Seat No. ________________

H-003-1164001

M. Sc. (Sem. IV) (CBCS) Examination

April - 2023

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003
Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions: (1) There are five questions.
(2) Answer all the questions.
(3) Each question carries 14 marks.

1 Answer any **seven** of the following: **7×2=14**

(1) Define (i) Singular linear transformation (ii) Regular linear transformation.

(2) Define characteristic root of a linear transformation.

(3) Define similar linear transformations and similar matrices for a finite dimensional vector space V over F.

(4) Let $T \in A_F(V)$. When the subspace W of V is invariant under T? Justify the answer with an example.

(5) Define (i) Nilpotent linear transformation (ii) Index of nilpotence of a nilpotent linear transformation.

(6) For A, B $\in F_n$, prove that tr (A + B) = tr (A) + tr (B).

(7) For A $\in \mathbb{C}_n$, prove that $(A^*)^* = A$.

(8) Define unitary liner transformation.

(9) Define bilinear form.

(10) Define homomorphism between two algebras A and B over a field F.

2 Answer any **two** of the following: **2×7=14**

(1) Let V be a finite dimensional vector space over a field F, then prove that $T \in A_F(V)$, is invertible if and only if the constant term of the minimal polynomial for T is not zero.

(2) Let $\lambda \in F$ is a characteristic root of $T \in A_F(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of q(T).

(3) Let V be a finite dimensional vector space over a field F and $T_1, T_2 \in A_F(V)$ are such that T_1 and T_2 are similar, then prove that there exists bases B_1, B_2 of V over F such that the matrix of T_1 in B_1 = the matrix of T_2 in B_2.

3 Answer the following **both** questions: **2×7=14**

(1) For A, B $\in F_n$, prove that (i) (A')' = A (ii) (A + B)' = A' + B' (iii) (AB)' = B'A'.

(2) Let V be an n-dimensional vector space over a field F and $T \in A_F(V)$. Let V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 are linear transformations induced by T on V_1 and V_2 respectively. If the minimal polynomial of T_1 over F is $p_1(x)$ while that of T_2 is $p_2(x)$, then prove that the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$.

OR

3 Answer the following both questions: **2×7=14**

(1) Let A, B $\in F_n$ are similar then prove that det A = det B.

(2) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \in F_3$ then prove that the matrix is nilpotent and find the index of nilpotence.

4 Answer the following questions: **2×7=14**

(1) Find the solution using Cramer's rule

$$x + y + z = 6$$
$$y + 3z = 11$$
$$x - 2y + z = 0$$

(2) State and prove Jacobson's lemma.

5 Answer any **two** of the following : **2×7=14**

(1) Let V be a finite dimensional vector space over a field F. If $T \in A_F(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

(2) Let V be a finite dimensional vector space over a field F and $T \in A_F(V)$. Let $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ are the distinct characteristic roots of T in F and $v_1, v_2 \ldots. v_k$ are the characteristic vectors of T belonging to $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ respectively, then prove that $v_1, v_2 \ldots. v_k$ are linearly independent over F.

(3) Let (V, < >) be a finite dimensional inner product space over $\mathbb{C}$. Let $T \in A_{\mathbb{C}}(V)$ then prove that T is unitary if and only if $\langle T(u), T(u) \rangle = \langle u, u \rangle, \forall u \in V$.

(4) Prove that the determinant of a triangular matrix is equal to the product of its all the entries of the main diagonal.

DM-003-1164001 Seat No. 4020.

M. Sc. (Sem. IV) (CBCS) Examination

March – 2022

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are total five questions.

(2) All questions are mandatory.

(3) Each question carries equal marks.

1 Answer any **seven** of the following :

(1) Define with example: Singular linear transformation.

(2) Define with example: Companion matrix.

(3) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1,x_2,x_3)=(4x_1,2x_2,3x_3)$. Justify whether 4 is a characteristic root of T or not?

(4) Define with example: Nilpotent linear transformation.

(5) Define Jordan Canonical Form.

(6) Define with example: Transpose of a matrix.

(7) Define with example: Normal linear transformation.

(8) Define with example: Matrix of a bilinear form.

(9) Define with example: Non-degenerate bilinear form.

(10) Define with example: Unitary Linear Transformation.

2 Answer any **two** of the following :

(1) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Prove that, T is regular if and only if T maps V onto V.

(2) Let V be an n-dimensional vector space over F. Prove that, $T \in A_F(V)$ is invertible if and only if $m(T)$ is has inverse in F_n.

(3) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Let $\lambda_1, \lambda_2, \ldots \lambda_k$ are the distinct characteristic roots of T in F and $v_1, v_2, \ldots v_k$ are the characteristic vectors of T beloging to $\lambda_1, \lambda_2, \ldots \lambda_k$ respectively. Prove that, $v_1, v_2, \ldots v_k$ are linearly independent over F.

3 Answer the following :

(1) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. If T is nilpotent, then prove that, $\alpha_0 Id_v + \alpha_1 T + \ldots + \alpha_m T^m$, where the $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.

(2) Let V be an n-dimensional vector space over F and $T \in A_F(V)$. Suppose $V = V_1 \oplus V_2$, where V_1 and V_2 are T-invariant subspaces of V. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2, respectively. Let $p_1(x)$ and $p_2(x)$ be the minimal polynomials of T_1 and T_2, respectively. Prove that, the minimal polynomial of T is the least common multiple of $p_1(x)$ and $p_2(x)$.

OR

(1) Let the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \in F_3$. Prove that, A is nilpotent and find the invariants of A.

(2) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Let $p(x) = x^r + \gamma_{r-1}x^{r-1} + \ldots + \gamma_1 x + \gamma_0 \in F[x]$ be the minimal polynomial of T over F. If V is cyclic $F[x]$-module, then prove that, there exists a basis B of V over F such that the matrix of T in B equals $C(p(x))$.

4 Answer the following :

(1) Let $A = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \in \mathbb{R}_3$. Determine the Jordan form of A.

(2) State and prove, Cayley-Hamilton Theorem.

5 Answer any two of the following :

(1) Let $A, B \in F_n$. Prove that, $\det(AB) = dt(A)\det(B)$.

(2) Using Cramer's rule find the solutions, in the real field, of the system of equations gioven below :

$x + y + z = 1$

$2x + 3y + 4z = 1$

$x - y - z = 0.$

(3) Let $A \in \mathbb{C}_n$ be a hermitian matrix. Prove that, any characteristic root of A must be real.

(4) Let V be an n-dimensional inner product space over $\mathbb{C}$ and $T \in A_F(V)$. Prove that,

(a) $(S+T)^* = S^* + T^*$

(b) $(\lambda S)^* = \bar{\lambda} S^*$

(c) $(ST)^* = T^* S^*.$

BBA-003-1164001 Seat No. 45130

M. Sc. (Sem. IV) Examination

July - 2021

Mathematics : CMT - 4001

(Linear Algebra)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions:

1) Attempt any five questions from the followings.

2) There are total ten questions.

3) Each question carries equal marks.

1 Answer the following seven: $7 \times 2 = 14$

1. Define with example: Algebra over a field.
2. Define with example: Homomorphism between two algebras.
3. Define with example: Invertible Linear Transformation.
4. Define with example: Minimal Polynomial.
5. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (x_1, 3x_2)$.
   Justify, whether 3 is a characteristic root of T or not?
6. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_3, x_2, 0)$.
   Justify, whether $W = \{(0,0,z): z \in \mathbb{R}\}$ is invariant under T or not?
7. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (0, x_1)$.
   Justify, whether T would be nilpotent or not?

2 Answer the following seven: **7 X 2 = 14**

1. Define with example: Cycle with respect with to a linear transformation.
2. Define Jordan form of a linear transformation.
3. Define with example: Companion matrix.
4. Define with example: Characteristic Polynomial of a linear transformation.
5. State Primary Decomposition Theorem.
6. Let A' denotes the transpose a matrix $A \in \mathbb{F}_n$. Justify, whether $(A')' = A$ or not?
7. State Cramer's Ruie.

3 Answer the following both: **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Prove that, T is invertible if and only if the constant term of the minimal polynomial is non-zero.

b) Let V be a finite dimensional vector space over $\mathbb{F}$ and $S, T \in A(V)$ with S invertible. Prove that, $r(ST) = r(TS)$.

4 Answer the following both: **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Let $\lambda \in \mathbb{F}$ be a characteristic root of T. Prove that, λ is a root of a minimal polynomial of T over $\mathbb{F}$.

b) Let V be an n-dimensional vector space over $\mathbb{F}$. Prove that, $T \in A(V)$ is invertible if and only if $m(T)$ is has inverse in $\mathbb{F}_n$.

5 Answer the following both: **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$.

Let W be a T-invariant subspace of V. Prove that, T induces a linear transformation $\bar{T}$ on V/W defined by $\bar{T}(v + W) = T(v) + W$.

Also prove that, the minimal polynomial of $\bar{T}$ divide the minimal polynomial of T.

b) Let $\mathbb{F}$ be a subfield of a field K. Let $n \in \mathbb{N}$,and $A \in \mathbb{F}_n$. Prove that, A is invertible in $\mathbb{F}_n$ if and only if A is invertible in K_n.

6 Answer the following both: **2 X 7 = 14**

a) Let V be an n-dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Suppose all the characteristic roots of T lies in $\mathbb{F}$. Prove that, T satisfies a polynomial of degree n over $\mathbb{F}$.

b) Let V be a finite dimensional vector space over $\mathbb{F}$. Let $T \in A(V)$ be nilpotent with index of nilpotence k. Let $v \in V$ be such that, $T^{k-1}(v) \neq 0$. Prove that, the vectors $v, T(v), \ldots, T^{k-1}(v)$ are linearly independent over $\mathbb{F}$.

7 Answer the following both: **2 X 7 = 14**

a) State and prove, Cayley-Hamilton Theorem.

b) Let the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{F}_3$. Prove that, A is nilpotent and find the invariants of A.

PR-003-1164001

PR-003-1164001

Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

August - 2020

Mathematics : CMT - 4001

(Linear Algebra)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.

(2) There are 5 questions.

(3) Figures to the right indicate full marks.

1 Answer any **seven** : **14**

(1) If a linear transformation T is left invertible then show that T is invertible.

(2) Let $T : P_2[x] \to P_3[x]$ be a linear transformation defined by $T(p(x)) = \int p(x)\,dx$. Find matrix of T in the standard bases.

(3) Suppose that T is a nilpotent linear transformation and $\alpha \in F$ is non-zero then prove that $\alpha I + T$ is regular.

(4) For any $A \in M_n(\mathbb{C})$ show that tr $(AA^*) \geq 0$.

(5) Prove or disprove :

(1) $tr(AB) = tr(A)tr(B)$

(2) $\det(A+B) = \det(A) + \det(B)$.

(6) Let V be an inner product space. Then show that $(S+T)^* = S^* + T^*$ and $(ST)^* = S^*T^*$.

(7) Characterise eigen values of a unitary transformation.

(8) Define : Bilinear form and non-degenerate bilinear form.

(9) Prove that any orthonormal subset of an inner product space is linearly independent.

(10) Suppose that f is a non-zero skew-symmetric bilinear form and $v, w \in V$ be such that $f(v, w) = 1$. Then show that v and w are linearly independent.

2 Attempt any **two** : **14**

(1) Prove that $p(x)$ is a minimal polynomial for T over F if and only if whenever $h(x) \in F[x]$ such that $h(T) = 0, p(x)$ divides $h(x)$.

(2) State and prove :

(1) Jacobson's lemma

(2) Polarization identity.

(3) Suppose T is nilpotent with index of nilpotence n_1 and let $v \in V$ be such that $T^{n_1-1}(v) \neq 0$. Then show that, $V_1 = L(\{v, Tv, \ldots, T^{n_1-1}v\})$ is a subspace of V with dimension n_1 and it is invariant under T. Also find the matrix of $T \mid v_1$.

3 Answer the following : **14**

(1) Prove that : A linear transformation T is invertible if and only if the constant term in the minimal polynomial for T is not 0.

(2) Prove that T is unitary if and only if T maps an orthonormal basis of V to an orthonormal basis of V.

OR

3 Answer the following : **14**

(1) Prove that T is regular if and only if $\ker(T) = \{0\}$.

(2) State and prove : Cramer's rule.

4 Answer the following : **14**

(1) Suppose V_1 and V_2 are invariant subspaces of V under T such that $V = V_1 \oplus V_2$. If $p_1(x), p_2(x) \in F[x]$ are minimal polynomial for $T|v_1$ and $T|v_2$ respectively. Then prove that minimal polynomial for T is the LCM of $p_1(x)$ and $p_2(x)$.

(2) Let $A, B \in M_n(F)$. Then prove that, $\det(AB) = \det(A)\det(B)$.

5 Attempt any **two** : **14**

(1) Prove that λ is characteristic root of T if and only if λ is a root of minimal polynomial for T.

(2) Suppose that V is a cyclic $F[x]$–module and $p(x) \in F[x]$ is the minimal polynomial for T. Then prove that there exists a basis of V over F such that matrix of T is companion matrix of $p(x)$.

(3) Show that any eigen value of a Hermitian matrix over $\mathbb{C}$ is real. Using this result deduce that if the matrix is of the form AA^* for some $A \in M_n(\mathbb{C})$, then its eigen value is non-negative.

(4) Let f be a bilinear form then prove that f is symmetric if and only if $[f]_B$ is symmetric for any basis B of V over F.

RBE-003-1164001

Seat No. 15004

M. Sc. (Mathematics) (Sem. IV) (CBCS) Examination

April / May - 2019

Maths : Linear Algebra : CMT - 4001

(Old & New Course)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer the following questions : (any seven) 7×2=14

(1) Define : (i) Algebra over a field (ii) Algebra of a linear transformation.

(2) Prove that : $tr(A+B)=tr(A)+tr(B)$, where $A \in F_n$, and $B \in F_n$.

(3) Prove that: For $S \in A_F(V)$, if $S^*S(v)=0$, then $S(v)=0$.

(4) Explain in brief Cramer's rule.

(5) Prove that for $S, T \in A_F(V), u, v \in V$,

(i) $\left(T^*\right)^* = T$

(ii) $(S+T)^* = S^* + T^*$

(6) Define: (i) Canonical form over $\mathbb{R}$ (ii) Companion matrix of $f(x)$, where $f(x) \in F[x]$.

(7) Define :(i) Non-degenerate Bilinear forms (ii) Symmetric bilinear forms.

(8) Define :(i) Nilpotent transformation (ii) Jordan block.

(9) Define :(i) Positive Semidefinite matrix (ii) Negative definite matrix.

(10) Define: (i) Normal transformation (ii) Unitary transformation.

[Contd...

2 Answer the following questions : (any **two**) **2×7=14**

(1) If α is a characteristic root of $T \in A_F(V)$, then prove that for any polynomial $q(x) \in F[x], q(\alpha)$ is a characteristic root of $q(x)$.

(2) If V is a finite dimensional vector space over a field F, then prove that $T \in A_F(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

(3) Prove that an element $\alpha \in F$ is a characteristic root of $T \in A_F(V)$ if and only if $\exists\, v \neq 0$ in the vector space V such that $T(v) = \alpha(v)$.

(4) For $T \in A_F(V), \forall\, v \in V$, if $\langle T(v), T(v) \rangle = \langle v, v \rangle$, then prove that T is unitary.

3 Answer the following questions : **2×7=14**

(a) For a normal transformation N, if $N(v) = 0$, then prove that $N^*(v) = 0$.

(b) Find the real symmetric matrix of $x_1^2 + 4x_2^2 + 2x_3^2 + 6x_1x_2 + 14x_2x_3$.

OR

(a) Prove that T is unitary if and only if $T^*T = I$.

(b) If V is a finite dimensional vector space over a field F, and if $T \in A_F(V)$ is right invertible, then prove that T is invertible.

4 Answer the following questions : (any **two**) **2×7=14**

(1) State and prove Sylvester's law of Inertia.

(2) For $T \in A_F(V)$, prove that if $\langle T(v), T(v) \rangle = 0 \; \forall\, v \in V$, then $T = 0$.

(3) (i) State and prove the Polarization Identity.

(ii) State and prove the Jacobson lemma.

5 Answer the following questions : (any two) 2×7=14

(1) Let N be a normal transformation and α and β be the distinct characteristic roots of N. If $u, v \in V$ are the corresponding characteristic vectors, then show that $\langle u, v \rangle = 0$.

(2) Let α be a basis for a finite dimensional vector space over a field k and if $\dim(V) = n$, then, prove that any bilinear form f on V is determined by the matrix $[f]_\alpha$. Moreover, show that for $v, w \in V$, $f(v, w) = [v]_\alpha^{t} [f]_\alpha [w]_\alpha$.

(3) Find the solution using Cramer's rule.

$2x + 5y + 7z = 12$

$3x + 4y + 9z = 13$

$x + y + z = 19$

(4) Prove that for real symmetric matrices, congruence is an equivalence relation.

(5) For $T \in A_F(V)$, if T is Hermitian, then prove that all its characteristic roots are real.

* M B S - 0 0 3 - 1 1 6 4 0 0 1 - 1 0 0 - 3 *

MBS-003-1164001 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2018

Mathematics : MATH.CMT - 4001

(Linear Algebra)

(New Course)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions

(2) Each question carries 14 marks.

(3) Vector spaces considered here are finite-dimensional

1 Answer any **seven** : **7×2=14**

(a) When is an element $T \in A_F(V)$ said to be invertible? If $T \in A_{\mathbb{R}}\left(\mathbb{R}^{(7)}\right)$ is invertible, then find $r(T)$.

(b) Why does there exist no vector space V over $\mathbb{R}$ such that $\dim_{\mathbb{R}} A_{\mathbb{R}}(V) = 85$?

(c) Let $T \in A_F(V)$ and let $p(x)$ be the minimal polynomial of T over F. If $\lambda \in F$ is a characteristic root of T, then show that $p(\lambda) = 0$.

(d) When are $T, S \in A_F(V)$ said to be similar?

(e) Let $T : \mathbb{Q}^{(3)} \to \mathbb{Q}^{(3)}$ be defined by $T(1,0,0) = (0,1,0)$, $T(0,1,0) = (0,0,1), T(0,0,1) = (0,0,0)$ and extend T linearly to the whole of $\mathbb{Q}^{(3)}$. Verify that T is nilpotent and find the index of nilpotence of T.

(f) Let $A \in \mathbb{R}_5$. When is A said to be a basic Jordan block belonging to $\sqrt{13}$?

(g) State Cramer's rule.

(h) Let $(V,<,>)$ be an inner product space over $\mathbb{C}$. Let $N \in A_{\mathbb{C}}(V)$ be normal. If u $v \in KerN$, then show that $v \in Ker\, N^*$.

(i) Let $(V,<,>)$ be as in (h). If $T \in A_{\mathbb{C}}(V)$ is Hermitian, then show that $< T(v), v > \in \mathbb{R}$ for any $v \in V$.

(j) State the polarization identity.

2 Answer any **Two** : **7×2=14**

(a) (i) Let $T \in A_F(V)$. Prove that T satisfies a nontrivial polynomial $q(x) \in F[x]$.

(ii) If $T \in A_F(V)$ is invertible, then show that T^{-1} is a polynomial expression in T over F.

(b) If V is a n–dimensional vector space over a field F, then prove that $A_F(V)$ and F_n are isomorphic as algebras over F.

(c) Let $T, S \in A_F(V)$. If S is regular, then show that T and STS^{-1} have the same minimal polynomial.

3 (a) If n_1 is the index of nilpotence of a nilpotent $T \in A_F(V)$ and if $v \in V$ is such that $T^{n_1 - 1}(v) \neq 0$, then prove that $\{v, T(v) ..... T^{n_1 - 1}(v)\}$ is linearly independent over F. **5**

(b) Let $V = V_1 \oplus V_2$, where V_1 and V_2 are invariant under $T \in A_F(V)$. If $p_i(x) \in F[x]$ is the minimal polynomial of $T|V_i$ for each $i \in \{1,2\}$, then show that the minimal polynomial of T over F is the least common multiple of $p_1(x)$ and $p_2(x)$. **5**

(c) Let $A, B \in F_n$. Show that $tr(AB) = tr(BA)$. **4**

OR

3 (a) Let $T, S \in A_F(V)$ be similar. Show that given a basis B_1 of V over F, there exists a basis B_2 of V over F such that the matrix of T in B_1 equals the matrix of S in B_2. **5**

(b) Prove that any $T \in A_F(V)$ satisfies its characteristic polynomial. **5**

(c) Let $A \in \mathbb{C}_n$ be Hermitian. Show that any characteristic root of A is real. **4**

4 Answer any **Two** : **7×2=14**

(a) If $\dim_F(V) = n$ and if $T \in A_F(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

(b) Let $A \in F_n$. Show that $\det(A) = \det(A')$.

(c) Let $A \in F_n$ and suppose that K is the splitting field of the minimal polynomial of A over F. Show that there is an invertible matrix $C \in K_n$ such that CAC^{-1} is in Jordan form.

5 Answer any **Two** : **7×2=14**

(a) Let $T \in A_F(V)$. If V is cyclic relative to T, then prove that there exists a basis B of V over F such that the matrix of T in B is $C(p(x))$, where $p(x)$ is the minimal polynomial of T over F.

(b) Let $(V, <,>)$ be an inner product space over $\mathbb{C}$. Let $T \in A_{\mathbb{C}}(V)$. Show that T is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

(c) Let V be a vector space over $\mathbb{R}$ and let f be a symmetric bilinear form on V. Prove that there is a basis B of V that the matrix of f in B is diagonal.

(d) Let $n \geq 1$. Show that the mapping $f : F_n \to F_n$ defined by $f(A) = A'$ is an adjoint of F_n.

NBU-003-016409 Seat No. ______

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003
Subject Code : 016409

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1 Answer any **seven** : **7×2=14**

(i) Define similar linear transformations and similar matrices for a finite dimensional vector space V over F.

(ii) Define an algebra over a field F. Also give an example of an algebra over the field F.

(iii) Let $T \in A_F(V)$ and $p(x) \in F[x]-\{0\}$. When $p(x)$ becomes a minimal polynomial of T over F ?

(iv) Let $T_1 S \in A_F(V)$. In standard notation verify that $r(ST) \le r(T)$.

(v) Define homomorphism between two algebras A_1, A_2 over a field F.

(vi) Let $A, B \in F_n$. In standard notation verify that $tr(A+B) = tr(A) + tr(B)$, where $tr: F_n \to F$ and it is trace of the matrix.

(vii) Let $T \in A_F(V)$. When T is said to a nilpotent element of $A_F(V)$?

(viii) Let $T \in A_F(V)$ and T is a nilpotent linear transformation. Define index on nilpotence for T.

(ix) Let $A \in \not\subset n$. When A is called Hermitian ? When A is called skew Hermitian ?

(x) Let $A \in F_n$ and A is invertible. Prove that $\det\left(A^{-1}\right) = \frac{1}{\det A}$.

2 Answer any **two** : **2×7=14**

(a) Let $T \in A_F(V)$ and T is invertible. Prove that T^{-1} has polynomial expression in T over F. i.e. there is $f(x) \in F[x]$ such that $T^{-1} = f(T)$.

(b) Let λ be a characteristic root of T, where $T \in A_F(V)$. Prove that $p(\lambda) = 0$, where $p(x) \in F[x]$ and it is a minimal polynomial of T over F.

(c) Let $T_1, T_2 \in A_F(V)$ and they both have same invariants. Prove that they are Similar.

(d) Let $A \in F_n$. Prove that the interchanging two rows of A change the sign of its determinant.

3 Answer any **one** : **1×14=14**

(a) Let $tr\left(T^k\right) = 0, \ \forall k \in \mathbb{N}$. Prove that T is nilpotent.

(b) Let $A, B \in F_n$. Prove that $\det(AB) = \det(A) \cdot \det(B)$.

(c) Let $\dim_F V = n$ and $L(V, V, F)$ is the vector space consisting, all the bilinear forms on V. Prove that $\dim_F\left(L(V,V,F)\right) = n^2$.

4 Answer any two : **2×7=14**

(a) State and prove - Cayley-Hamilton theorem.

(b) State and prove - Jacobson Lemma.

(c) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Prove that T is invertible iff the constant term of any minimal polynomial of T over F is non-zero term.

(d) Prove that the determinant of triangular matrix is equal to the product of its all the entries of the main diagonal.

5 Answer any two : **2×7=14**

(a) Let V be a finite dimensional vector space over F. Let $T \in A_V(F)$ and $p(x) \in F[x]$ be a minimal polynomial of T over F. Let $f(x) \in F[x] - \{0\}$ be such that $f(T)$ is a zero mapping. Prove that ${}^{p(x)}\!/_{f(x)}$ in $F[x]$.

(b) Let V be a finite dimensional vector space over F. Let $T \in A_V(F)$. Prove that there is a unique monic polynomial $g(x) \in F[x]$ of least degree such that $g(T)$ is a zero mapping.

(c) Let F be a field with char $F = 0$. Let $A = (\alpha_{ij})$, $B = (B_{ij}) \in F_n$. Let $\alpha_{ij} = 1$, $\forall\, i, j \in \{1, 2, \ldots.., n\}$ and $\beta_{11} = n, \beta_{ij} = 0$, $\forall\, i, j \in \{1, 2, \ldots.., n\}$ and $i \neq 1$ or $j \neq 1$. Verify that A and B are similar.

(d) Solve by Cramer's rule :

$$x_1 + 2x_2 + 3x_3 = -5,$$

$$2x_1 + x_2 + x_3 = -7 \text{ and}$$

$$x_1 + x_2 + x_3 = 0.$$

DLL-003-016407 Seat No. ________

M. Sc. (Mathematics) (Sem. IV) (CBCS) Examination

May / June – 2015

Maths CMT - 4001 : Linear Algebra

(New Course)

Faculty Code : 003

Subject Code : 016407

Time : Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.
(3) Unless otherwise specified, vector spaces consdiered here are finite - dimensional.

1. Answer any **Seven** $7 \times 2 = 14$

(a) Define an algebra over a field F and illustrate it with an example.

(b) When is an element $\lambda \in F$ said to be a characteristic root of $T \in A_F(V)$? Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be given by $T(1,0) = (1,1)$ and $T(0,1) = (1,-1)$. Determine the characteristic roots of T.

(c) Let $n \geq 1$ and $A \in F_n$. Define $tr(A)$. Verify that the map $tr : F_n \to F$ is linear.

(d) State Cramer's rule.

(e) Let $(V, <, >)$ be an inner product space over $\mathbf{C}$. If a linear transformation $T : V \to V$ is unitary, then prove that $T^*T = Id_V$.

(f) Let $T \in A_F(V)$. When is a subspace W of V said to be invariant under T? Verify that $T^2(V)$ is a subspace of V and is invariant under T.

(g) Let $T \in A_F(V)$ be nilpotent. Define the concept of the invariants of T.

(h) Let V be a vector space over $\mathbf{R}$. Let $f : V \times V \to \mathbf{R}$ be bilinear. When is f said to be skew-symmetric? If f is skew-symmetric, then show that $f(v,v) = 0$ for any $v \in V$.

(i) Let V be a vector space over a field F and let $\mathcal{B} = \{v_1, \ldots, v_n\}$ be a basis of V over F. Let $T \in A_F(V)$. Define $[T]_{\mathcal{B}}$, the matrix of T in $\mathcal{B}$. For any $T, S \in A_F(V)$, verify that $[T+S]_{\mathcal{B}} = [T]_{\mathcal{B}} + [S]_{\mathcal{B}}$.

(j) Let $n \geq 1$ and $A \in F_n$. Define the secular equation of A. Let $A \in F_3$ be any diagonal matrix. Determine the secular equation of A.

2. Answer any **Two** $2 \times 7 = 14$

(a) Let V be a finite-dimensional vector space over a field F. Let $T \in A_F(V)$. Prove the following:

(i) If T is invertible, then T^{-1} is a polynomial expression in T over F.

(*ii*) If T is singular, then there exists a nonzero $S \in A_F(V)$ such that $ST = TS =$ zero map from V into V.
(*b*) If A is an algebra, with unit element, over a field F, then prove that A is isomorphic to a subalgebra of $A_F(V)$ for some vector space V over F.
(*c*) If $T \in A_F(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

3. (*a*) Let $\lambda \in F$ be a characteristic root of $T \in A_F(V)$. Prove that for any polynomial $q(X) \in F[X]$, $q(\lambda)$ is a characteristic root of $q(T)$. 5
(*b*) If V is finite-dimensional over F and if $T \in A_F(V)$ is onto, then prove that T is regular. 5
(*c*) If $T \in A_F(V)$, then prove that $tr(T)$ is the sum of the characteristic roots of T (using each characteristic root as often as its multiplicity). 4

OR

3. (*a*) Let $A \in F_n$. If two rows of A are equal, then prove that $det A = 0$. 5
(*b*) Let $(V, <,>)$ be a finite-dimensional inner product space over $\mathbf{C}$. Let $T \in A_{\mathbf{C}}(V)$. Then given $v \in V$, prove that there exists an element $w \in V$, depending on v and T such that $< T(u), v >=< u, w >$ for all $u \in V$. 5
(*c*) Let V be as in (*b*). Let $T, S \in A_{\mathbf{C}}(V)$. Prove that $(ST)^* = T^*S^*$. 4.

4. Answer any **Two** $2 \times 7 = 14$
(*a*) Let $T \in A_F(V)$ be nilpotent and let n_1 be its index of nilpotence. Let $v \in V$ be such that $T^{n_1-1}(v) \neq 0$. Let V_1 be the subspace of V spanned by the vectors $v, T(v), \ldots, T^{n_1-1}(v)$. Then prove the following:
(*i*) $dim_F(V_1) = n_1$.
(*ii*) If $u \in V_1$ is such that $T^{n_1-k}(u) = 0$ for some k with $0 < k \leq n_1$, then $u = T^k(u_0)$ for some $u_0 \in V_1$.
(*b*) Let $(V, <,>)$ be a finite-dimensional inner product space over $\mathbf{C}$. Let $T \in A_{\mathbf{C}}(V)$. Prove that T is unitary if and only if T maps an orthonormal basis of V into an orthonormal basis of V.
(*c*) Let $f : V \times V \to F$ be a bilinear form on an n-dimensional vector space V over F. If $\mathcal{B}$, $\mathcal{B}'$ are any two basis of V over F, then prove that there exists an invertible matrix $C \in F_n$ such that $[f]_{\mathcal{B}'} = C[f]_{\mathcal{B}}C'$.

5. Answer any **Two** $2 \times 7 = 14$
(*a*) Prove that any $A \in F_n$ satisfies its secular equation.
(*b*) Let $(V, <,>)$ be as in 4(*b*). If $N \in A_{\mathbf{C}}$ is normal, then prove that there exists an orthonormal basis of V consisting of characteristic vectors of N, in which the matrix of N is diagonal.
(*c*) Let V be a finite-dimensional vector space over a field F and let $T \in A_F(V)$. Prove that T is invertible if and only if the constant term of the minimal polynomial for T over F is nonzero.
(*d*) (*i*) Let $A, B \in F_n$. Prove $tr(AB) = tr(BA)$.
(*ii*) Let F be a field of characteristic 0. If $T, S \in A_F(V)$ are such that $ST - TS$ commutes with S, then prove that $ST - TS$ is nilpotent.

HA-003-1164002

Seat No. ____________

HA-003-1164002

M. Sc. (Sem. IV) Examination

April - 2023

Mathematics : CMT-4002

(Integration Theory)

Faculty Code : 003

Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions:
(1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries equal marks.

1 Answer any seven of the following: **7×2=14**

(1) Define with example: Measure on measurable space.

(2) Define Positive and Negative set with respect to a signed measure.

(3) Show that, every measurable subset of a negative set with respect to signed measure on measurable space is a negative set.

(4) If $(A_i, B_i), i = 1, 2$ are Hahn-decomposition of X with respect to signed measure μ then show that, $A_1 \Delta A_2$ is null sets with respect to μ.

(5) Define with example: A measure absolutely continuous with respect to another measure.

(6) Let γ_1, γ_2 and μ be measures on measurable space $(X, \mathcal{A})$ and $\gamma_1 \perp \mu$, $\gamma_2 \perp \mu$ then show that, $c_1\gamma_1 + c_2\gamma_2 \perp \mu$.

(7) Define with example: Complete measure.

(8) State, Riesz-Representation Theorem for bounded linear functional on $L^p(\mu)$.

(9) State, Fubini's Theorem without proof.

(10) Define Baire sets in a locally compact Hausdorff space.

2 Answer any two of the following: **2×7=14**

(1) Let μ_1, μ_2 be two measures on measurable space $(X, \mathcal{A})$ such that at least one of them is finite then show that, $\mu : \mathcal{A} \to [-\infty, \infty]$ defined by $\mu = \mu_1 - \mu_2$ is a signed measure on $(X, \mathcal{A})$.

(2) State and prove, Hahn Decomposition Theorem.

(3) Let $(X, \mathcal{A})$ be a measurable space, $D \subseteq \mathbb{R}$ be dense in $\mathbb{R}$ and $B_\alpha, \alpha \in D$ be measurable such that $B_\alpha \subset B_\beta, \forall \alpha, \beta \in D$ with $\alpha < \beta$ then show that, there exists a unique measurable function $f : X \to [-\infty, \infty]$ such that $f(x) \leq \alpha, \forall x \in \beta_\alpha$ and $f(x) \geq \alpha, \forall x \in X \setminus \beta_\alpha, \forall \alpha \in D$.

3 Answer the following: **2×7=14**

(1) State and prove, Lebesgue decomposition theorem.

(2) State and prove, Radon-Nikodym theorem for signed measures.

OR

3 Answer the following : **2×7=14**

(1) State and prove, Caratheodory Extension Theorem.

(2) Let $(X, \mathcal{A}, \mu)$ be a complete measure space and $1 \leq p < \infty$. Show that, S is dense in $L^p(\mu)$. Where

$S = \{s / s \text{ is simple measurable on } X \ \& \ \mu(\{x \in X / s(x) \neq 0\}) < \infty\}$

4 Answer the following: **2×7=14**

(1) Let $F:\mathbb{R}\to\mathbb{R}$ be monotonically increasing and continuous on the right and $(a,b]\subset\bigcup_{n=1}^{\infty}(a_n,b_n]$ then show that,

$$F(b)-F(a)\le\sum_{n=1}^{\infty}(F(b_n)-F(a_n))$$

Where $F(-\infty)=\lim_{x\to-\infty}F(x)$ and $F(\infty)=\lim_{x\to\infty}F(x)$.

(2) Let μ be a measure on an algebra A of subset of a set X and μ^* be the outer measure on X induced by μ then prove that, every element $E\in\mathcal{A}$ is μ^* measurable.

5 Answer any two of the following. **2×7=14**

(1) Let $(X\times Y, F, \mu\times\gamma)$ be the product measure space of two σ-finite complete measure spaces. $\Re$ be the semi algebra of measurable rectangles in $X\times Y$, $E\in\Re_{\sigma\delta}$ and $x\in X$ then show that, E_x is a measurable subset of Y.

(2) Describe by example that the hypothesis "the non negativity of f" in Tonnelli's theorem can not be dropped.

(3) Let X be a locally compact Hausdorff space. Then show that, $C_c(x)=\{f:X\to\mathbb{R}\,/\,f$ is continuous and *Supp* f is compact in $X\}$ is vector space over $\mathbb{R}$ with respect to point wise addition and scalar multiplication.

(4) Describe by example that Hahn decomposition of X is not unique with respect to a signed measure on $(X, \mathcal{A})$.

NBV-003-016402 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT-4002

(Integration Theory)

Faculty Code : 003

Subject Code : 016402

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Answer all questions.

(2) Each question carries **14** marks.

1 Answer any **seven** questions. Each question carries two marks : **2×7=14**

(i) Define σ-finite measure on a measurable space (X, A) and give an example of a σ-finite measure.

(ii) If μ is the Lebesgue measure on $[0,1]$ and v is the atomic measure on [0, 1] concentrated at $\frac{1}{2}$ then $\left(0,\frac{1}{2}\right)$ is a ________ set w.r.t. $\mu-\gamma$. Justify.

(iii) Define complete measure on a measurable space and give an example of a complete measure.

(iv) If γ is a signed measure on $(X, \mathscr{A})$ then $A \in \mathscr{A}$ is a null set w.r.t. γ iff $|\gamma|(E) =$ ________. Justify.

(v) True or False ? Justify. If γ is a signed measure on $(X, \mathscr{A}), A \in \mathscr{A}$ and $\gamma(A) = 0$ then A is null set w.r.t. γ.

(vi) Define μ^*-measurable subset of a set x with outer measure μ^*. Prove that every $E \subset X$ with $\mu^* E = 0$ is μ^*-measurable.

(vii) Prove that the product of two σ-finite complete measures is σ-finite.

(viii) If $f \in L^1(X, \mathcal{A}, \mu)$ then prove that $f(x)$ is finite are $x \in X$.

(ix) If x is a to topological space and $E \subset X$ then find supp (x_E).

(x) Give an example of a space which is locally compact but not compact. Justify.

2 Answer any **two** questions : **2×7=14**

(a) State and prove Hahn decomposition theorem.

(b) Define Jordan decomposition of a signed measure on a measurable space and prove that it is unique.

(c) If μ_1, μ_2 are two measures on a measurable space $(X, \mathcal{A})$ and at least one of them is finite then prove that $\mu_1 - \mu_2$ is a signed measure on $(X, \mathcal{A})$.

3 (a) Define the concept of a measure absolutely continuous w.r.t. another measure. If γ is the lebesgue measure on $\mathbb{R}$ and γ is the counting measure on $\mathbb{R}$ then prove that $\gamma << \mu$. If $\mu << \gamma$? Justify. **7**

(b) Define mutually singular measures on a measurable space $(X, \mathcal{A})$ and give an example of mutually singular measures. **7**

OR

3 (c) If μ is the counting measure on a countable set X then prove that $L^p(\mu) \cong l^p, \forall\, 1 \leq p \leq \infty$. **7**

(d) State, without proof, caratheodary extension theorem. Prove that the assumption "μ is σ-finite" in the theorem can not be dropped. **7**

4 Answer any **two** questions : **2×7=14**

(a) Define Baire measure on the real line. Prove that the cumulative distribution function "F" of a finite signed measure on the real line is bdd, monotonically increasing and $\lim_{x \to -\infty} f(x) = 0$.

(b) If μ is a measure on an algebra $\mathcal{A}$ of subsets of a set X and μ^* is the outer measure on X induced by μ then prove that every $E \in \mathcal{A}$ is μ^*-measurable.

(c) Give an example of a compact Hausdorff space X

$st\, Ba(X) \subsetneqq Bo(X)$.

5 Answer any **two** questions : **2×7=14**

(a) If X is a locally compact separable metric space then prove that $Bo(X) = Ba(X)$.

(b) State, without proofs, Fubini's theorem and Tonelli's theorem.

(c) Define σ-compact set in a locally compact Hausdorff space. Prove that every σ-compact open set in a locally compact Hausdorff space is a Baire set.

(d) Give an example of baire measure on a locally compact Hausdorff space which is not regular. Justify.

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MBT-003-1164002 Seat No. ________

M. Sc. (Mathematics) (Sem. IV) (CBCS) Examination

April / May - 2018

Maths : Integration Theory (CMT - 4002)

(New Course)

Faculty Code : 003
Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks

1 Answer the following : **7×2=14**

(a) __________ is a lower semi continues on a topological space X.

(b) Define complete measure on a measurable space and give an example of a complete measure.

(c) True or False? Justify. If γ is a signed measure on $(X, A,)$, $A \in A$ and $\gamma(A) = 0$ then A is the null set w.r.t γ.

(d) The lebesuge measure on $\mathbb{R}$ is __________.

(e) The cumulative function F of a finite barie measure on the real line is __________.

(f) If (X, A, μ) is a complete measure space then {s / s is simple measurable on X and $\mu\{x \in X / s(x) \neq 0\} < \infty\}$ is dense in __________.

(g) Every closed set in a metric set is a __________.

2 Answer any **two** : **2×7=14**

(a) Define signed measure on a measurable space. If μ_1, μ_2 are two signed measures on a measurable spaces (X, A) then state and prove the condition under which $\mu_1 - \mu_2$ is a signed measures on (X, A).

(b) Define positive set w.r.t. a signed measure. Prove that the countable union of positive sets is positive.

(c) State and prove Lebesgue decomposition theorem for a σ–finite measure w.r.t. another σ–finite measure on a measurable space.

3 Answer the following : **2×7=14**

(a) State, without proof, Hahn decomposition theorem. Is Hahn decomposition is unique? Justify.

(b) Prove that if (X, A) is a measurable space and $f : X \to [0, \infty]$ be measurable then there exists a sequence $\{S_n\}_{n=1}^{\infty}$ of simple measurable function such that

(i) $0 \le S_1 \le S_2 \le \cdots .... \le S_n ...... \le f$; on X.

(ii) $\lim_{n \to \infty} S_n = f(x); \forall x \in X$.

OR

(a) State without proof, Jordan decomposition theorem. Is Jordan decomposition is unique? Justify.

(b) Prove that if X be a countable set and μ be the counting measure then $L^P(\mu) \cong l^P; \forall\, 1 \le P < \infty$.

4 Answer any **two** : **2×7=14**

(a) State, without proof, Caratheodary extension theorem. Give an example to show that σ–finite assumption in the theorem cannot be dropped.

(b) State, without proof, Fubim's theorem and Tonelli's theorem.

(c) If μ, γ are measures on a measurable space then with usual notation prove that $\mu << \gamma$ and $\mu \perp \gamma \Rightarrow \mu = 0$. Does $\mu << \gamma \Rightarrow \gamma << \mu$? Justify.

5 Answer any **two** : **2×7=14**

(a) (i) True or False? Justify.

μ is outer regular $\Rightarrow \mu$ is inner regular.

(ii) Define G_δ sets and F_σ sets.

(b) Define :

(i) a locally compact and

(ii) a hausdorff space. Is the set of rationals in $\mathbb{R}$ is locally compact ?

(c) Let X is a locally compact hausdroff space. Prove that Ba(X) = the σ–algebra generated by compact G_δ sets in X.

(d) Define σ–bdd set in a locally compact hausdorff space X. If $E \in Ba(X)$ then prove that either E or X \ E is σ–bdd.

04500
RBF-003-1164002 Seat No. ____

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2019

Mathematics : CMT - 4002

(Integration Theory)

Faculty Code : 003

Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer any **seven** questions : 7×2=14

(i) Prove that every measurable subset of a negative set is negative.

(ii) Give the example of measure zero set need not be a null set with required justification.

(iii) Define: L^P - space.

(iv) State only the statement of Radon - Nikodym Theorem for signed measure.

(v) Give only statement of Monotone Convergence Theorem.

(vi) Define: Outer measure on set X.

(vii) Give only statement of Fubini's Theorem.

(viii) If μ^* is an outer measure on a set X and $\beta \in P(X)$ be such that $\mu^*\beta = 0$ then prove that β is μ^* measurable.

(ix) Give the example of locally compact space which is not compact with required justification.

(x) Define: Support of a real valued function on a topological space with an example.

2 Answer any **two** questions : 2×7=14

(a) Define : σ-algebra of subset of a set X. If X is any set then prove that $\mu : P(X) \rightarrow [0, \infty]$ defined by.

$$\mu(A) = \begin{cases} \text{number of element} & \text{if } A \in P(X) \text{ is finite} \\ \infty & \text{if } A \in P(X) \text{ is infinite} \end{cases}$$

is a measure on $(X, P(X))$.

(b) Define: Hahn Decomposition. Prove that Hahn Decomposition is unique except for the null set.

(c) Define positive set with respect to signed measure. Prove that the countable union of positive sets is positive.

3 Answer any **one** question : 1×14=14

(i) (a) Let M be the σ-algebra of all Lebesgue measurable subsets of $\mathbb{R}, \mu$ be the Lebesgue measure on $(\mathbb{R}, M)$ and f be a non-negative Lebesgue measurable function on $\mathbb{R}$. Prove that $\gamma : M \rightarrow [0, \infty]$ defined by $\gamma(E) = \int_E f d\mu, \forall\, E \in M$ is a measure on $(\mathbb{R}, M)$.

(b) Give an example of signed measure μ on a measurable space (X, A) such that Hahn-Decomposition of (X, A) with respect to μ is not unique.

(ii) (a) State and prove Lebesgue Decomposition Theorem for σ-finite measure with respect to another σ-finite measure on a measurable space.

(b) If μ_1, μ_2 are two measures on a measurable space (X, A) and at least one of them is finite then prove that $\mu_1 - \mu_2$ is a signed measure on (X, A).

4 Answer any **two** questions : 2×7=14

(a) Define: Measure absolutely continuous with respect to another measure and mutually singular measures. If (X, A) is a measurable space and γ, μ are signed measures on (X, A), $\gamma \perp \mu, \gamma \ll \mu$ then prove that $\gamma = 0$.

(b) If X is a countable set and μ is the counting measure on $(X, P(X))$. Then prove that $L^p(\mu) \cong l^p, 1 \le p \le \infty \gamma$.

(c) Define: Jordan Decomposition. Prove the uniqueness of Jordan Decomposition of signed measure.

5 Answer any **two** questions : 2×7=14

(a) If μ^* is an outer measure on a set X and $B = \{E \subseteq X / E \text{ is } \mu^* - \text{measurable}\}$. Then prove that B is σ - algebra of subset of X.

(b) Let μ be a measure on (X, A). Then prove that $\mu^* : P(X) \to [0, \infty]$ defined by $\mu^*(E) = \inf\left\{\sum_{n=1}^{\infty} \mu(B_n) / \{B_n\}_{n=1}^{\infty} \subseteq A \text{ such that } E \subseteq \cup_{n=1}^{\infty} B_n\right\}$ is outer measure on X induced by μ and $\mu^*(E) = \mu(E), \forall E \in A$.

(c) If X is a locally compact separable metric space then prove that $B_0(X) \subseteq B_a(X)$.

(d) Define: Upper and Lower semi-continuous function. If X is a topological space then prove that the function $f : X \to [-\infty, \infty]$ is continuous if and only if $f : X \to [-\infty, \infty]$ is upper semi-continuous and lower semi-continuous.

PS-003-1164002

Seat No. ______

M. Sc. (Mathematics) (Sem. IV) Examination

August - 2020

CMT-4002 : Integration Theory

Faculty Code : 003
Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each question carry 14 marks.
(3) Figures on the right indicates marks.

1 Answer any seven questions : **7×2=14**

(1) Give only statements of Fatou's lemma.

(2) Define the simple function and also write its canonical representation.

(3) State only the statement of Radon-Nikydom Theorem for signed measure space.

(4) Give only statement of Fubini's Theorem.

(S) Write the statement of Uryson's lemma.

(6) Write the definition of Borel σ-algebra on $\mathbb{R}$.

(7) Write the definition of locally compact space.

(8) Define the measurable function also write one example of measurable function.

(9) Write the definition of positive set and negative set for a signed measure space.

(10) Define the word saturated measure space and also write one example of saturated space.

2 Answer any **two** questions : **2×7=14**

(a) Let f and g be nonnegative measurable function on a measurable space (X, A, μ) then $\int f + g d\mu = \int f d\mu + \int g d\mu$ for every measurable subset E of X.

(b) Let $\mu_1, \mu_2, ..., \mu_k$ be measure on (X, A) and let $\alpha_1, \alpha_2, ..., \alpha_k$ be nonnegative real numbers then $\alpha_1\mu_1 + \alpha_2\mu_2 + ... + \alpha_k\mu_k$ is a measure on (X, A).

(c) State and Prove the Lebesgue Dominated Convergence Theorem.

3 Answer the following both. **14**

(a) State and Prove Hahn-Decomposition also if X is any nonempty set and $v = \delta_{x_0} - \eta$ defined on $P(X)$, Where $x_0 \in X$ and η is the counting measure, then find Hahn-Decomposition of v.

(b) State and Prove Monotone Convergence Theorem.

OR

3 Answer the following both : **14**

(a) Let (X, A, μ) be a σ-finite measure space and let v be a finite signed measure on (X, A) that is absolutely continuous with respect to μ then show that there is an integrable function f on X (with respect to μ) such that $v(E) = \int_E f d\mu$ for every $E \in A$.

(b) State and prove Jordan Decomposition and also prove the Uniqueness of Jordan Decomposition of signed measure.

4 Answer any two questions : **2×7=14**

(a) Define Measure absolutely continuous with respect to another measure and mutually singular measure also show that if (*X*, *A*) is a measure space and v and μ be a signed measure on (*X*, *A*) if $v \perp \mu$ and $v << \mu$ then prove that $\mu = 0$.

(b) State without proof Fubini's Theorem and Tonelli's Theorem.

(c) Let *X* be a Locally compact separable metric space then prove that $B_0(X) = B_a(X)$.

(d) Let μ^* be an outer measure on *X* and let $\{E_n\}$ be a sequence of pair wise disjoint measurable subset of *X* then show that $\Sigma_n \mu^* (A \cap E_n) = \mu^* (A \cap (\cup_n E_n))$.

5 Answer any **two** of the following questions : **2×7=14**

(a) Let μ^* be an outer measure on *X* then show that collection *B* of all μ^* measurable subset of *X* is a σ-algebra.

(b) Define the word Locally compact Hausdorff space also show that if *X* is locally compact Hausdorff space then $B_a(X) \subset B_0(X)$.

(c) State and Prove The Simple Approximation Theorem.

(d) State without proof Caratheodary extension theorem Give an example to show that σ-finite assumption in the theorem can not be dropped.

BBB-003-1164002 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

July - 2021

Mathematics : CMT - 4002

(Integration Theory)

Faculty Code : 003

Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions: (1) Answer any **five** questions.
(2) Each question carries 14 marks.
(3) There are **10** questions.

1 Answer the following *seven* questions: (7 X 2 = 14) **[14]**

(1) Define: σ −algebra of subsets of a nonempty set X. Also give an example of an algebra which is not a σ −algebra.

(2) If $\mathcal{A}$ is a σ −algebra of subsets of X and $\mathcal{A} \neq \phi$ then prove that $\phi, X \in \mathcal{A}$.

(3) Define: Atomic Measure.

(4) Give only statement of Lebesgue Decomposition Theorem.

(5) Define: Positive set, negative set and null set.

(6) Prove that, every compact subset of K of a Hausdorff space is closed.

(7) Let $(X, \mathcal{A}, \mu), (Y, \mathcal{B}, \gamma)$ be complete measure spaces. Then prove that, $\mathcal{R} = \{A \times B \subset X \times Y \,/\, A \in \mathcal{A} \text{ and } B \in \mathcal{B}\}$ is a semi-algebra.

2 Answer the following *seven* questions: (7 X 2 = 14) **[14]**

(1) Define: Measure of a measurable space with example.

(2) Define: Signed Measure.

(3) Give the statement of Radon – Nikodym Theorem for signed measure.

(4) Give the statement of Tonelli's Theorem.

(5) Define: Mutually singular measures with example.

(6) Let X be a locally compact T_2 −space and K be a compact G_δ −set in X then prove that, $K \in B_a(X)$.

(7) Prove that, a function is continuous if and only if it is lower semi continuous as well as upper semi continuous.

3 Answer the following *two* questions: (2 X 7 = 14) **[14]**

a) Define: Algebra of subsets of a set X. If X is any set, prove that, $\mu: P(X) \to [0, \infty]$ defined by

$$\mu(A) = \begin{cases} \text{the number of elements} & ; \text{if A is finite} \\ \infty & ; \text{if A is infinite} \end{cases}$$

is measure on $(X, P(X))$.

b) State and prove: Hahn Decomposition Theorem.

4 **Answer the following *two* questions: (2 X 7 = 14)** **[14]**

1. Define: Measure absolutely continuous with respect to another measure and mutually singular measures. If $(X, \mathcal{A})$ is a measurable space and γ, μ are signed measures on $(X, \mathcal{A})$, $\gamma \perp \mu$, $\gamma \ll \mu$ then prove that, $\gamma = 0$.
2. Let γ be a signed measure on $(X, \mathcal{A})$. Then prove that, $\exists$ unique measures γ^+ and γ^- on $(X, \mathcal{A})$ such that $\gamma = \gamma^+ - \gamma^-$ on $\mathcal{A}$, $\gamma^+ \perp \gamma^-$, where γ^+ and γ^- are positive and negative part of γ respectively.

5 **Answer the following *two* questions: (2 X 7 = 14)** **[14]**

1. Let $(X, \mathcal{A}, \mu)$ be a finite complete measure space, p, q be extended non-negative real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$, g be integrable on $(X, \mathcal{A}, \mu)$ and $\left|\int_X g \, \phi \, d\mu\right| \leq M \cdot ||\phi||_p$, for all simple measurable function ϕ on X for some $M > 0$. Prove that, $g \in L^q(\mu)$.
2. If μ^* is an outer measure on a set X and $B = \{E \subseteq X / E \text{ is } \mu^* - \text{measurable}\}$. Prove that, B is σ –algebra of subsets of X.

6 **Answer the following *two* questions: (2 X 7 = 14)** **[14]**

1. Define: Baire measure on the real line. Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be monotonically increasing and continuous function on the right. Prove that, $\exists$ a baire measure μ on the real line such that $\mu(a, b] = f(a) - f(b)$, $\forall a, b \in \mathbb{R}$ and $a < b$.
2. If X is a countable set and μ is the counting measure on $(X, P(X))$. Prove that, $L^p(\mu) \cong l^p$, $1 \leq p \leq \infty$.

7 **Answer the following *two* questions: (2 X 7 = 14)** **[14]**

1. Let $(X, \mathcal{A})$ be a measurable space, $\mathcal{D} \subseteq \mathbb{R}$ be dense and B_α, $\alpha \in \mathcal{D}$ be measurable in $(X, \mathcal{A})$ such that $B_\alpha \subseteq B_\beta$, $\forall \alpha, \beta \in \mathcal{D}$ such that $\alpha < \beta$. Prove that, $\exists$ a unique measurable function $f: X \longrightarrow [-\infty, \infty]$ such that $f(x) \leq \alpha$, $\forall x \in B_\alpha$ and $f(x) \geq \alpha$, $\forall x \in X - B_\alpha$.
2. Let m be the σ –algebra generated by all lebesgue measurable subsets of $\mathbb{R}$ and μ be the lebesgue measure on $(\mathbb{R}, m)$. Prove that, μ is regular.

8 **Answer the following *two* questions: (2 X 7 = 14)** **[14]**

1. Let X be a topological space. Prove that,
   a) For $F \subseteq X$, $\chi_F: X \longrightarrow \{0,1\}$ is upper semi continuous if and only if F is closed in X.
   b) If $f_\alpha: X \longrightarrow \{0,1\}$ are upper semi continuous, $\forall \alpha \in \Lambda$ then prove that, $\inf_{\alpha \in \Lambda} f_\alpha$ is also upper semi continuous on X.
2. Let X be a locally compact T_2 –space. Then prove that, $B_a(X)$ is the σ –algebra generated by all compact G_δ –sets in X.

9 **Answer the following *one* questions: (1 X 14 = 14)** [14]

1. State and prove, Fubini's theorem.

10 **Answer the following *one* questions: (1 X 14 = 14)** [14]

1. Let X be a locally compact T_2 −space and $E \in B_a(X)$. Then prove that, either E is σ −bounded or $X - E$ is a σ −bounded set in X. Also prove that, E and $X - E$ both are σ −bounded then X must be σ −compact.

Darshan.. N. Gohil

DN-003-1164002 Seat No. 4020

M. Sc. (Sem. IV) (CBCS) Examination

March - 2022

Mathematics : CMT - 4002

(Integration Theory)

Faculty Code : 003

Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Each question carries 14 marks.
(2) There are 5 questions in total.

1 Answer the following questions. 14

(A) Define : Counting measure. Also find the counting measure of a set $A = \{\mathbb{R}^- \cup [0,2022]\} \cap \mathbb{N}$

(B) Give the example of measure zero set need not be a null set with required justification.

(C) Give only statement of Jordan Decomposition Theorem.

(D) Prove that, every measurable subset of a negative set is negative.

(E) Let X be a locally compact T_2 – space and K be a compact G_δ – set in then prove that, $K \in B_a(X)$.

(F) If μ^* is an outer measure on a set X and $\beta \in P(X)$ be such that $\mu^* \beta = 0$ then prove that, β is μ^* measurable.

(G) Prove that, every compact subset of K of a Hausdorff space is closed.

2 Answer *any two* questions : 14

(A) Let μ be a signed measure on $(X, \mathcal{A})$. Then prove that $\exists$ a positive set A with respect to μ and a negative set B with respect to μ such that $X = A \cup B, A \cap B = \phi$.

(B) Let γ be a signed measure on $(X, A), (\gamma^+, \gamma^-)$ be Jordan decomposition of γ. Prove that, for $E \in A$.

(a) E is a positive set with respect to γ if and only if $\gamma^-(E) = 0$.

(b) E is a negative set with respect to γ if and only if $\gamma^+(E) = 0$.

(c) E is a null set with respect to γ then $|\gamma|(E) = 0$.

(C) State and prove : Lebesgue Decomposition theorem.

3 Answer the following questions : 14

(A) If μ_1, μ_2 are two measure on a measurable space (X, A) and at least one of them is finite then prove that, $\mu_1 - \mu_2$ is a signed measure on (X, A).

(B) If μ^* is an outer measure on a set X and $B = \{E \subseteq X / E \text{ is } \mu^* - \text{measurable}\}$. Prove that, B is $\sigma -$ algebra of subset of X.

OR

3 Answer the following questions : 14

(A) If X is a countable set and is the counting measure on (X, P(X)). Prove that, $L^p(\mu) \cong l^p, 1 \le p \le \infty$.

(B) Define : Measure absolutely continuous with respect to another measure and mutually singular measures. If (X, A) is a measurable space and γ, μ are signed measures on $(X, A), \gamma \perp \mu \ll \mu$ then prove that, $\gamma = 0$.

4 Answer *any two* questions : 14

(A) State and prove : Radon - Nikodym theorem for measure.

(B) Let $\mathcal{C}$ be a semi algebra of subset of a set S and $\mu : \mathcal{C} \to [0, \infty]$ be such that

(a) $c \in C, = \bigcup_{i=1}^{n} c_i$. prove that, $\eta(c) = \sum_{i=1}^{n} \mu(c_i), \forall n \in \mathbb{N}, c_i \in C$ and $c_i \cap c_j = \phi, \forall i, j$

(b) $c \in C, c = \bigcup_{n=1}^{\infty} c_n$. prove that, $\mu(c) = \sum_{i=1}^{\infty} \mu(c_n), \forall c_i \in C$ and $c_i \cap c_j = \phi, \forall i, j$.

(C) Let (X, A, μ), $(Y, \mathcal{B}, \gamma)$ be $\sigma -$finite complete measure space, R be the semi algebra of all measurable rectangles in $X \times Y$ and $E \in R_{\sigma\delta}$ such that $(\mu X \gamma)(E) < \infty$. Prove that, a function $g : X \to [0, \infty]$ defined by $g(x) = \gamma(E_x), \forall x \in X$ is measurable and $\int_x g \, d\mu = (\mu \times \gamma)(E)$.

14

5 Answer ***any two*** questions :

(1) Let X be a topological space. Prove that,

(a) For $F \subseteq X, X_F : X \to \{0,1\}$ is upper semi continuous if and only if F is closed in X.

(b) If $f_a : X \to \{0,1\}$ are upper semi continuous, $\forall a \in \wedge$ then in $f_{a \in \wedge} f_a$ is also upper semi continuous on X.

(2) Let m be the σ-algebra generated by all legesgue measurable subsets of $\mathbb{R}$ and μ be the lebesgue measure on $(\mathbb{R}, m)$. Prove that, μ is regular.

(3) Let X be a locally compact spareable metric space. Prove That, $B_0(X) = B_a(X)$.

(4) Let X be a locally compact T_2-space. Prove that, $B_a(X)$ is the σ-algebra generated by all compact G_δ-sets in X.

DII-003-016403

DII-003-016403

Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

May / June – 2015

Mathematics : Course No. 4003 : (Number Theory - 2)

(New Course)

Faculty Code : 003
Subject Code : 016403

Time : 150 Minutes] [Total Marks : **70**

Instructions: (1) There are five questions.
(2) All questions are compulsory.
(3)Each question carries 14 marks.

Q.1 Fill in the blanks: (Each question carries two marks)

(i) If a/b and c/d are consequetive Farrey fractions in the n^{th} row then _______is the unique rational between a/b and c/d with the smallest denominater.

(ii) If r is a rational multiple of π then the sum of all rational values of $\cos r\pi$=_________

(iii) If m and n are positive integers and θ is a rational solution of x^n=m then θmust be____________

(iv) If x_0 is a positive integer such that $1+2+3+\ldots+x_0=y^2$ then$(2x_0+1,y)$ is a solution of Pell's equation______________.

(v) If the continued fraction expansionof θ is periodic then θ= __________.

(vi) If θ >1 and $\theta + \theta^{-1}<\sqrt{5}$ then θ <________ and θ^{-1}>_____________

(vii) If θ is an irrational number and a/b is a rational number with b>0,(a,b)=1 and $|\theta - a/b| < \frac{1}{2b^2}$ then a/b=____________

Q.2 Attempt any two.

(a) State and Prove Hurwitz's Theorem using Farrey Fraction Method. 7

(b) If θ is an irrational number and $\frac{h_n}{k_n}$ denotes the nth convergent corresponding to the continued fraction expansion of θ then prove that $|\theta k_{n-1} - h_{n-1}| > \frac{1}{k_{n+1}} > |\theta k_n - h_n|$,$\forall$ n $\geq$1. 7

(c) If $a_0,a_1,a_2,\ldots,a_n$are positive integers and $x > 1$then prove that$<a_0,a_1,a_2,\ldots a_{n-1},x> = \frac{xh_{n-1}+h_{n-2}}{xk_{n-1}+k_{n-2}}$. Deduce that $< a_0,a_1,\ldots a_n >= \frac{h_n}{k_n}$ for all n$\geq$ 1 7

Q.3 All are compulsory:

(a) Suppose $a_0, a_1, a_2, \ldots, a_n$ are positive integers and $\frac{h_n}{k_n}$ are the nth convergents corresponding to this intergers then prove that for all $n \geq 1$, $h_n k_{n-2} - h_{n-2} k_n = (-1)^n$. 5

(b) Prove that every irrational number θ can be uniquely expressed as an infinite simple continued fraction. 6

(c) Suppose θ is an irrational number, $\frac{a}{b}$ is a rational number with $b>0$ and $\frac{h_n}{k_n}$ are the nth convergent for θ. If $|\theta b - a| < |\theta k_n - h_n|$ implies $b \geq k_{n+1}$ then prove that $\left|\theta - \frac{a}{b}\right| < \left|\theta - \frac{h_n}{k_n}\right|$ implies $b \geq k_n$. 3

OR

Q.3 All are compulsory:

(a) Prove that if θ is a rational multiple of π and $\cos\theta$ is rational then $|cos\theta|$ cannot be different from $0, \frac{1}{2}$ and 1. 7

(b) Suppose (x_1, y_1) is the smallest positive solution of $x^2 - dy^2 = 1$ (d is a positive integers with is not a perfect square). Prove that all the positive solutions of the above equaition are given by $x_n + \sqrt{d}\, y_n = (x_1 + \sqrt{d}\, y_1)^n$ $(n = 1,2,3,..)$. 7

Q.4 Attempt any two of the following:

(a) Prove that the Diophantine equation $15x^2 - 7y^2 = 9$ has no solution in integers. 7

(b) Suppose (x,y,z) is a primitive Pythagorean triplet. Prove that there are positive integers r and s such that $r>s$, $(r,s) = 1$ and $x = r^2 - s^2$, $y = 2rs$ and $z = r^2 + s^2$. 7

(c) Prove that for every integer $n \geq 1$ there is a polynomial $f_n(x)$ with integer coefficients and leading coefficient 1such that $f_n(2cos\theta) = 2cosn\theta$ for all θ. 7

Q.5 Do as directed: (Each question carries two marks)

(i) Find the simple continued fraction expansion of $\sqrt{2} - 1$.

(ii) Find the simple continued fraction expansion of $\frac{1741}{31}$.

(iii) Find the irrational number θ whose continued fraction expansion is $< 4,4,8,4,8,4,8, \ldots >$.

(iv) Find the primitive pythagorean tripletes for which $z < 60$.

(v) Find first three positive solutions of $x^2 - 19y^2 = 1$.

(vi) Find the simple continued fraction expansion of $\frac{1}{\sqrt{2}}$.

(vii) Find first three positive solutions of $x^2 - 2y^2 = -1$.

NBW-003-016403 Seat No. _______

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : 4003

(Number Theory - II)

Faculty Code : 003

Subject Code : 016403

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions in this paper

(2) Each question carries 14 marks

(3) All questions are compulsory

(4) Figures to the right indicate full marks.

1 Fill in the blanks : (Each question carries two marks)

(a) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the nth row and $\frac{a}{b}$ is less than $\frac{c}{d}$ then ...... and $\frac{c}{d}$ are consecutive Farey fractions in the $(n+1)^{th}$ row.

(b) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the n^{th} row then $\left|\frac{a}{b}-\frac{a+c}{b+d}\right| \leq$ ....

(c) There is a polynomial $f_n(x)$ with degree n, leading coefficient 1 and integer coefficients such that $f_n(2\cos\theta) =$ ...... for every $n \geq 1$ and real θ.

(d) If θ is an irrational number and $\frac{a}{b}$ is a rational number such that $b > 0$ and $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$ then $\frac{a}{b}$ = ... for some n.

(e) If continued fraction expansion of an irrational θ is purely periodic then $\frac{1}{\theta}$ lies between _______ and _______.

(f) If θ is an irrational, $\frac{a}{b}$ is a rational number such that $|\theta b - a| < |\theta k_n - h_n|$ for some $n \geq 1$ then $b \geq$ ______.

(g) The Diophantine equation $ax + by = c$ has a solution if and only if _________ divides c.

2 Attempt any two of the following :

(a) If θ is an irrational number then prove that there are infinitely many rational numbers $\frac{a}{b}$ such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{b^2}$. 7

(b) Prove that π is irrational using elementary method. 7

(c) Suppose $f(x) = a_n X^n + a_{n-1} X^{n-1} + ... + a_0$ is a polynomial of degree n, with integer coefficients and suppose $\frac{a}{b}$ is a rational number with $b > 0$, $(a, b) = 1$ and $\frac{a}{b}$ is a root of $f(x)$. Prove that b divides a_n and a divides a_0. Deduce that if $a_n = 1$ then $\frac{a}{b}$ must be an integer and also deduce that if an integer x has a rational n^{th} root then it must be an integer. 7

3 All are compulsory :

(a) If θ is a quadratic irrational such that (i) $\theta > 1$ (ii) $-1 < \theta' < 0$ then prove that continued fraction expansion of θ is purely periodic. **6**

(b) If θ is irrational and $\theta = \langle a_0, a_1, ...a_n, ... \rangle$ then prove that $k_n < \theta_n k_{n-1} + k_{n-2} < k_{n+1}$ for all $n \geq 0$. **4**

(c) Prove that $x^2 = y^3 + 7$ has no solution in integers. **4**

OR

3 All are compulsory :

(a) Suppose θ is irrational and $\frac{a}{b}$ is a rational number such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b}$ is the n^{th} convergent for some n. **6**

(b) Find general solutions (if any) of the following Diophantine equations : **4**

(i) $2x + 5y = 11$

(ii) $100x + 101y = 2017$

(c) Find the value of following continued fractions : **4**

(i) $\langle 1, 3, 3, 3, 3, ... \rangle$

(ii) $\langle -1, 1, 1, 1, 1, 1, ... \rangle$

4 Attempt any two of the following :

(a) Prove that there are infinitely many positive integers n such that $n^2 + (n + 1)^2$ is a perfect square. **7**

(b) Prove that there are infinitely many positive integers n such that $1 + 2 + 3 + ... + n = m^2$ for some integer m. **7**

(c) If $\frac{h_j}{k_j}$ denote the j^{th} convergent of an irrational number θ then prove that for all $n \geq 1$: **7**

(i) $\left|\theta k_n - h_n\right| < \left|\theta k_{n-1} - h_{n-1}\right|$

(ii) $\left|\theta - \frac{h_n}{k_n}\right| < \left|\theta - \frac{h_{n-1}}{k_{n-1}}\right|$

5 Do as directed : (Each question carries two marks) **14**

(a) Give the definition of a purely periodic continued fraction.

(b) Express $\frac{2017}{101}$ as a simple continued fraction.

(c) Write down the values of $\frac{h_0}{k_0}, \frac{h_1}{k_1}$ for the continued fraction $\langle 1, 2, 1, 2, \ldots\rangle$.

(d) Write down all the Farey fractions between 0 and 1 in the rows up to 7^{th} row.

(e) Find first four positive solutions of $x^2 - 2y^2 = 1$.

(f) Find three primitive Pythagorean tripletes (x, y, z) for which $z > 40$.

(g) Find three positive integers n for which $1 + 2 + 3 + \ldots\ldots\ldots\ldots + n$ is a perfect square.

MBU-003-1164003 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2018

Mathematics - 4003

(Number Theory - II)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper
(2) Each question carries 14 marks
(3) All questions are compulsory

1 Fill in the blanks : (Each question carries two marks)

(a) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the n^{th} row and $\frac{a}{b}$ is less than $\frac{c}{d}$ then $\frac{a}{b}$ and $\frac{a+c}{b+d}$ are consecutive Farey fractions in the .... th row.

(b) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the n^{th} row then $\left|\frac{a}{b}-\frac{a+c}{b+d}\right| \leq$ ....

(c) If the simple continued fraction expansion of θ is finite then θ must be a .... number.

(d) If θ is an irrational and $\frac{a}{b}$ is a rational number such that $b>0$ and $\left|\theta-\frac{a}{b}\right|<\frac{1}{2b^2}$ then $\frac{h_n}{k_n}$ = ... for some n.

(e) If continued fraction expansion of an irrational θ is periodic and θ' lies between–1 and 0 then continued fraction expansion of θ is ....

(f) If θ is an irrational, $\frac{a}{b}$ is a rational number such that $\left|\theta - \frac{a}{b}\right| < \left|\theta - \frac{h_n}{k_n}\right|$ for some $n \geq 0$ then b is greater than ....

(g) The Diophantine equation $ax + by = c$ has a solution if and only if .... divides c.

2 Attempt any **two** of the following :

(a) Write the statement of Hurwitz theorem and prove it using Farey fraction method. **7**

(b) Prove that π is irrational using elementary method. **7**

(c) Prove that for each $n > 0$ there is a polynomial $f_n(x)$ of degree n, leading coefficient 1 and with integer coefficients such that $f_n(2\cos\theta) = 2\cos n\theta$. **7**

3 All are compulsory :

(a) State and prove the necessary and sufficient condition under which the continued fraction expansion of a quadratic irrational is purely periodic. **6**

(b) If θ is irrational and $\theta = \langle a_0, a_1, \dots\dots\dots\dots a_n, \dots\dots\dots \rangle$ then prove that $k_n < \theta_n k_{n-1} + k_{n-2} < k_{n+1}$ for all $n \geq 0$. **4**

(c) Prove that $15x^2 - 7y^2 = 9$ has no solutions in integers. **4**

OR

3 All are compulsory :

(a) Suppose θ is irrational and $\frac{a}{b}$ is a rational number such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b} = r_n$ for some n. **7**

(b) Find general solutions (if any) of the following Diophantine equations : **4**

(i) $2x + 9y = 11$

(ii) $100x + 101y = 2018$

(c) Find the value of following continued fractions : **4**

(i) $\langle 1,1,1,1,1 \rangle$

(ii) $\langle 0,2,2,2,2,2,..... \rangle$

4 Attempt any **two** of the following :

(a) Suppose θ is an irrational number whose continued fraction expansion is periodic. Prove that θ is quadratic irrational. **7**

(b) Prove that there are infinitely many positive integers n such that $1+2+3+....+n=m^2$ for some integer m. **7**

(c) If $\frac{h_j}{k_j}$ denotes the j[th] convergent of an irrational number θ then prove that for all $n \ge 1$. **7**

(i) $|\theta k_n - h_n| < |\theta k_{n-1} - h_{n-1}|$

(ii) $\left|\theta - \frac{h_n}{k_n}\right| < \left|\theta - \frac{h_{n-1}}{k_{n-1}}\right|$

5 Do as directed : (Each question carries two marks)

(a) Give the definition of Diophantine equation.

(b) Express $\frac{2018}{17}$ as a simple continued fraction.

(c) Write down the values of $\frac{h_o}{k_o}, \frac{h_1}{k_1}$ for the continued fraction $\langle 1,1,1,1,.... \rangle$.

(d) Write down all the Farey fractions between 0 and 1 in the rows up to 7[th] row.

(e) Find first four positive solutions of $x^2 - 8y^2 = 1$.

(f) Find three primitive Pythagorean triplets (x, y, z) for which $z > 50$.

(g) Find three positive integers n for which $1+2+3...+n$ is a perfect square.

4500H

RBG-003-1164003 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

April / May - 2019

CMT - 3003 : Number Theory - 2

(Old & New Course)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Answer **all** the questions.

(2) There are **five** questions.

(3) Figures to the **right** indicate full marks.

1 Answer any **seven** : 14

(1) Write down all farey fractions (between 0 and 1) of 5^{th} and 6^{th} row.

(2) Express the rational numbers $\frac{37}{2}$ and $\frac{554}{11}$ as simple continued fractions.

(3) Show that $gcd\ (x, y) = gcd\ (x, y, z)$ if (x, y, z) is a Pythagorean Triplet.

(4) Find three positive integers for which $1 + 2 + 3 + ... + n$ is a perfect square.

(5) Find the general solutions (if any) of the Diophantine equation $2x + 9y = 18$.

(6) Find three Primitive Pythagorean Triplet (x, y, z) for which $z > 50$.

(7) Find at least three positive solutions of $x^2 - 2y^2 = 1$.

(8) If r and s are positive integers and t is a rational solution of $x^r = s$ then t must be ______. Justify your answer.

(9) Define : (a) Primitive Pythagorean triplet and (b) Simple Continued Fraction Expansion.

(10) Show that there are infinitely many solutions (x, y) of $x^2 - dy^2 = 1$ in which k/y for $d > 1$ is not a perfect square and $k \geq 1$.

2 Answer any **two** : **14**

(1) Prove that θ is an irrational number if and only if θ can be expressed as an infinite simple continued fraction. **7**

(2) Prove that if θ is an irrational number then **7**

there are infinitely many rationales $\frac{a}{b}$ such that

$\left|\theta - \frac{a}{b}\right| < \frac{1}{b^2}$; $b > 0$ and $(a, b) = 1$.

(3) (a) Show that if θ is an rational multiple of π then the only rational values $\cos\theta$ can take are $0, \pm\frac{1}{2}, \pm 1$. **4**

(b) Prove or disprove that if $a_0, a_1, a_2, a_3, \ldots, a_n$ is a sequence of integers for $a_i \geq 1; \forall i \geq 1$ with **3**

$$\theta = \lim_{n\to\infty} r_{2n} \text{ then } \theta = \lim_{n\to\infty} r_n \text{ where } r_n = \frac{h_n}{k_n}.$$

3 Answer the following : 14

(1) (a) Find the values of <1, 1, 1, 1, 1, 2> and <–2, 2, 4, 3, 3>. 4

(b) Prove that there are infinitely many positive integers n such that $n^2 + (n + 1)^2$ is a perfect square. 3

(2) Suppose θ is a quadratic irrational such that $\theta > 1$ and $-1 < \theta' < 0$. Prove that its periodic continued fraction expansion is purely periodic. Confirm this result for $\frac{\sqrt{5}+1}{2}$. 7

OR

3 Answer the following : 14

(1) (a) State and prove necessary and sufficient condition under which the linear Diophantine equation has a solution. 4

(b) Prove that $h_n k_{n-2} - h_{n-2} k_n = (-1)^n a_n$. 3

(2) Prove that π is an irrational. 7

4 Answer the following : 14

(1) Prove that if (x_1, y_1) is a smallest positive solution of $x^2 - dy^2 = 1$ then every positive solution of the Pell's equation $x^2 - dy^2 = 1$ is of the form (x_n, y_n) for some $n \geq 1$ where $\left(x_n + \sqrt{d}\,y_n\right) = \left(x_1 + \sqrt{d}\,y_1\right)^n$ 7

(2) (a) Show that an irrational number is a quadratic irrational if its simple continued fraction expansion is periodic. 4

(b) Prove that $x^2 - 80\,y^2 = -1$ has no solution in integers. 3

[Contd...

5 Answer any two of the following : 14

(1) Show that $x^4 + y^4 = z^2$ has no non-trivial solution in integers. 7

(2) Show that the equation $x^2 = y^3 + 7$ has no solution in integers. 7

(3) Prove that if θ is an irrational number and suppose for some rational number $\frac{a}{b}$ for $b > 0$ and $(a, b) = 1$ with $|\theta b - a| < |\theta k_n - h_n|$ for some n then $b \geq k_{n+1}$. 7

(4) Prove that there are infinitely many positive integers n such that $\sum n$ is a perfect square. 7

PT-003-1164003 Seat No. ________

M. Sc. (Sem. IV) Examination

August - 2020

Mathematics : CMT - 4003

(Number Theory - 2)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Answer any **seven** of the following : **14**

(1) Find the value of r_0, r_1, r_2 and r_3 for the continued fraction expansion $<1,1,1,1,2>$.

(2) Find the general solution of Diophantine equation $4x+8y=42$.

(3) Find two Primitive Pythagorean triplet (x,y,z) for which $z<59$.

(4) Define with examples : (a) Quadratic irrational and (b) Pell's Equation.

(5) Find two positive integers n such that $1+2+3+\cdots+n$ is a perfect square.

(6) Express the rational numbers $\frac{101}{7}$ and $\frac{1437}{11}$ as simple contiued fractions.

(7) Find the continued expansion of $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$.

(8) Show that $\gcd(x,y)=\gcd(y,z)$, where (x,y,z) is a Pythagorean Triplet.

(9) Write down the Farey fractions between 0 and 1 in the rows upto 7^{th} row.

(10) Find the value of $<1,1,2,2,2,\ldots\ldots>$ and $<-2,2,4,3,3>$.

2 Answer any **two** of the followigng : **14**

(1) State and prove Hurwitz Inequality for simple continued fractions. **7**

(2) (a) Prove that for an irrational number x the infinite continued expansion is always unique. **4**

(b) If $a_0 a_1 a_2 \ldots\ldots, a_n, \ldots$ is sequence of integers with $a_i \geq 1$; for $i = 1, 2, 3, \ldots,$ then show that $|xk_n - h_n| < \left|\frac{1}{k_{n+1}}\right|; \forall n,$ where x is in irrational number. **3**

(3) Suppose $u \neq 0$ is an integer and $v > 1$ is not a perfect square with $|u| < \sqrt{v}$. If (a, b) is the positive solution of $x^2 - vy^2 = u$ then $a = h_n$ *and* $b = k_n$, for some n provided $(a, b) = 1$. **7**

3 Answer the following : **14**

(1) (a) Find three positive solutions of the equation $x^2 - 3y^2 = 1$. **3**

(b) Justify : Is $x^2 - 15y^2 = -1$ has a solution in integers ? **4**

(2) Suppose $< a_0, a_1 a_2 \ldots\ldots, a_n, \ldots >$ be an infinite sequence of integers with $a_i \geq 1$; for $i = 1, 2, 3, \ldots\ldots n$ then prove that the subsequences r_{2j} and r_{2j-1} both converge to the same point, where h_j, k_j and r_j are defined as usual. **7**

OR

3 Answer the following : **14**

(1) Prove that if θ is an irrational number and suppose for some rational number $\frac{a}{b}$ for $b > 0$ and $(a, b) = 1$ with $|\theta b - a| < |\theta k_n - h_n|$ for some n then $b \geq k_{n+1}$. **7**

(2) (a) Prove that the equation $30x^2 - 14y^2 = 18$ does not have solutions in integer. **3**

(b) Find four solution of the equation $x^2 - 29y^2 = 1$. **4**

4 Answer the following : **14**

(1) Write an algorithm to find the sequence of integers $a_0, a_1 a_2 \ldots\ldots, a_n, \ldots..$ when an irrational number is given and then explain with an example. **7**

(2) Prove that the value of $f(x) = x^4 + x^3 + x^2 + x + 1$ is a perfect square only for $x = -1, 0, 3$ otherwise $f(x)$ is not a perfect square. **7**

5 Answer any **two** of the following : **14**

(1) If $x > 1$ and $-1 < x' < 0$ the show that the continued fraction expansion of *x* is purely periodic provided *x* is a quadratic irrational. **7**

(2) Show that if the triplet (x, y, z) is a Primitive Pythagorean triplet then there exists *r* and *s* such that $r > s \geq 1.(r, s) = 1$ and *r* is even then *s* is odd and vice–versa. **7**

(3) Show that the equation $x^2 = y^3 + 7$ has no solution in integers. **7**

BBC-003-1164003 Seat No. ________

M. Sc. (Sem. IV) Examination

July - 2021

Mathematics : CMT - 4003

(Number Theory - 2)

Faculty Code : 003

Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions:

(1) *Attempt any five questions from the followings.*

(2) *There are total ten questions.*

(3) *Each question carries equal marks.*

1 Answer the following: [7 X 2 = 14] **14**

a) Express the rational numbers $\frac{1055}{8}$ and $\frac{554}{11}$ in continued fraction expansion.

b) Find the value of $< -2,4,4,3,3,1 >$ and $< 1,1,1,1,1,2 >$.

c) Find out, the values of $, r_3, r_4$ of $< 2,4,3,4,1,1 >$.

d) Find, four Primitive Pythagorean triplet (x, y, z) for which $z > 35$.

e) Express the numbers $\sqrt{2} + 1$ and $\frac{\sqrt{5}-1}{2}$ in continued fraction expansion.

f) Define: a) Diophantine Equation and b) Simple Continued Fraction Expansion

g) If (x, y, z) is a Pythagorean Triplet then, show that, $gcd\,(y, z) = gcd(x, y, z)$

2) Answer the following: [7 X 2 = 14] **14**

a) Prove that, $h_n k_{n-1} - h_{n-1} k_n = (-1)^{n-1}$.

b) Define: i) Quadratic irrational and ii) Pell's equation with examples.

c) Write down the farey fractions [between 0 and 1] of 5^{th} and 6^{th} row.

d) Show that, there are infinitely many solutions (x, y) of $x^2 - dy^2 = 1$ in which k/y for $d > 1$ is not a perfect square and $k \geq 1$.

e) Find the number, whose expansion is $< 0,4,4,8,4,8,4,8, \ldots\ldots >$ and $< 2,2,2,2. \ldots\ldots >$.

f) Prove that, the g.c.d (x, y) where $\frac{x}{y}$ is a farey fraction of the n^{th} row is 1.

g) If $a > 1$ is a real and if $x + x^{-1} < \sqrt{5}$ then, show that,

$x < \frac{\sqrt{5}+1}{2}$ and $x^{-1} > \frac{\sqrt{5}-1}{2}$.

3 Answer the following: [2 X 7 = 14] **14**

1) State and prove, Hurwitz Inequality for continued fractions.
2) Prove that, if x is a rational number then its finite simple continued fraction expansion is always unique provided the last term is not equals to 1.

4 Answer the following: [2 X 7 = 14] **14**

1) Prove that, for any $n \geq 1$, there is a polynomial $f_n(x)$ with integer co-efficient of degree n and leading co-efficient 1 such that $f_n(2\cos\theta) = 2\cos n\theta$.
2) Prove that, the value of $f(x) = x^4 + x^3 + x^2 + x + 1$ is a perfect square for $x = -1, 0$ and 3 and for some values of $x, f(x)$ is not a perfect square.

5 Answer the following: [(5+4) + 5 = 14] **14**

1) i) Show that, if θ is a rational multiple of π then the only rational values $\cos\theta$ can take are $0, \mp\frac{1}{2}, \mp 1$.

   ii) Check, whether the equation $x^2 - 18y^2 = 1$ and $x^2 - 18y^2 = -1$ has a solution or not. Justify your answer.
2) Prove that, $x^2 - 11y^2 = -1$ has no solution in integers.

6 Answer the following: [2 X 7 = 14] **14**

1) State and prove, the necessary and sufficient condition under which the continued fraction expansion of quadratic irrational is purely periodic.
2) Find, the first four positive solution of $x^2 - 7y^2 = 1$.

7 Answer the following: [2 X 7 = 14] **14**

1) Prove that, there are infinitely many positive integers n such that $\sum n$ is a perfect square.
2) Show that, the sequence $< a_0, a_1, a_2, \ldots, a_n >$ is a finite simple continued fraction if and only if its value is a rational number.

14

8 **Answer the following: [2 X 7 = 14]**

1) Suppose (x_1, y_1) is a smallest positive solution of $x^2 - dy^2 = 1$ then prove that,

   i) $(x_1 + \sqrt{d}y_1)^n$ is a solution of it for $n \geq 1$.

   ii) Every positive solution is of the form $(x_1 + \sqrt{d}y_1)^n$ for some n.

2) If x is an irrational number and $\frac{a}{b}$ is a rational number with $(a, b) = 1$ and $b > 0$ such that, $\left|x - \frac{a}{b}\right| < \frac{1}{2b^2}$ then show that, $a = h_n$ and $b = k_n$; for some n.

14

9 **Answer the following: [2 X 7 = 14]**

1) Prove that, if (x, y, z) is a primitive Pythagorean triplet then either

   i) x is odd and y is even or x is even and y is odd.

   ii) z is always odd.

2) Show that, the equation $x^4 = z^2 - y^4$ has no solution in integers.

14

10 **Answer the following: [2 X 7 = 14]**

1) Suppose $< a_0, a_1 a_2, \ldots \ldots, a_n, \ldots \ldots >$ be an infinite sequence of integers with $a_i \geq 1$; for $i = 1,2,3, \ldots \ldots n$ then prove that, the subsequences r_{2j} and r_{2j-1} both converges to the same point, where h_j, k_j and r_j are defined as usual.

2) Suppose $C_n x^n + C_{n-1} x^{n-1} + \cdots + C_0$ is a polynomial with integer co-efficient and $\frac{s}{t}$ is a rational number with t is a positive and $(s, t) = 1$. If $\frac{s}{t}$ is a root of this polynomial then prove that, s *divide to* C_0 *and* t *divide to* C_n. Hence, deduce that, if a is an integer and $x^n = a$ has a rational root then it must be an integer.

Seat No. 4030

M. Sc. (Sem. IV) Examination

March / April - 2022

Mathematics : CMT-4003

(Number Theory-2)

Faculty Code : 003

Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are five questions.

(2) All the questions are compulsory.

(3) Each question carries 14 marks.

1 Do as directed : (answer any **seven**) 14

(a) If $x = <2, 2, 2, 2, \ldots\ldots\ldots>$ then, $x^2 =$ ______.

(b) If n is the smallest positive integer such that $\frac{14}{349}$ appears in the n^{th} row then $n =$ ______.

(c) Prove that, $h_n k_{n-2} - h_{n-2} k_n = (-1)^n a_n$.

(d) If (x, y, z) is a Pythagorean Triplet show that, $\gcd(x, z) = \gcd(y, z)$.

(e) Show that, there are infinitely many solutions (x, y) of $x^2 - dy^2 = 1$ in which k/y for d >1 is not a perfect square and $k \geq 1$.

(f) Express $\frac{2022}{29}$ and $\frac{1992}{5}$ in contined fraction expansion.

(g) Find the value of <1,3,3,3,3,...., ...> and <–1,2,3,4,5>.

(h) Find the irrational number having continued fraction expansion $<8, \overline{1, 16}>$.

(i) Define : (i) Periodic continued fraction form and (ii) Primitive pythagorean triplet with examples.

(j) Write down any two solution of $3x + 5y = 8$.

2 Answer any two of the following : 14

(1) Prove that, π is an irrational.

(2) State and prove, Hurwitz Inequality for Farey Fractions.

(3) State and prove, necessary and sufficient condition under which the linear diophantine equation has a solution.

3 Answer the following : 14

(1) Prove that, if θ is an irrational number then there are infinitely many rationales $\frac{a}{b}$ such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{b^2}; b > 0$ and $(a, b) = 1$.

(2) If the continued fraction expansion of an irrational number x is periodic then prove that, the number x is quadratic irrational.

OR

3 Answer the following : 14

(1) Suppose $\frac{x}{y}$ is a rational number where y is positive and $(x, y) = 1$. Prove that, $\frac{x}{y}$ appears in the y^{th} row of farey fractions and subsequent rows.

(2) Suppose $C_n x^n + C_{n-1} x^{n-1} + \cdots + C_0$ is a polynomial with integer co-efficient and $\frac{s}{t}$ is a rational number with t is a positive and $(s, t) = 1$. If $\frac{s}{t}$ is a root of this polynomial then prove that, s divide C_0 and t divide to C_n. Hence deduce that, if a is an integer and $x^n = a$ has a rational root then it must be an integer.

4 Answer the following : 14

(1) (i) Let $\frac{a}{b}$ and $\frac{c}{d}$ be consecutive farey fractions in the n^{th} row. Show that, $|ad - bc| = 1$.

(ii) Show that, $15x^2 - 7y^2 = 9$ does not have a solution in integers.

(2) (i) Prove that, there are infinitely many positive integers n such that $n^2 + (n+1)^2$ is a perfect square.

(ii) Prove or disprove that, if $a_o, a_1, a_2, a_3, ..., a_n$ is a sequence of integers for $a_i \geq 1; \forall i \geq 1$ with $0 = \lim_{n \to \infty} r_{2n}$ then $0 = \lim_{n \to \infty} r_n$ where $r_n = \frac{h_n}{k_n}$.

5 Answer any **two** of the following : 14

(1) Show that, if the triplet (x, y, z) is a primitive pythagorean triplet then there exists r and s such that $r > s \geq 1, (r, s) = 1$ and r is even then s is odd and vice-versa.

(2) Show that, the equation $a^4 + b^4 = c^2$ has no solution in integers.

(3) Prove that, if θ is an irrational number and suppose for some rational number $\frac{a}{b}$ for b>0 and (a, b)=1 with $|\theta b - a| < |\theta k_n - h_n|$ for some n then $b \geq k_{n+1}$.

(4) Show that, the equation $x^2 = y^3 + 7$ has no solution in integers.

Seat No. ______________

HB-003-1164003

M. Sc. (Sem. IV) Examination

April - 2023

Mathematics : CMT-4003

(Number Theory-II)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Do as directed : (answer any **seven**) **14**

(a) Find the value of $\langle 0, 3, 3, 3, 3, ..., ...\rangle$ and $\langle -2, 1, 2, 6, 14\rangle$.

(b) Express the numbers $\sqrt{7}+2$ and $\sqrt{3}-1$ in continued fraction expansion.

(c) Write the statement of Hurwitz's Inequality for $x, y > 0$ for Farey fractions.

(d) Find out the values of r_2, r_3 of $\langle 0, 2, 2, 2, 2, 2\rangle$.

(e) Find the period of $\sqrt{18}$.

(f) Find four positive integers for which $1 + 2 + 3 + 4 \dots \dots + n$ is a perfect square.

(g) Find the general solution if any of the equation $131x + 211y = 2$.

(h) If (x, y, z) is a Pythagorean Triplet then show that g.c.d. (y, z) = g.c.d. (x, z).

(i) There is a polynomial $f_n(x)$ with degree n, leading co-efficient 1 and integer coefficients such that $f_n(\sin x) =$ ________, $\forall n \geq 1$ and real x.

(j) Show that, the g.c.d. (x, y) where $\frac{x}{y}$ Farey fraction of the n^{th} row is 1.

2 Answer any two of the following : **14**

(1) Prove that, π is an irrational number.

(2) Prove that, $x^2 - 143y^2 + 1 = 0$ has no solution in integers.

(3) Suppose (x_1, y_1) is the smallest positive solution of $x^2 - dy^2 = 1$ then prove that:

(i) $(x_1 + \sqrt{d}\, y_1)^n$ is a solution of it for $n \geq 1$.

(ii) Every positive solution is of the form $(x_1 + \sqrt{d}\, y_1)^n$ for some n.

3 Answer the following: **14**

(1) (i) Suppose r and s are positive integer such that $r > s \geq 1$, $(r, s) = 1$ and r is even then s is odd and vice-versa then prove that, the triplet (x, y, z) is a Primitive Pythagorean triplet where $x = r^2 - s^2$, $y = 2rs$ and $z = r^2 + s^2$.

(ii) If $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ be a polynomial with integer coefficient with degree n and if $\frac{a}{b}$ satisfies $f(x)$ then show that, $b \mid a_n$ and $a \mid a_0$ provided $(a, b) = 1$ and $b \geq 0$.

(2) Solve the linear Diophantine equation $172x + 20y = 1000$ by usual method.

OR

3 Answer the following: **14**

(1) Prove that, the value of $f(x) = x^4 + x^3 + x^2 + x + 1$ is a perfect square for $x = -1, 0$ and 3 and for some values of x, $f(x)$ is not a perfect square.

(2) Suppose θ is a quadratic irrational such that $\theta > 1$ and $-1 < \theta' < 0$. Prove that, its periodic continued fraction expansion is purely periodic. Justify the result for $\frac{\sqrt{3}+1}{2}$.

4 Answer the following: **14**

(1) If p, q is a positive solution of $x^2 - ny^2 = 1$, then show that, $\frac{p}{q}$ is a convergent of the continued fraction of $\sqrt{n}$.

(2) (i) Prove that, $x^2 - 80y^2 = -1$ has no solution in integers.

(ii) Prove that, $\lim_{n \to \infty} r_n$ provided its subsequence $\{r_{2n}\}$ has its limit θ, where θ is an irrational. number.

5 Answer any two of the following: **14**

(1) If $a_0, a_1, a_2, ..., a_n, ...$ are integers with $a_i \geq 1; \forall i \geq 1$ then prove that, the number θ is and irrational number if and only if the expansion $\langle a_0, a_1, a_2, ..., a_n, ...\rangle$ is infinite.

(2) If θ is an irrational number and $\frac{a}{b}$ is a rational number then prove that, $b \geq k_{n+1}$ provided $|\theta b - a| < |\theta k_n| - h_n$ for some $n \geq 0$.

(3) Find first four positive solution of $x^2 - 19y^2 = 1$.

(4) Prove that, if x is a rational number then its finite simple continued fraction expansion is always unique provided the last term is not equal to 1.

Seat No. ________________

HC-003-1164004

M. Sc. (Sem. IV) Examination

April - 2023

CMT-4004 : Mathematics

(Graph Theory)

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks (14).

1 Answer any **seven** questions : **7×2=14**

(1) Define terms:
  (1) Self loop
  (2) Parallel edges and
  (3) Simple graph

(2) Draw a simple graph on three vertices and a graph with self loop as well as parallel edges on two vertices.

(3) Define terms:
  (1) Odd vertex and
  (2) Even vertex

(4) Define term:
  (1) Pendent vertex and
  (2) Pendent edge

(5) Define term: Tree and draw a tree on five vertices with indication of its all pendent vertices as well as pendent edges.

(6) Define term: Minimally connected graph. Also draw a minimally connected graph on two vertices.

(7) Define terms:
  (1) Hamiltonian cycle
  (2) Hamiltonian path and
  (3) Hamiltonian graph

(8) Define term: Cut-set and draw a simple graph on four vertices with indication of its two or three cut-sets.

(9) Define terms:
   (1) Fundamental cycle and
   (2) Fundamental cut-set

(10) Define terms:
   (1) Weighted graph and
   (2) Minimal spanning tree.

2 Answer any two questions: **2×7=14**

(a) Let $G=(V,E)$ be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) $V=V_1 \cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. Prove that, $q \le \frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be be connected grpah. Prove that, G is an Euler grpah $\Rightarrow d_G(v)=\text{even}, \forall, v \in V(G)$.

3 Answer following two questions: **2×7=14**

(1) Let G be a simple graph and G has atleast two vertices. Suppose $d_G(v) \ge \frac{n}{2}, \forall v V(G)$. Prove that G, is a Hamiltonian graph.

(2) Let G be a connected graph. Prove that, G is an open Euler graph if and only if G has precisely two odd vertices and remaining are even vertices.

OR

3 Answer following two questions: **2×7=14**

(1) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n-k$ edges.

(2) Let G be a simple graph on n vertices, Let $u, v \in V(G)$ be two non-adjacent vertices of G such that, $d_G(u)+d_G(v) \geq n$. Prove that, G is Hamiltonian graph if and only if its super graph $G+\{uv\}$ is Hamiltonian.

4 Answer following two questions: **2×7=14**

(a) For a tree T, with $|V(T)|=n$, prove that, T has $n-1$ edges.

(b) Prove that, a graph G is a minimally connected graph if and only if it is a tree.

5 Answer any two questions: **2×7=14**

(i) Let T be a tree with $|V(T)| \geq 2$. Prove that, T is a 2-chromatic graph.

(ii) Define adjacency matrix for a graph G. Write down adjacency matrix for C_6. Also write down atleast four properties for the adjacency matrix $X(G)$, for a graph G.

(iii) For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$.

(iv) Let T be a tree with n vertices $(n \geq 2)$. Prove that, T has either one center or two centers. Also prove that, in the case of T admits two centers, they must be adjacent by an edge in T.

DP-003-1164004 Seat No. 4020

M. Sc. (Sem. IV) Examination

March - 2022

Mathematics : CMT-4004

(Graph Theory)

Faculty Code : 003
Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All the questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks (14).

1 Answer any **seven** questions : 7×2=14

(1) Define terms : Isomorphism of two graphs, Subgraph, Vertex disjoint subgraph and Edge disjoint subgraph.

(2) Define terms : Hamiltonian Cycle, Hamiltonian graph, Eulerian graph and Open Eulerian graph.

(3) Define term : Minimally connected graph. Also draw a graph G, with $|V(G)| \geq 5$ and G is a minimally connected graph.

(4) Give two non isomorphic graphs G_1 and G_2, which are having properties $|V(G_1)| = |V(G_2)|, |E(G_1)| = |E(G_2)|$ and for any non-negative integer t, the number of vertices in G_1 with degree t and the number of vertices in G_2 with degree t are same.

(5) State and prove, First Fundamental Theorem of Graph Theory.

(6) State Euler's Theorem.

(7) Write down terms : Fundamental cycle and Fundamental cut-set of a connected graph G with respect to a spanning tree T.

(8) Define term : Weighted graph and Minimal spanning tree.

(9) Define term: Edge connectivity and Vertex connectivity.

(10) Draw a graph G, so that the vertex connectivity for $G = 2$, the edge connectivity for $G = 3$ and $\delta(G) = \min_{v \in V(G)} d_G(v) = 4$.

2 Answer any **two** questions : 2×7=14

(a) Let G be a finite graph. Prove that there are subgraphs $g_i = (V_i, E_i), i = 1, 2, ..., k$, for some $k \geq 1$ such that,

(i) Each g_i is a maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi, i \neq j$ and $i, j \in \{1, 2, ..., k\}$.

(iii) $V = V_1 \cup .... \cup V_k$ and $E = E_1 \cup .... \cup E_k$.

(iv) If $g = (W, F)$ be any connected sub graph of G, then g must be a subgraph g_i, for some $i \in \{1, 2, ..., k\}$.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. Prove that, $q \leq \frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be graph and it does not contain any self loop. Suppose for any pair of vertices $u, v \in V(G)$, there is a unique path between u and v in G. Prove that, G is a tree.

3 Answer following **one** questions : 1×14=14

(1) Let G be a connected graph with $E(G) \neq \phi$. Prove that G is an Eulerian graph if and only if it can be decomposed into edge disjoint cycles.

(2) Define term : Maximal non-Hamiltonian graph. Let G be a simple graph, $|V(G)| > 2$ and $d_G(v) \geq \frac{n}{2}, \forall v \in V(G)$. Prove that, G is a Hamiltonian graph.

4 Answer following **two** questions : 2×7=14

(a) Let T be a tree and it has at least two vertices. Let $P = u_0 - u_1 - u_2 - .....u_n$ be a longest path in T. Prove that, u_0 and u_n both are pendent vertices in T.

(b) For a tree T, with $|V(T)| = n$, prove that T has $n-1$ edges.

(c) Prove that, a graph G is a minimally connected graph if and only if it is a tree.

5 Answer following **two** questions : 2×7=14

(i) Let T be a tree with $|V(T)| \geq 2$. Prove that, T is a 2-chromatic graph.

(ii) Define adjacency matrix for a graph G. Write down adjacency matrix for C_6. Also write down at least four properties for the adjacency matrix $X(G)$, for a graph G.

(iii) Let G be a connected graph. Prove that, G is an Open Euler graph if and only if G has precisely two odd vertices and remaining are even vertices.

(iv) Let T be a tree with n vertices $(n \geq 2)$. Prove that, T has either one center or two centers. Also prove that, in the case of T has two centers, they must be adjacent by an edge in T.

BBD-003-1164004 Seat No. 45130

M. Sc. (Sem. I) Examination

July - 2021

Mathematics : CMT - 4004

(Graph Theory)

Faculty Code : 003

Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1 Answer following seven questions : 7×2=14

1. Define terms : Odd vertex, Even vertex, Regular graph, k–Regular graph.

2. Define terms : Graph, Degree of a vertex, Self loop, Parallel edges (Multiple edges).

3. Define terms : Tree, Acyclic graph, Pendent vertex, Pendent edge.

4. Give two non isomorphic graphs G_1 and G_2, which are having properties $|V(G_1)| = |V(G_2)|$, $|E(G_1)| = |E(G_2)|$ and for any non-negative integer t, the number of vertices in G_1 with t degree and the number of vertices in G_2 with t degree are same.

5. Give two example of Eulerian graphs G_1 and G_2, so that G_1 is Hamiltonian graph, while second graph G_2 does not admits any Hamiltonian cycle.

6. Define term: Arbitrary traceable from a vertex in an Euler graph.

7. Give an example of an Eulerian graph, which has two arbitrary traceable vertices.

5 Answer following two questions : $2 \times 7 = 14$

1. Prove that, for a separable graph G, v is a cut vertex in G if and only if there are two vertices $x, y \in V(G) - \{v\}$ such that every path in G between x and y passes through v.

2. Let $X(G) = \begin{vmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{vmatrix}$

be an adjacency matrix of a graph G. Find $Y = X + X^2 + X^3 + X^4$, where $X = X(G)$. Determine G is a connected graph or it is not a connected graph.

6 Answer following two questions : $2 \times 7 = 14$

(a) Let G be a connected graph and S is a cut set for G. Let T be a spanning tree for G. Prove that, $S \cap E(T) \neq \phi$..

(b) Let G be a connected graph and S be a cut set for G. Let F be any cycle in G. Prove that, $|E(F) \cap S|$ is even.

7 Answer following two questions : $2 \times 7 = 14$

1. Prove that, a graph G is a minimally connected graph if and only if it is a tree.

2. Prove that, a connected graph G, admits a spanning tree.

8 Answer following two questions : $2 \times 7 = 14$

(a) For a tree T, with $|V(T)| = n$, prove that, T has $n - 1$ edges.

(b) Let G be a connected graph and it satisfy $|E(G)| = |V(G)| - 1$. Prove that, G is a tree.

2 Answer following seven questions : 7×2=14

(i) Prove or disprove that, any tree T with $|V(T)| \geq 2$ has atleast two pendent edges.

(ii) Prove or disprove that, any tree T with $|V(T)| \geq 2$ has precisely two pendent vertices.

(iii) Define distance between two vertices in a connected graph.

(iv) Define term: Eccentricity of a vertex in a connected graph and center of a connected graph.

(v) Write down atleast three facts about dual of a planner graph.

(vi) Define term: Incidence matrix of a graph.

(vii) Write down atleast three properties of an incidence matrix of a graph G.

3 Answer following two questions : 2×7=14

(a) Let G be a finite graph. Prove that there are subgraphs $g_i = (V_i, E_i)$, $i = 1, 2, \ldots\ldots, k$, for some $k \geq 1$ such that,

(i) Each g_i is a maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi$, $i \neq j$ and $i, j \in \{1, 2, \ldots, k\}$

(iii) $V = V_1 \cup \ldots \cup V_k$ and $E = E_1 \cup \ldots \cup E_k$.

(iv) If $g = (W, F)$ be any connected subgraph of G, then g must be a subgraph g_i, for some $i \in \{1, 2, \ldots, k\}$.

(b) Let $G = (V, E)$ be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) $V = V_1 \cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

4 Answer following one question : 1×14=14

Let G be a connected graph. Prove that, it is an Euler graph if and only if its all the vertices have even degree.

9 Answer following one questions : 1×14=14

Define term: Maximal non-Hamiltonian graph. Also State and prove Dirac's Theorem.

10 Answer following two questions : 2×7=14

(i) Let u and v be distinct vertices of a tree T. Prove that, there is a unique path P between u and v in T.

(ii) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n - k$ edges.

PU-003-1164004

Seat No.

M. Sc. (Sem. IV) (CBCS) Examination

August - 2020

CMT - 4004 : Mathematics

(Graph Theory)

Faculty Code : 003

Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer any seven questions : 7×2=14

(i) Define terms : Degree of a vertex, pendent vertex in graph G and null graph.

(ii) Define subgraph of a graph G and draw a graph G with its two subgraphs H_1, H_2 so that H_1 and H_2 have five common vertices, but they have no common edges. $\left(i.e. \left|V(H_1)\cap V(H_2)\right| = \text{ and } E(H_1)\cap E(H_2)=\phi\right)$.

(iii) Define k-regular graph. Also define terms $\delta(G)$ and $\Delta(G)$.

(iv) Define isomorphism of two graphs. Write down at least two properties for two isomorphic graphs G_1 and G_2.

(v) Define Hamiltonian cycle and draw wheel graph W_7 with its Hamiltonian cycle.

(vi) Define Eulerian graph. Draw a graph G, which admits an Eulerian line and draw another graph H, which can't admit any Eulernin line.

(vii) Define incidence matrix and write down the incidence matrix for the cycle C_4.

(viii) Write down at least three properties of adjacency matrix $X(G)$ for a graph G.

$2\times7=14$

2 Answer any two questions :

(a) Let G be a graph and it contains exactly two odd vertices, say $x, y \in V(G)$. Prove that x and y both lies in the same component of G.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. In standard notation prove that $q \le \frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be a connected graph with $E(G) \neq \phi$. Prove that G is an Eulerian graph if and only if it can be decomposed into edge disjoint cycles.

(d) State and prove Euler's Theorem.

$1\times14=14$

3 Answer any one question :

(a) For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$ and also prove that (i) $e \ge \frac{3f}{2}$ (ii) $e \le 3n - 6$. Using these prove that K_5 and $K_{3,3}$ both are non–planner graphs, where e = number of edges in G, n–number of vertices in G and f = number of faces in the planner graph G.

(b) Let G be a simple graph, $|V(G)| > 2$ and $d_G(v) \ge \frac{n}{2}, \forall v \in V(G)$. Prove that G is a Hamiltonian graph. Also define closure of a graph G and write down closure for K_4, C_5.

4 Answer any two questions : $2\times7=14$

(a) Define minimally connected graph. Prove that a graph G is a minimally connected graph if and only if it is a tree.

(b) Let T be a tree with $V(T) \neq \phi$. Prove that T has either one center two centers. In the case it has two centers they must be adjacent in T.

(c) Prove that a connected graph G, admits a spanning tree.

5 Answer any two questions : 2×7=14

(a) Define weighted graph and minimal spanning tree. Write down two algorithms to obtain minimal spanning tree for a weighted connected graph G, in detail.

(b) Let G be a connected graph with $|V(G)>2.|$ Prove that the vertex connectivity for $G \leq$ the edge connectivity for G.

(c) Define a separable graph. Prove that for a separable graph G, v is a cut vertex in G if and only if there are two vertices $x, y \in V(G)-\{v\}$ such that every path in G between x and y passes through v.

(d) Let T be a tree with $|V(T)| \geq 2$. Prove that T is a 2-chromatic graph.

RBH-003-1164004 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May – 2019

Mathematics : Paper - CMT - 4004

(Graph Theory)

Faculty Code : 003
Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are **compulsory**.
(2) Each question carries 14 marks.

1 Answer any **seven** questions : 7×2=14

(i) Define following terms :
Simple graph, Null graph, Open Walk and Cycle.

(ii) Define subgraph of a graph and draw a graph G with its two subgraphs H_1, H_2, so that H_1 and H_2 have one common vertex, but they have no common edges.
(i.e. $|V(H_1) \cap V(H_2)| = 1$ and $E(H_1) \cap E(H_2) = \phi$).

(iii) Define k-regular graph. Draw a simple 4-regular graph on 9 vertices.

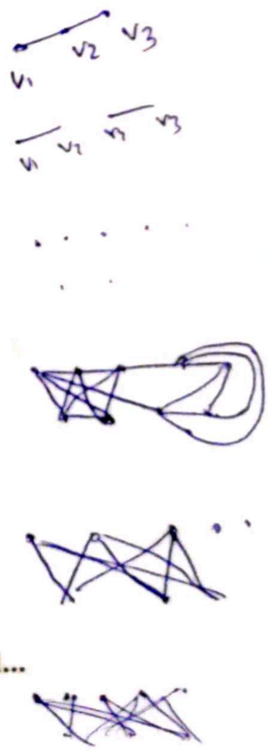
(iv) Define Euler graph and Hamiltonian graph. Give an example of a graph which is an Euler graph, but it is not a Hamiltonian graph.

(v) Define Hamiltonian cycle and draw wheel graph W_7 with it's a Hamiltonian cycle.

(vi) Define complete graph and complete bipartite graph. Draw the complete graph K_6 and complete bipartite graph $K_{3,\,5}$.

(vii) Define incidence matrix and write down the incidence matrix for path P_4 .

(viii) Write down atleast two properties for the incidence matrix $A(G)$ for a graph G.



2 Answer any **two** questions : 2×7=14

(a) Let G be a simple graph with n vertices, q edges and k number of components in G. In standard notation prove that $q \leq \frac{1}{2}(n-k)(n-k+1)$.

(b) State and prove first fundamental Theorem of Graph Theory and also state and prove its application (Second fundamental Theorem of Graph Theory).

(c) State and prove Max flow min cut Theorem.

(d) Let G be- a finite graph. Prove that there are subgraphs $g_i = (V_i, E_i)$, $i = 1, 2, \ldots, k$, for some $k \geq 1$ such that

(i) Each g_i is maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi$, $i \neq j$ and $i, j \in \{1, 2, \ldots, k\}$.

(iii) $V = V1 \cup \ldots \cup Vk$ and $E = E1 \cup \ldots \cup Ek$.

(iv) If $g = (W, F)$ be any connected sub graph of G, then g must be a subgraph g_i, for some $i \in \{1, 2, \ldots, k\}$.

3 Answer any **one** question : 1×14=14

(a) For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$ and also prove that (i) $e \geq \frac{3f}{2}$ (ii) $e \leq 3n - 6$. Using these prove that K_5 and $K_{3,3}$ both are non-planner graphs, where e = number of edges in G, n = number of vertices in G and f = number of faces in the planner graph G.

(b) Let G be a connected graph. Prove that it is an Euler graph if and only if its all the vertices have even degree.

(c) Define Maximal non - Hamiltonian graph. Also state and prove Dirac's Theorem.

4 Answer any **two** questions : 2×7=14

(a) Let u and v be distinct vertices of a tree T. Prove that there is a unique path P, between u and v in T.

(b) Let G be graph and it does not contain any self loop. Suppose for any pair of vertices $u, v \in V(G)$, there is a unique path between u and v in G. Prove that G is a tree.

(c) Define tree. For a tree T, prove that T has $n - 1$ edges, where $n = |V(T)|$.

(d) Define acyclic graph. Let G be an acyclic graph with n vertices and k components. Prove that T has $n - k$ edges.

5 Answer any **two** questions : **2×7=14**

(a) Let G be a connected graph with n vertices and $S \subseteq E(G)$. Prove that S is a cutest for G if and only if rank of $G - S$ is $n - 2$ and rank of $G - S_1$ is $n - 1$, $\forall$ S_1 proper subset of S.

(b) Let G be a connected graph and S is a cutest for G. Let T be a spanning tree for G. Prove that

$S \cap E(T) \neq \phi$.

(c) Let G be a connected graph and S be a cut set for G. Let F be a cycle in G. Prove that $|E(F) \cap S|$ is even.

(d) Define Tree graph of a connected graph G. Write down all the spanning trees of the cycle C_5 and draw Tree graph for C_5.

$E(T) = E(T') + 1$

$E(T') = E(T) - 1$

$=$

$E(T) = V(T) + 1$

$= (V(T') + 1$

$= E(T')$

$= |E(T')| - 1$

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MBV-003-1164004

Seat No. ______

M. Sc. (Sem. IV) Examination

April / May - 2018

Mathematics : CMT - 4004

(Graph Theory) (New Course)

Faculty Code : 003

Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Each question carries 14 marks.

(2) All the question are compulsory.

1 Answer any **seven** from following short questions : **7×2=14**

(i) Define following terms :

Self loop, Parallel edges, null graph and simple graph.

(ii) Define subgraph of a graph. Draw a graph G with its two subgraphs H_1, H_2 so that $V(H_1) \cap V(H_2)$ is a single ton set and $E(H_1) \cap E(H_2)$ = empty set.

(iii) Define closed walk and cycle. Give an example of a closed walk of a graph G which is not a cycle in G.

(iv) Define Eulerian graph. Draw an Eulerian graph G, which is a simple graph but it is not a k-regular graph for any $k \in \{1, 2, ..., |V(G)| - 1\}$.

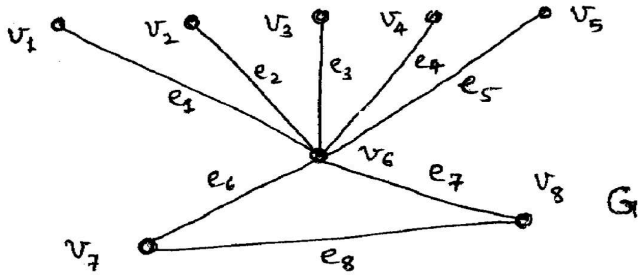
(v) Define Hamiltonian cycle and Hamiltonian graph. Draw a wheel graph Wn with its a Hamiltonian cycle, for some integer $n \geq 3$.

(vi) Draw a simple graph G with following properties :

Number of components of G is atleast three, no component of G is a null graph, $|V(G)| = 9$ and $|E(G)| \leq 7$.

(vii) Define incidence matrix of a self loopness graph G. Also write down the incidence matrix K_3 (Complete graph on three vertices).

(viii) Write down all the spanning trees of following graph G.



2 Attempt any **two** : **2×7=14**

(a) Define connected graph G. Prove that a graph G is disconnected if there are two non-empty disjoint subsets V_1, V_2 of $V(G)$ such that

(1) $V_1 \cup V_2 = V(G)$ and

(2) There is no edge $e \in E(G)$, whose one end vertex lies in V_1 and another end vertex lies in V_2.

(b) For a connected graph G, prove that G is an open Eulerian graph if G has exactly two odd vertices and remaining all vertices are even vertices if exist.

(c) Let G be a graph and it contains exactly two odd vertices say x and y. Let for any $V \in V(G) - \{x, y\}$, $d_G(V)$ = even. Prove that there must be a path in G between x and y.

(d) Let T be a tree. Prove that any two distinct vertices u and v of T, there is a unique path between u and v in T.

3 Attempt any **one** : **1×14=14**

(a) For a connected planner graph G, prove that $f = e - n + 2$. Also derive following :

(1) $e \geq \frac{3f}{2}$

(2) $e \leq 3n - 6$.

Where $e = |E(g)|, n = |V(G)|$ and f = the number of faces in the graph G.

(b) For a connected graph G, prove that the ring sum of two cutsets of G is either a cutset for G or it is an edge disjoint union of two cutsets.

(c) State and prove Eulerian theorem.

4 Attempt any **two** : **2×7=14**

(a) State and prove Max flow min cut theorem.

(b) Prove that Kuratowaski's first graph K_5 and second graph $K_{3,3}$ both are non-plannar graphs.

(c) For a tree T, prove that $|E(T)| = |V(T)| - 1$.

(d) For an acyclic graph G, Prove that $|E(G)| = |V(G)| - k$, where k is the number of components for the graph G.

5 Attempt any **seven** : **7×2=14**

(1) Define minimally connected graph. Draw a minimally connected graph G with $|V(G)| = 4$.

(2) Define eccentry of a Vertex and distance between two vertices in a connected graph.

(3) State the statement of Konig's theorem. Also write down number centers for a path P_6 on six vertices.

(4) Write definitions of fundamental cycle and fundamental cut-set of a graph G.

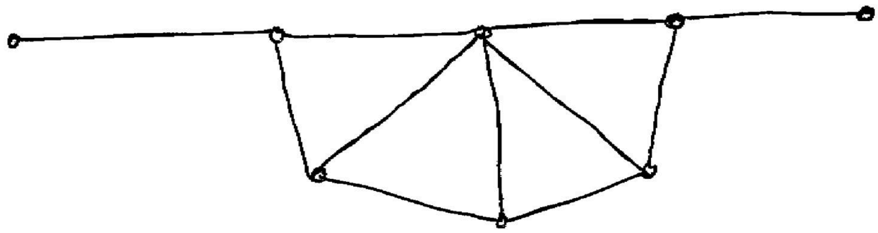
(5) Define weighted graph and minimal spanning tree.

(6) Draw dual graphs of K_3 and K_4.

(7) Draw a simple graph whose adjacency matrix is given by

$$X(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(8) Write down proper edge coloring and proper vertex coloring of following graph :



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NBX-003-016404 **Seat No.** ______

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT - 4004

(Graph Theory)

Faculty Code : 003
Subject Code : 016404

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Each question carries **equal** marks.

(2) All the questions are **compulsory**.

1 Answer the following short questions : **7×2=14**

(i) Define walk, trail, path and cycle.

(ii) Define adjacency matrix $X(G)$ for a graph G.

(iii) Define isomorphism of two graphs.

(iv) Give example of two graphs G_1, G_2 which satisfy

(i) $|V(G_1)| = |V(G_2)|$, (ii) $|E(G_1)| = |E(G_2)|$ and (iii) for any integer $m \geq 0$, the number of vertices in G_1 with degree m is same as the number of vertices in G_2 with degree m, however G_1 and G_2 are not isomorphic graphs.

(v) Write down all the spanning trees for following graph G :

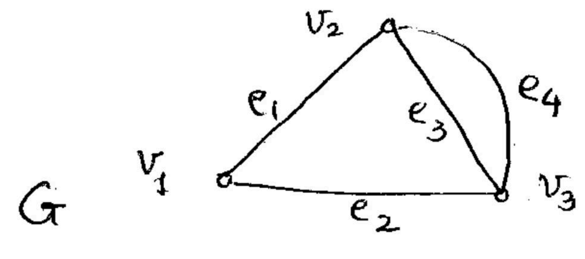

Fig. 1

(vi) Give definitions of k-regular graph and simple graph. Draw a 3-regular simple graph.

(vii) Give statement of Euler's theorem and draw a graph which is Eulerian graph and its all vertices have not same degree.

2 Attempt any **two** : **2×7=14**

(a) Let G be a simple graph with n vertices, q edges and k components. In standard notation prove that

$$q \le \frac{1}{2}(n-k)(n-k+1).$$

(b) Let G be a connected graph and $d_G(v) = even, \forall v \in V(G)$. Prove that G must be an Eulerian graph.

(c) Let u and v be two vertices of a tree $T(u \ne v)$. Prove that there is a unique path P between u and v in T.

3 Attempt any **one** : **1×14=14**

(a) Let $G = (V, E)$ be a connected graph. Prove that the ring sum of two cut sets of G is either a cutset for G or it is an edge disjoint union of two cut sets of G.

(b) Let G be a simple graph with n vertices and $n \ge 3$. Let G satisfies $d_G(v) \ge \frac{n}{2}.\ \forall\ v \in V(G)$. Prove that G is a Hamiltonian graph. Also define Hamiltonian graph and maximal non-Hamiltonian graph.

(c) Prove Max-flow min-cut theorem : In a given transport network G, the maximum value of a flow from s to t is equal to the minimum value of the capacities of all the cuts in G that separate s from t.

4 Attempt any **two** : **2×7=14**

(a) Let $G = (V, E)$ be a graph which does not contains any loop. Also every pair $u, v \in V$, there is a unique path between u and v in G. Prove that G must be a tree.

(b) Let T be a tree on n vertices. Prove that T has either one or two centers. Also deduce that in the case T has two centers, they must be adjacent by an edge in T.

(c) Define separable graph and cut-vertex. For a separable graph $G=(V, E)$, prove that $v \in V$ is a cut-vertex for G if and only if there is atleast two vertices $x, y \in V-\{v\}$ such that every path between x and y in G passes through v.

(d) Let G be a connected plannar graph with n vertices, e edges and f regions (faces). In standard notation derive the formula $f = e - n + 2$.

5 Attempt any **two** : **2×7=14**

(a) Let T be a tree with $|V(T)| \geq 2$. Prove that T is a 2-chromaticgraph.

(b) Find out the adjacency matrix X for following graph G. Also find the matrix $Y = X + X^2 + X^3 + X^4$. Is G connected ?

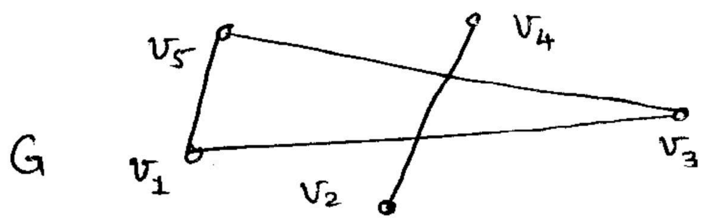

Fig. 2

(c) Prove that Kuratowaski's first graph K_5 and second graph $K_{3,3}$ both are not plannar graphs.

(d) Let G be a connected graph, S be a cut set in G and F be a cycle in G. In standard notation prove that $|E(F) \cap S|$ is even.

 Seat No. ________

M. Sc. (Sem. IV) (Maths) (CBCS) Examination

May / June – 2015

CMT-4004 : Graph Theory

Faculty Code : 003
Subject Code : 016404

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Each question carries equal marks.
(2) Attempt all the questions.

1 Choose appropriate alternative : (any **seven**) **7×2=14**

(1) Let G be a null graph. What is sum of degree of all vertices $\sum_{v \in V(G)} d(v)$ for G ?

(A) 0 (B) $|V(G)|$

(C) 1 (D) $2|V(G)|$

(2) Let $G=(V,E)$ be a disconnected graph and $K =$ number of components of G. Then which of following is true for G ?

(A) $k=1$ (B) $k \le |V|$

(C) $k > |V|$ (D) None of these

(3) Which of following is not an Euler graph ?

(A) C_{99}

(B) K_{99}

(C) P_{99}

(D) 4-regular connected graph

(4) Which of following is not a Hamiltonian graph ?

(A) C_9 (B) C_{10}

(C) K_{10} (D) P_{11}

(5) What is the number of vertices in a tree with 20 edges ?

(A) 20 (B) 19

(C) 21 (D) 40

(6) Let $G=(V,E)$ be a connected graph with $|V|=10$ and $|E|=20$. What is number of edges in a spanning tree T for G ?

(A) 10 (B) 20

(C) 19 (D) 9

(7) Which of following is not a planar graph ?

(A) K_6 (B) P_{19}

(C) C_{20} (D) $K_{2,3}$

(8) Let $G=(V,E)$ be a connected planar graph with 10 vertices and four faces (regions). What is number of edges in the graph G ?

(A) 14 (B) 12

(C) 9 (D) 40

2 Attempt any two : **2×7=14**

(1) Let $G=(V,E)$ be a graph. Let $x, y \in V$ be such that $d_G(x)$ and $d_G(y)$ both are odd. If $d_G(w)=\text{even}$, $\forall\, w \in V-\{x,y\}$, then prove that there must be a path between x and y in G.

(b) Let $G=(V,E)$ be a finite graph. Then prove that there are subgraphs $G_1, \ldots\ldots, G_k$ of G such that $G_i=(V_i, E_i), \forall\, i=1,2,\ldots\ldots,k$ and

(i) Each G_i is maximal connected subgraph of G.

(ii) $V_i \cap V_j=\phi,\ \forall\, i,j \in \{1,\ldots\ldots,k\}$ and $i \neq j$

(iii) $V=V_1 \cup V_2 \cup \ldots\ldots \cup V_k,\ \ E=E_1 \cup E_2 \cup \ldots\ldots \cup E_k$.

(c) Let $G=(V,E)$ be a connected graph. Prove that there is a spanning tree $T=(T,F)$ for *G*.

(d) Let $G=(V,E)$ be a connected graph. Prove that *G* is a tree *iff* adding an edge between any two vertices of *G* creates exactly one circuit.

3 Attempt any one : **1×14=14**

(a) State and prove the Euler's theorem.

(b) Let $G=(V,E)$ be a connected graph. Prove that ring sum of two distinct cut-sets of *G* is either a cut-set of *G* or it is an edge disjoint union of two cut-sets of *G*.

(c) Let $G=(V,E)$ be a simple graph with $n=|V|\geq 3$. If $d_G(v)\geq \frac{n}{2}$, $\forall\, v\in V$ then prove that *G* is a Hamiltonian graph.

4 Attempt any **two** : **2×7=14**

(a) Define closure of a graph *G*. For a simple graph *G*, prove that *G* is Hamiltonian *iff* its closure $C(G)$ is Hamiltonian.

(b) Let *u, v* be two distinct vertices of a tree *T*. Prove that there is a unique path between *u* and *v* in *T*.

(c) Define minimally connected graph *G*. Prove that a connected graph *G* is minimally connected graph *iff* it is a tree.

(d) Define incidence matrix $A(G)$ for a graph *G*. Also write atleast five properties for the incidence matrix $A(G)$.

5 Attempt any seven : **7×2=14**

(1) Write adjacency matrix $X(G)$, where $G=C_3$, cycle on three vertices.

(2) Define chromatic number for a graph *G*. Give an example of a graph *G* which is a 1-chromatic graph.

(3) Define diametrer of a connected graph and draw a connected graph G whose diameter $D(G) = 3$.

(4) Draw a simple connected graph $G = (V, E)$ with $|V| = 5$ and G is a Hamiltonian graph as well as G is an Eulerian graph.

(5) Define a connected graph and draw a disconnected graph G with $|V(G)| = 3$.

(6) Define a self loop and parallel edges in a graph G.

(7) Give definition : Regular graph, Simple graph.

(8) Define isomorphism of graphs.

DKK-003-016405 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

May/June – 2015

EMT-4011 : Financial Mathematics

Faculty Code : 003

Subject Code : 016405

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt all the questions.

(2) Each question carries equal marks.

1 Choose the most appropriate alternative / alternatives : (any seven)

(1) In European call option a holder can purchase a prescribed asset

(A) only at the time of expiry

(B) any time before time of expiry

(C) after three months of expiry

(D) after six months of expiry

(2) 'NIFTY' is one of the index from

(A) Sri Lanka (B) Nepal

(C) India (D) Japan

(3) In Asian option the price depends upon

(A) past history

(B) financial condition of the company

(C) political situation

(D) some form of average

(4) The volatility is defined by

(A) θ (B) σ

(C) δ (D) none of these

(5) The randomness can be eliminates by

(A) $\sigma = 0$ (B) $r = 0$

(C) $\Delta = \frac{\partial f}{\partial s}$ (D) none of these

(6) Which of the following is/are not financial market

(A) Grocery market (B) Jewellery market

(C) Currency market (D) Bond market

(7) Hedging is an action to prevent

(A) loss

(B) profit

(C) financial irregularities

(D) waste of food

(8) The Black-Scholes differential equation is

(A) forward parabolic

(B) backward parabolic

(C) of order 2

(D) of order 3

(9) According to simple model of asset price

(A) holder's edge is important

(B) past history is fully reflected

(C) the financial condition is fully reflected

(D) date of expiry is fully reflected

(10) The value of portfolio is defined as

(A) S+P+C (B) S+P–C

(C) S–P–C (D) S+P

2 Attempt any two :

(a) Explain : What are options for ?

(b) Explain : Higher the exercise price more is received for the asset at the time of expiry of put option.

(c) What is an American option ? Why it is worth in comparison to an European option.

3 (a) Define the following terms :

(1) Asian option

(2) Look-back option

(3) Volatility

(4) Holder of option

(5) Hedging

(6) Asset price

(7) Excercise price.

(b) State and prove Ito's lemma.

4 Attempt any two :

(a) State the assumptions of Black-Scholes analysis and derive the Black-Scholes partial differential equations.

(b) Derive the stochastic differential equation and also give the economically reasonable justification of the derived equation.

(c) Explain the terms 'bid-ask' and 'bid-offer'.

5 Attempt any two :

(a) Explain : Forward and future contracts.

(b) Find the stochastic differential equation for $f(s) = s^n$.

(c) Explain in detail the elimination of randomness.

(d) Explain discrete dividend pay structure and derive the jump conditions for the same.

NBY-003-016405 Seat No. _______

M. Sc. (Maths.) (Sem. IV) (CBCS) Examination

April / May - 2017

EMT - 4011 : Financial Mathematics

Faculty Code : 003
Subject Code : 016405

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are 5 questions.
(2) Attempt all the questions.
(3) Figures to right indicate full marks.

1 Attempt the following : (Any Seven) **14**

1. Define up-front premium and speculative price.
2. Explain the terms bid-ask and bid-offer.
3. Giving the examples explain the terms Asian option and Look- back option.
4. Name the two indices of the Indian stock market.
5. Obtain the stochastic differential equation for f (s) = AS.
6. Explain the terms: Arbitrage and Risk- free investment.
7. Name any two financial markets and their dealing.
8. Explain the term financial derivatives.
9. Distinguish between European option and American option in minimum two points each.
10. Distinguish between call option and put option in minimum two points each.

2 AttemAt the following : **14**

a. Suchitra holds an option on 23rd January 2016 to purchase hundred shares of APL industries for Rs 3 500 per share after one year. If the cost of option is Rs 100 per share and price of share is Rs 8000 per share on 23rd January 2017 then find the total profit to Suchitra on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

b. Explain: How the call option value is a function of exercise price and time to expiry ?

OR

b. Explain: Higher the asset price on expiry of call option, greater the profit.

3 Attempt the following : **14**

a. Explain in detail the forward and future contracts.

OR

a. How much one should pay now to receive a guaranteed amount at the future time T ?

b Explain the simple model of asset prices. .

4 Attempt the following. 14

a. State and prove Ito's lemma and extend the result for $f \equiv f(S, t)$.

b. Derive the Black- Scholes partial differential equation.

5 Attempt the following : (Any Two). **14**

a. Discuss the mathematical significance of Black-Scholes equation and derive the boundary and final conditions for the same.

b. Solve the Black-Scholes differential equation.

c. What are dividends? Also define the term dividend yield and explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

MBW-003-1164005 Seat No. ______

M. Sc. (Sem. IV) Examination

April / May - 2018

Mathematics : EMT - 4011

(Financial Mathematics) (New Course)

Faculty Code : 003

Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

(3) There are 5 questions.

1 Attempt the following : (any **seven**) **14**

(1) Define up-front premium and speculative price.

(2) Distinguish between call option and put option in minimum two points each.

(3) Giving the examples explain the terms Asian option and Look back option.

(4) Name the two indices of the Indian stock market.

(5) Obtain the stochastic differential equation for $f(s) = S^n$.

(6) Explain the terms :

(i) Arbitrage

(ii) Risk and their types.

(7) Name any two financial markets and their dealing.

(8) Explain the term financial derivatives.

(9) Distinguish between European option and American option in minimum two points each.

(10) Explain the terms bid-ask and bid-offer.

2 Attempt the following : **14**

(a) Akshar holds an option on 1^{st} March 2017 to purchase 200 shares of Pioneer industries for Rs. 5,500 per share after one year. If the cost of option is Rs 100 per share and price of share is Rs 8000 per share on 1^{st} March 2018 then find the total profit to Akshar on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

(b) Explain: How the call option value is a function of exercise price and time to expiry.

OR

(b) Explain: Higher the exercise price more is received for the asset at expiry of put option.

3 Attempt the following : **14**

(a) Explain the simple model of asset- prices.

(b) How much one should pay now to receive a guaranteed amount at the future time T.

OR

(b) State and prove Ito's lemma and extend the result for $f \equiv f(S,t)$.

4 Attempt the following : **14**

(a) Explain in detail :

(i) Forward and future contracts

(ii) Portfolio and Hedging

(iii) Smaller order effect on portfolio

(iv) Sensitivity to volatility

(b) Derive the Black- Scholes partial differential equation.

5 Attempt the following : (any **two**) **14**

(a) Discuss the mathematical significance of Black-Scholes equation and derive the boundary and final conditions for the same.

(b) Solve the Black-Scholes differential equation.

(c) Define the term dividend yield and explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

(d) Explain: discrete dividend structure and derive the jump conditions for the same.

45004

RBI-003-1164005 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May – 2019

Mathematics : Paper - EMT - 4011
(Financial Mathematics) (New Course)

Faculty Code : 003
Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt **all** the questions.
(2) There are **5** questions.
(3) Figures to the **right** indicate full marks.

1 Attempt the following : (any **seven**) 14

1. Name any two indices of Indian stock markets.
2. Explain the terms bid-ask and bid-offer.
3. Giving the examples explain the terms Asian option and Barrier option.
4. Define up-front premium and speculative price.
5. Obtain the stochastic differential equation for $f(S) = AS$.
6. Explain the terms: Arbitrage and Risk- free investment.
7. Distinguish between call option and put option in minimum two points each.
8. Explain the term financial derivatives.
9. Distinguish between European option and American option in minimum two points each.
10. What are dividends ?

2 Attempt the following : 14

a. Sachin holds an option on 23rd February, 2017 to purchase hundred shares of PLA logistics for Rs. 3,000 per share after one year. If the cost of option is Rs 100 per share and price of share is Rs 8000 per share on 23rd February, 2018 then find the total profit to Sachin on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

b. Explain in brief the central idea behind theory and practice of option pricing.

OR

b. Explain : How the call option value is a function of exercise price and time to expiry.

3 Attempt the following : **14**

a. How much one should pay now to receive a guaranteed amount at the future time T.

OR

a. Explain in detail the forward and future contracts.

b Explain the simple model of asset prices.

4 Attempt the following : **14**

a. State and prove Ito's lemma and extend the result for $f \equiv f(s, t)$.

b. Derive the Black-Scholes partial differential equation.

5 Attempt the following : (any two) **14**

a. Discuss the mathematical significance of Black-Scholes equation and derive the boundary and final conditions for the same.

b. Solve the Black-Scholes differential equation.

c. What are dividends ? Also define the term dividend yield and explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

d. Explain: Higher the asset prices on expiry of call option, greater the profit.

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PV-003-1164005 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

August - 2020

EMT - 4011 : Mathematics

(Financial Mathematics)

(New Course)

Faculty Code : 003

Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.

(2) There are 5 questions.

1 Attempt the following : (Any **Seven**) **14**

(1) State minimum two differences between forward and futures contracts.

(2) Explain the terms : bid-ask or bid-offer

(3) What are look-back options? Give an example.

(4) Obtain the stochastic differential equation for $f(S) = AS$.

(5) Name two popular indices each of India and America.

(6) Explain the terms : (i) Risk free investment (ii) Hedging.

(7) Explain the term financial derivatives.

(8) What are foreign exchange markets? What they dealt with?

(9) State minimum three differences each between call option and put option.

(10) Define American options and explain why they are popular in compare to European options.

2 Attempt the following : **14**

(a) Define exercise price and explain higher the exercise price more is received for the asset at expiry of put option.

(b) Define call option and explain bow the call option value is a function of exercise price and time to expiry.

(c) Atul Holds an option to purchase 50 shares of Amrita industries at Rs. 400 per share. If the asset price is Rs. 300 per share after one year and up-front premium is Rs. 10 per share then will Atul exercise his option? Why? Explain with reasons.

3 Attempt the following : **14**

(a) Establish the relation $M = E\, e^{-\int_t^T r(s)ds}.$

(b) What is put-call parity?

OR

3 (a) Explain the simple model of asset pricing. **14**

(b) State and prove *Itôs* lemma and extend the result when the function f is also a function of t.

4 Attempt the following : **14**

(a) Describe the procedure to eliminate the randomness from *Itôs* lemma.

(b) Derive the Black-Scholes partial differential equation.

5 Attempt the following : (Any **Two**) **14**

(a) Explain the situation of a call option and put option at the time of expiry of options.

(b) Solve the Black-Scholes differential equation.

(c) Define the terms :

(i) Portfolio

(ii) Dividends (iii) discrete dividend structure. Also derive the jump conditions for the same.

(d) What is dividend yield? Explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

BBE-003-1164005 Seat No. ________

M. Sc. (Sem. IV) Examination

July - 2021

Mathematics : EMT-4011

(Financial Mathematics)

Faculty Code : 003
Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Attempt the following. 14
(1) Explain the term holder.
(2) Define cost of option.
(3) Define Market price.
(4) Define exercise price.
(5) Obtain the stochastic differential equation for $f(S) = S^{100}$.
(6) Name any two popular indices of the world with the names of respective countries.
(7) When any investment is called a Risk free? Also give two examples of it.

2 Attempt the following. 14
(1) Name any three popular indices from India.
(2) Distinguish between European option and American option in minimum three points each.
(3) Define smaller order effects on the portfolio.
(4) What are financial derivatives?
(5) Which options are used by oil refiners?
(6) Define the term: Sensitivity to interest rate.
(7) Name any two financial markets and their dealing.

3 Attempt-the following. 14

(a) What are options for?

(b) Explain: Higher the asset price on expiry of call option, greater the profit.

4 Attempt the following. 14

(a) Explain the simple model of asset pricing.

(b) State and prove It's lemma and extend the result for $f = f(S, t)$.

5 Attempt the following. 14

(a) How much one should pay now to receive a guaranteed amount at the future time T.

(b) Explain in brief the central idea behind the theory and practice of option pricing.

6 Attempt the following. 14

(a) Derive the Black- Scholes partial differential equation.

(b) Discuss mathematical significant Black - Scholes equations. Also derive the boundary and final conditions for European options.

7 Attempt the following. 14

(a) Solve the Black-Scholes differential equation.

(b) Explain the situation of a call option and put option at the time $t = T$.

8 Attempt the following 14

(a) Explain: Higher the exercise price more is received for the asset at expiry of put option.

(b) Define call option and explain how the call option value is a function of exercise price and time to expiry.

9 Attempt the following 14

(a) Avani holds an option on 1st June 2016 to purchase 500 shares of Suman industries for Rs 5000 per share after one year. If the up-front premium is Rs 100 per share and price of share is Rs 10000 per share on 1st June 2017 then find the total profit to Avani on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

(b) A company whose share price is Rs 500 offers bonus shares in the ration 1:1. What will be the asset price and how should option be altered?

10 Attempt the following 14

(a) Explain discrete dividend structure. Also derive the jump conditions for the same.

(b) Define the term dividend yield and explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

DQ-003-1164005

Seat No. A020

M. Sc. (Sem. IV) Examination

April - 2022

Mathematics : EMT-4011

(Financial Mathematics)

Faculty Code : 003

Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Attempt the following : (Any **seven**) **14**

(1) Define : Put option.

(2) State minimum three differences between option and contracts.

(3) Define: Smaller order effects on the portfolio.

(4) Name any two financial markets.

(5) Define: Asian option and give an example.

(6) Obtain the stochastic differential equation for $f(S) = S^{999}$.

(7) Define: Dividend and their types.

(8) Name any three popular indices of the world with the names of respective countries.

(9) Define with examples: Barrier options.

(10) Define: Market price.

2 Attempt the following : (Any two) **14**

(a) Explain: Higher the exercise price more is received for the asset at expiry of put option.

(b) What are options for? Also explain how the options reduce the risk to investors.

(c) Define call option and explain how the call option value is a function of exercise price and time to expiry.

3 Attempt the following : 14

(a) Explain: Higher the asset price on expiry of call option, greater the profit.

OR

(a) Derive the Black- Scholes partial differential equation.

(b) State and prove Itô 's lemma and extend the result for $f \equiv f(S,t)$.

OR

(b) Explain in detail the elimination of randomness from Itô 's lemma.

4 Attempt the following : 14

(a) What is put-call parity?

(b) Pratiksha holds an option on 28th October 2018 to purchase 100 shares of Narayan industries for Rs. 4000 per share after one year. If the up-front premium is Rs. 100 per share and price of share is Rs. 6000 per share on 28th October 2019 then find the total profit to Pratiksha on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

5 Attempt the following : (Any **two**) 14

(a) Why would anyone write an option?

(b) Explain the simple model for asset prices.

(c) State the Black-Scholes differential equation and show that $\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv$ where notations are being usual.

(d) What are dividends? Also explain discrete dividend structure and derive the jump conditions for the same.

Seat No. ______________

HD-003-1164005

M. Sc. (Sem. IV) Examination

April - 2023

Mathematics : EMT-4011

(Financial Mathematics)

Faculty Code : 003
Subject Code : 1164005

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) Attempt all the questions.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Attempt the following (Any Seven) : **14**

(1) Define exercise price and speculative price.

(2) State minimum three differences between option and contracts.

(3) What are look back options ? Give an example.

(4) Obtain the stochastic differential equation for $f(S) = S^{10}$.

(5) Define the term : Sensitivity to volatility and sensitivity to interest rate.

(6) Explain the terms:
(i) Risk free investment, (ii) Dividends

(7) Explain the term financial derivatives and give two examples of it.

(8) Name any two financial markets and their dealings.

(9) Distinguish between European option and American option in minimum three points each.

(10) Name any four popular indices of the world with the names of respective countries.

2 Attempt the following : 14

(a) How much one should pay now to receive a guaranteed amount E at the future time T.

(b) Explain : Higher the exercise price more is received for the asset at expiry of put option.

OR

(b) Akash holds an option on 1st April 2019 to purchase 300 shares of Milan industries for Rs. 5000 per share after one year. If the up-front premium is Rs. 50 per share and price of share is Rs. 6000 per share on 1st May 2020 then find the total profit to Akash on exercising the option. Also find the profit in percentage corresponding to up-front premium paid.

3 Attempt the following : 14

(a) Define call option and explain how the call option value is a function of exercise price and time of expiry.

(b) State and prove Ito's lemma and extend the result for $f \equiv (S, t)$.

OR

(a) Explain the simple model of asset pricing.

(b) Explain in brief the central idea behind the theory and practice of option pricing.

4 Attempt the following : 14

(a) What is put call parity ?

(b) Stating the assumptions of the Black-Scholes analysis, derive the Black-Scholes partial differential equation.

5 Attempt the following (Any Two) : 14

(a) Explain : Higher the asset prices on expiry of call option, greater the profit.

(b) Solve the Black-Scholes differential equation.

(c) Define the term dividend yield and explain in detail the constant dividend yield structure and derive the Black-Scholes partial differential equation corresponding to it.

(d) Explain discrete dividend structure. Also derive the jump conditions for the same.