ x^2,\ x^2+x+1\right\}.$$

(vii) Which of followings is not a cyclic extension ?

$$\left\{\mathbb{C}|\ \mathbb{R},\ Q\left(\cos\frac{2\pi}{p}+i\ \sin\ \frac{2\pi}{p}\right)|\ Q,\ Q\left(2^{1/4},\ i\right)|\ Q\right\}.$$