Shree H.N. Shukla College of Science

M. Sc (Mathematics) (Sem-1)

Prelims Test

MATH.CMT-1003: Topology-1

[Time: 2.5 Hours] [Total Marks: 70]

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1 Answer any seven 7x2=14

(a) Closure of irrational number in R is____.

(b) Limit point of (0,1)∪(2,3) is_____in R with standard topology .

(c) [0,1) is open set in _____topology on R.

(d) Prove or disprove : Arbitrary intersection of open set is open in R .

(e) If A is closed then closure of A =__ . .

(f) $\mathcal{T}$={ G ⊆ ℕ│ ℕ - G is finite }∪Ø is topology on ℕ. Give example of set which is Neither open nor closed in $\mathcal{T}$.

(g) Give example of set which locally connected but not connected in R.

2 Answer any two 2x7=14

(a) (1) Give definition of limit point of set A of topological space X.

(2) Let A= { $\frac{1}{n}$ / n ∈ ℕ } than show that A'=A∪ {0}.

(b) Suppose F1, F2, F3,..,Fn are closed sets. Prove that their union is closed.

(c) $\mathcal{T}$ = { G ⊆ ℕ│ ℕ - G is finite }∪Ø is topology on ℕ. Show that $\mathcal{T}$ be a topology on ℕ.

3

(a) Let be topology on X and Y ⊆ X then show that $\mathcal{T}_y$= { G∩Y│ G ∈ $\mathcal{T}$ } be topology on Y

(b) Prove that every component is a maximal connected set and it is closed set. 4

(c) $\mathcal{T}_\alpha$ be connected subset of X, for each $\alpha \in$ I and $\bigcap_{\alpha\in I}\mathcal{T}_\alpha \neq \emptyset$ then show that $\bigcup_{\alpha\in I}\mathcal{T}_\alpha$ is connected subset of X. 5

OR

3

(a) Suppose f : X → Y and g : Y → Z is continuous then show that gof : X → Z is continuous 4

(b) A, B ⊆ R then show that (1) $(A \cap B)^o = A^o \cap B^o$

(2) $(A \cup B)^o \neq A^o \cup B^o$. 5

(c) f : X → Y is continues on X ⇔ $f^{-1}(K)$ is closed in X whenever k is closed subset of Y. 5

4 Answer any two 2x7=14

(a) X and Y are locally connected if and only if X x Y are locally connected.

(b) Define path connected set and show that every path connected set on X is connected on X.

(c) Show that closure of Q = R. and show that $(A^o)^o = A^o$ where A ⊆ R.

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5 Answer any two 2x7=14

(a) Let X be space. (Y, d) be metric space and suppose f_n: X → (Y, d) is continuous for each n ∈ ℕ. Let f : X → Y be function such that (f_n)→f Uniformly on X. then f is continues.

(b) X is connected and locally path connected then X is path connected.

(c) X be any space and A ⊆ X , then show that

(1) $Cl_X A = A \cup A'$, where A'= limit point of A and $Cl_X A$ = closure of A

(2) $Cl_X(Cl_X A) = Cl_X A$.

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