**M.Sc. SEMESTER 1**

**Sub. Code**: **CMT-1001**

**Core Sub. 1:** **Abstract Algebra 1**

**Question Bank**

1)Let p be a prime number and n $\geq $ 1 . If a group G of order pn contains exactly one subgroup of order pi for each I = 1, 2, …., n-1 , then prove that G is cyclic .

2)Let R be a commutative ring with unity . If the only ideals of R are R and (0) , then prove that R is a field and find all the ideals of $Z$ ( the ring of integers ) .

3) Let f : R $\rightarrow $ S be a homomorphism of a ring R onto a ring S . Let N = ker f . Prove that the mapping F given by F(A) = f(A) is a bijection from the set of left ideals in R that contain N onto the set of left ideals in S .

4) State and prove first isomorphism theorem of groups.

5) For a finite group G with pn | O(G), prove that G contains a subgroup

H of order pn.

6) State and prove second isomorphism theorem of rings.

7) Prove that the product of two primitive polynomials is R[*x*] is also a

primitive polynomial, where R is UFD.

8) Define (1) a Euclidean Domain (ii) a principal ideal domain . Show that any Euclidean domain is a principal ideal domain .

9) Let R be a commutative ring with identity . Let M be an ideal of R . Prove that M is maximal ideal of R if and only if R/M is a field .

10) Define (1) a principal ideal domain (2) a Unique factorization Domain .Prove that every PID is a UFD .

11)Prove that every nonzero prime ideal in a Euclidean domain is a maximal .

12) State and prove sylow’s third theorem .

13 ) Prove that the ring of Gaussian integers is a Euclidean domain .

14) Let R be a UFD . If f(x) , g(x) $ϵ $R[x] are primitive , then prove that f(x)g(x) is also primitive .

15) Let R be a ring .Let M be an ideal of R , M$ \ne $ R .Prove that M is a maximal ideal of R if and only if M + (x) = R for any x $ϵ $R \ M where (x) is the principal ideal of R generated by x in R .

16) State and prove sylow’s first theorem .

17 ) Is the ideal ( x4 +4 ) a prime ideal in the polynomial ring $Q $[X] over the field of rational numbers ? Justify your answer .

18) Show that an ideal M in the ring of integers $Z$ is a maximal ideal if an only if M = (p) , where p is some prime .

19) Define the term content of a polynomial f(x)$ ϵ$ R[X] . For f(x) , g(x) $ϵ $R[x] , prove in usual notations that C(f g) = C(f ) C(g) . Here R is UFD .

20) State and prove sylow’s second theorem .