SHREE H.N.SHUKLA COLLEGE OF SCIENCE

T.Y.B.Sc. SEM-6

PAPER 601

GRAPH THEORY & COMPLEX ANALYSIS-II

UNIT-1:GRAPH THEORY

IMPORTANT DEFINITION

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DEFINITION: GRAPH

- Let $V = \{v_1, \quad v_2, \ldots\ldots\ldots\ldots v_n\}$

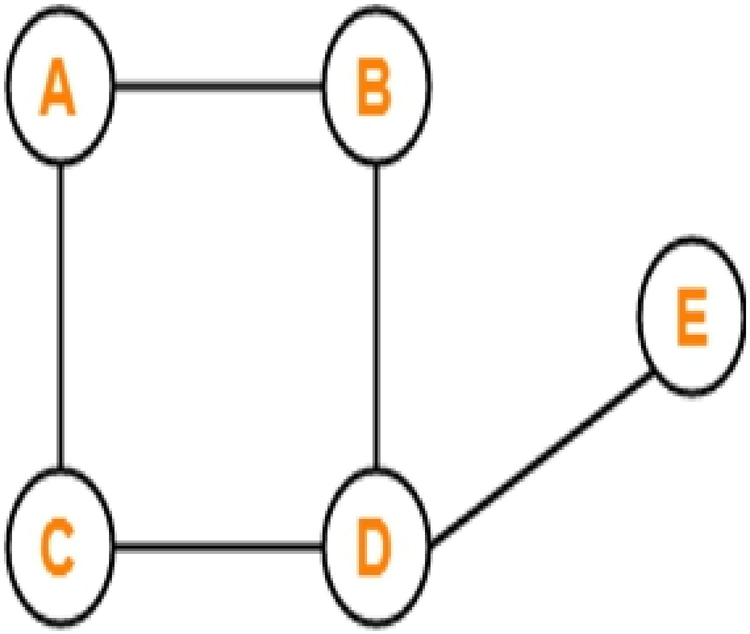
  and $E = \{e_1, \quad e_2, \ldots\ldots\ldots\ldots e_n\}$

  are set of vertices and edges respectively, A mapping $\emptyset : E \rightarrow V \times V$

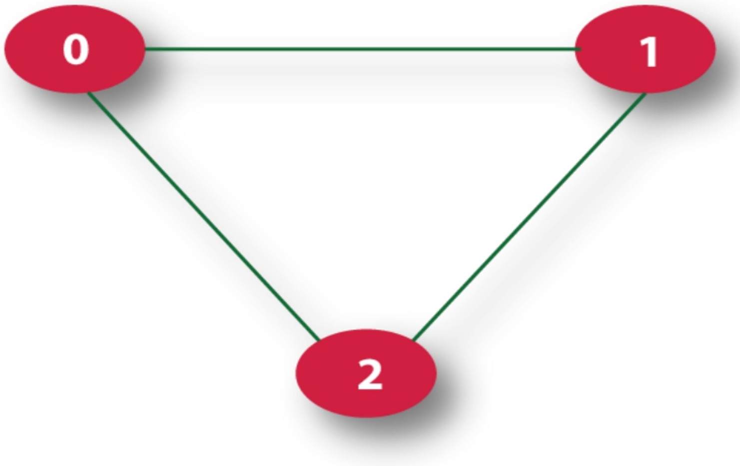
  is called graph and is denoted by G= (V, E).

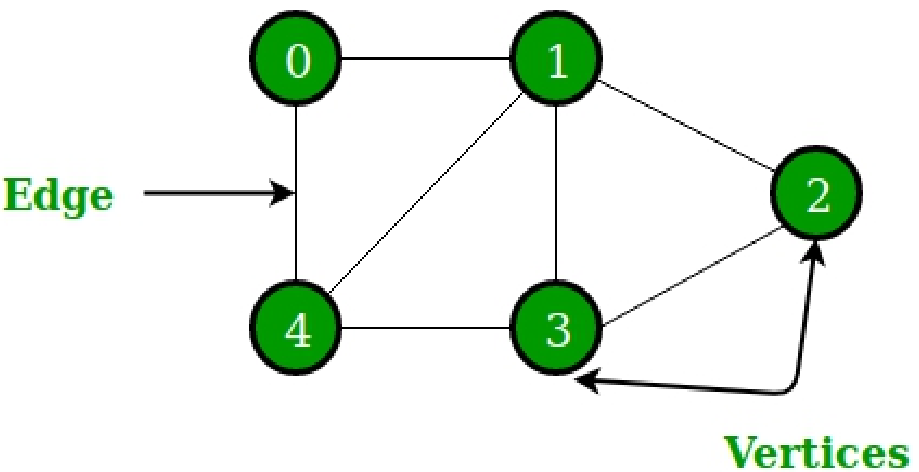
Where, V= the set of vertices

E= the set of edges



Example of Graph

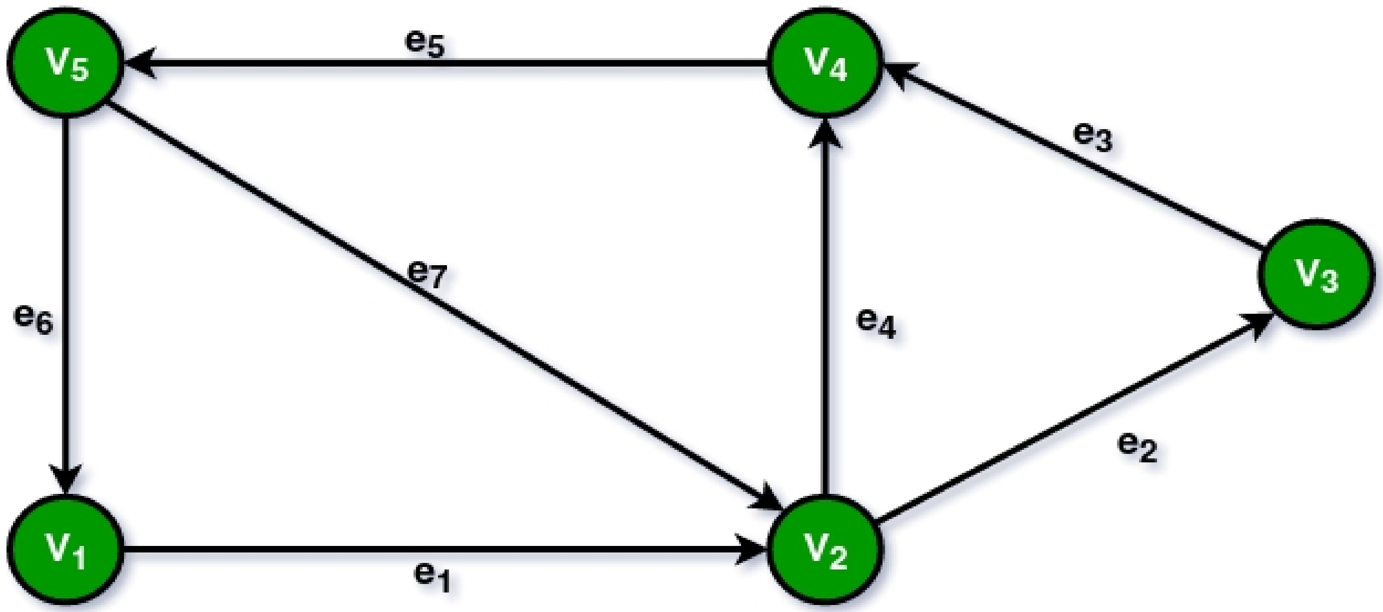




Examples of Graph

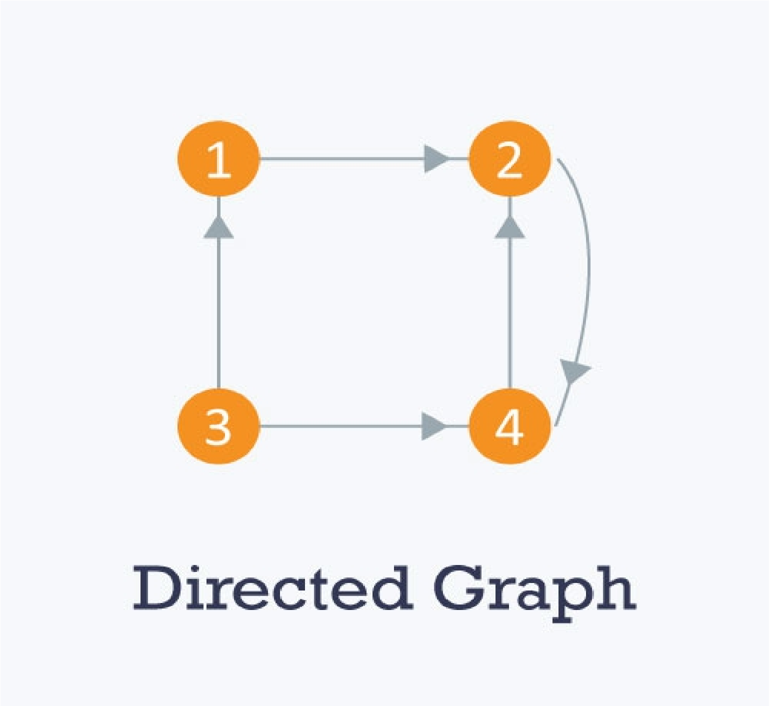
Definition: Directed edges

- Let G= (V, E) be a graph, Edge of graph is correspond to order pair of vertices is called directed edges.



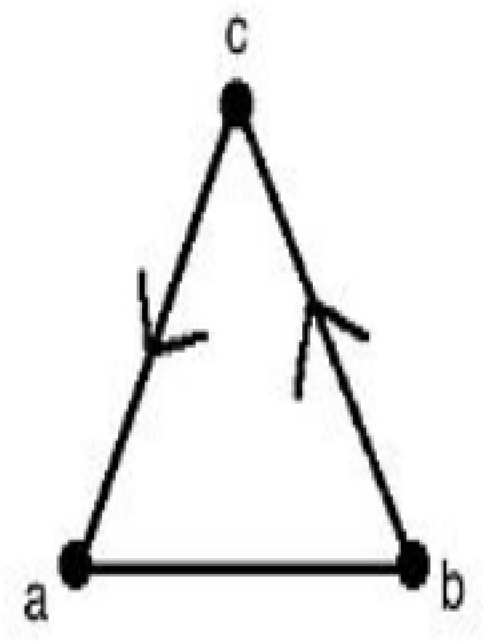
DEFINITION: DIAGRAPH OR DIRECTED GRAPH

- Let G= (V, E) be a graph, All edges of graph G are directed then graph G is called Diagraph OR Directed graph.



DEFINITION: MIXED GRAPH

- Let G= (V, E) be a graph, some edges of graph have direction and some edges of graph have no direction then graph G is called Mixed graph.



DEFINITION: (N, M) GRAPH

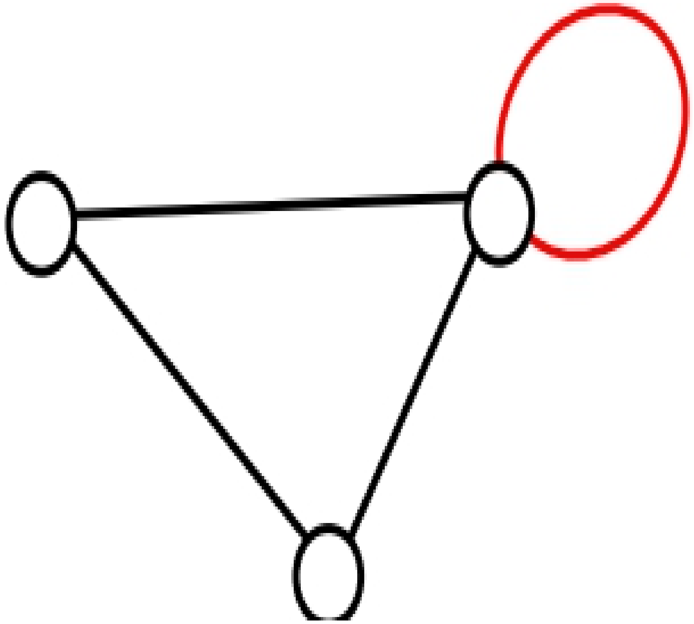
- Let G= (V, E) be a graph, number of vertices is n, i.e. n (V) = n is called order of G ⇒ o (G) = n and number of edges is m, i.e. n (E) = m is called size of graph,
- Then this is called (n, m) graph.

DEFINITION: FINITE GRAPH

- Let G= (V, E) be a graph, V and E are finite set then graph G is called finite graph.

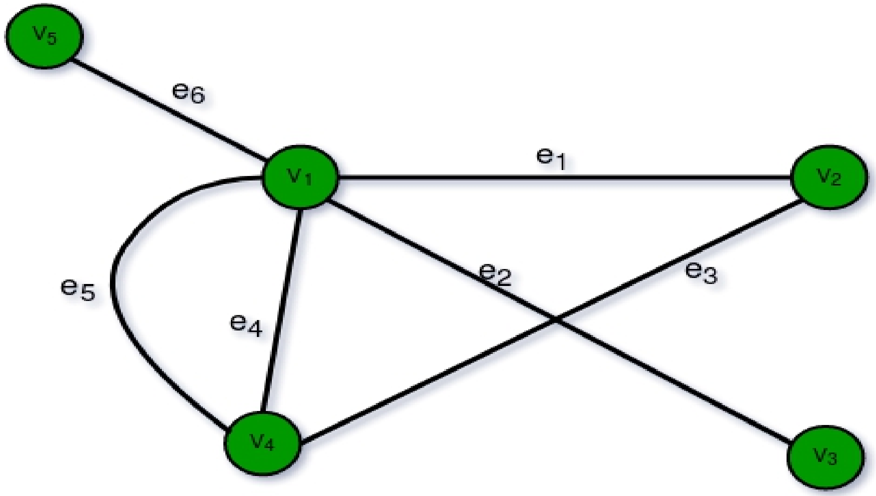
Definition: Loop OR Self loop

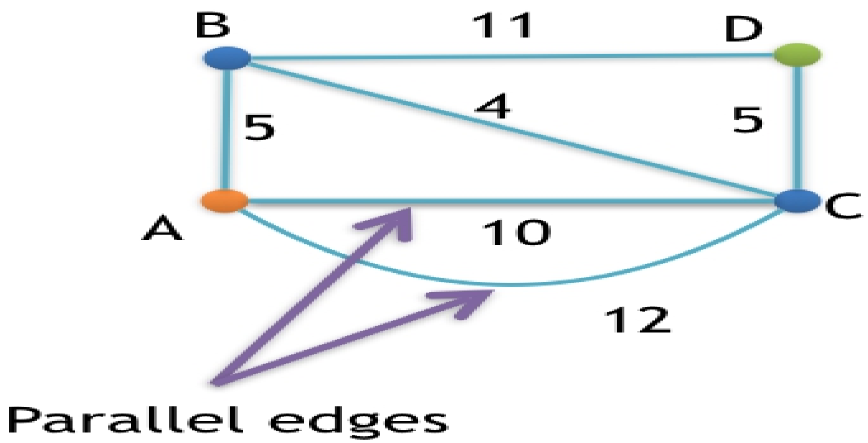
- Let G= (V, E) be a graph, e ϵ E is called loop OR self loop if terminal vertices of e are same



DEFINITION: PARALLEL EDGES

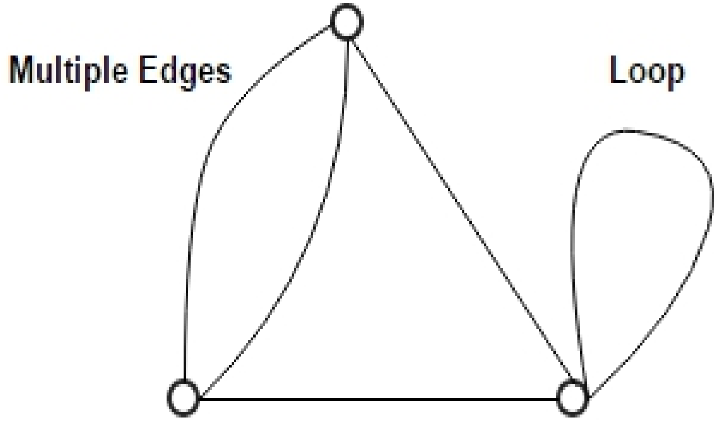
- Let G= (V, E) be a graph, two or more than two edges terminal vertices same which edges are called parallel edges.



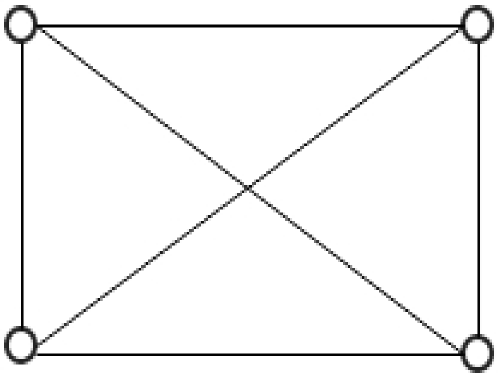


DEFINITION: SIMPLE GRAPH

- Let G= (V, E) be a graph as neither self loop nor parallel edges are called simple graph.



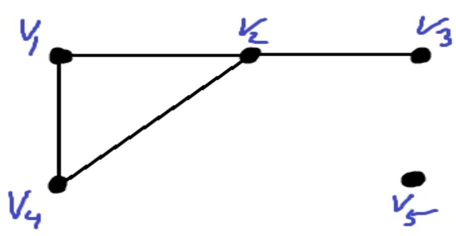
Not a Simple Graph



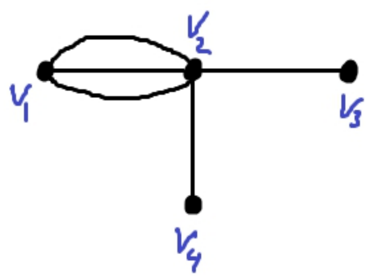
Simple Graph

<u>DEFINITION:</u> DEGREE OF VERTEX

- Let G= (V, E) be a graph, v ϵ V and no. of edges incidence on V is called degree of vertex and is denoted by d(v).
- If E is loop, it is incidence on d then degree of V count twice.



$deg(v_1) = 2$
$deg(v_2) = 3$
$deg(v_4) = 2$
$deg(v_3) = 1$
$deg(v_5) = 0$



$deg(v_1) = 3$
$deg(v_2) = 5$
$deg(v_3) = 1$
$deg(v_4) = 1$

DEFINITION: EVEN AND ODD VERTICES

Let G= (V, E) be a graph, v ϵ V and degree of V is even then V is called even vertices and degree of V is odd then V is called odd vertices.

DEFINITION: PENDENT VERTICES

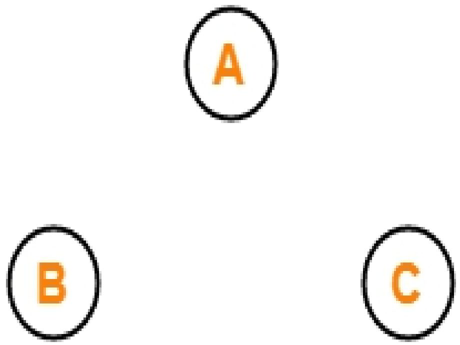
Let G= (V, E) be a graph in v ϵ V, degree of V is 1 [d(v)=1] then V is called pendent vertices.

DEFINITION: ISOLATED VERTICES

Let G= (V, E) be a graph in v ϵ V, degree of V is 0 [d(v)=0] then V is called pendent vertices.

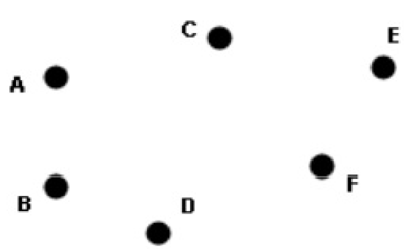
DEFINITION: NULL GRAPH

- Let G= (V, E) be a graph, E=ɸ is called null graph.



Example of Null Graph

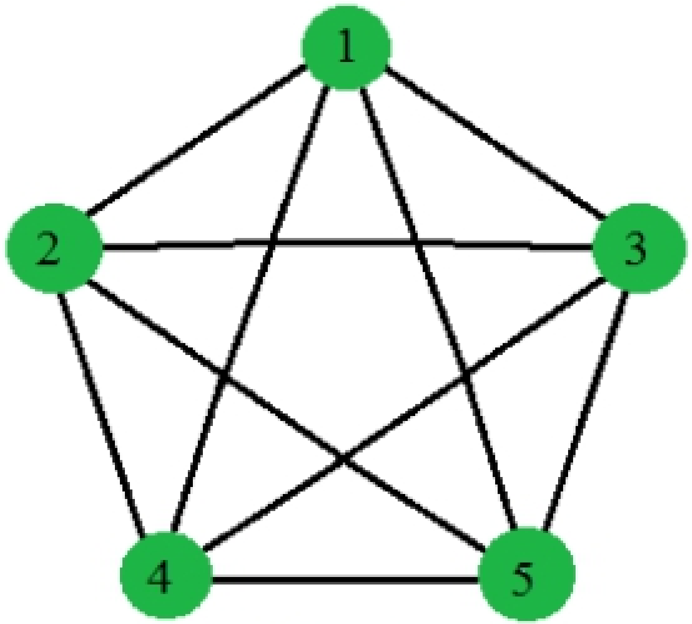
Null Graph



- Degree of all vertices in the null graph is zero.
- All vertices in the null graph is isolated vertex.

Definition: Regular graph

- Let G= (V, E) be a graph, degree of all vertices of G are same then G is called regular graph.
- If ∀ v ϵ V; d(v)=k then G is called k-regular graph.



4 Regular

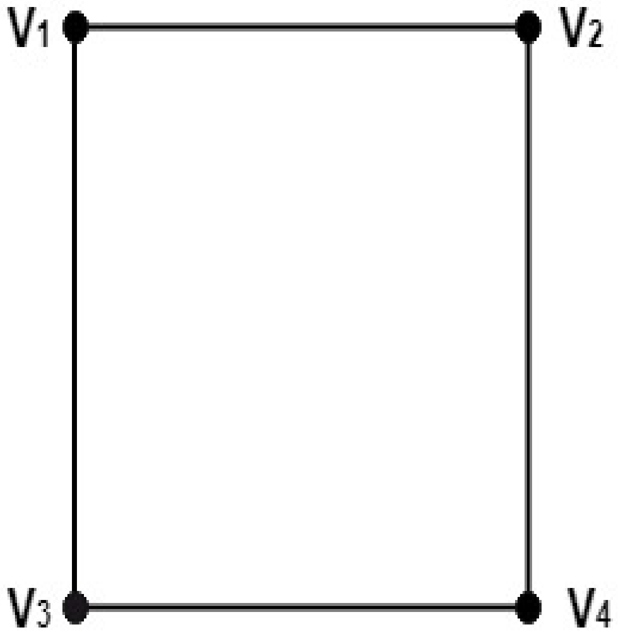


Fig:2-Regular Graph

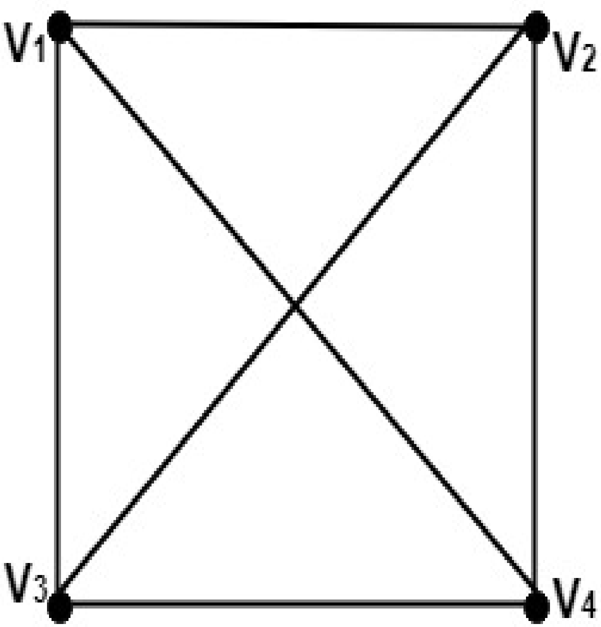


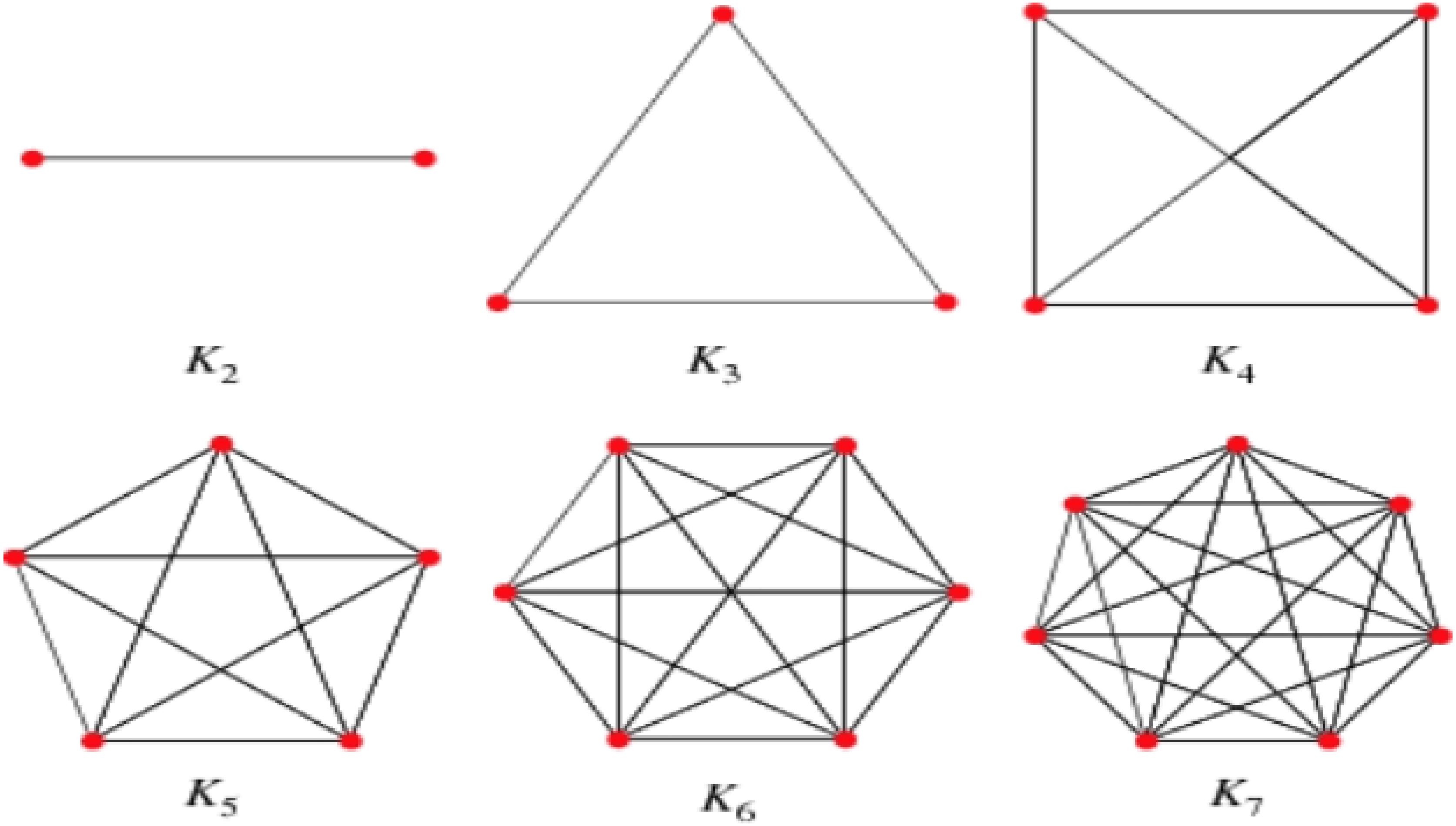
Fig:Regular Graph

DEFINITION: SERIES EDGES

- Let G= (V, E) be a graph, degree of common vertex of adjacent edge is two, which edges are known as series edges.

DEFINITION: COMPLETE GRAPH

A simple graph is said to be complete graph, if existence edges in between every pair of vertices in G and it is denoted by K_n.

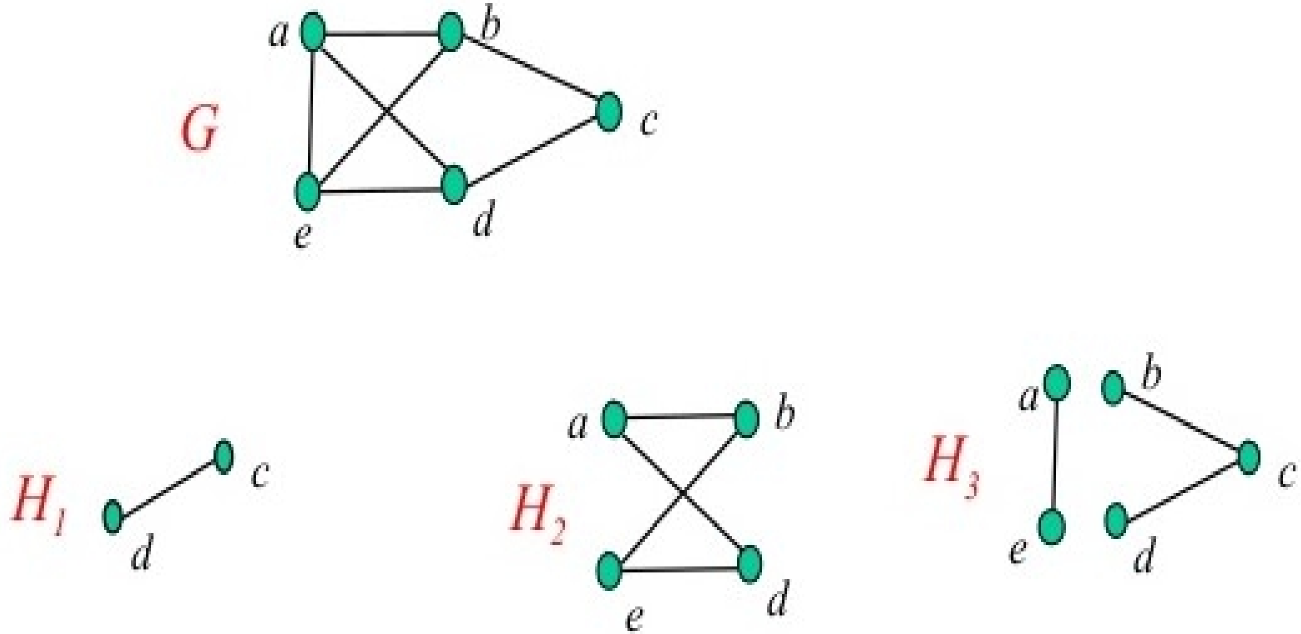


Examples of Complete Graph

DEFINITION: SUB GRAPH

- Let G= (V, E) and G'= (V', E') are two graph,
 If (i) V is subset of V' & E is subset of E'
 (ii) Terminal vertices of every edges in G is also in G'
 Then G is sub graph of G'.

- Example: H_1, H_2, and H_3 are subgraphs of G



DEFINITION: VERTEX DISJOINT SUB GRAPH
EDGES DISJOINT SUB GRAPH

- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are sub graph of G then

  (i) G_1 and G_2 are said to be vertex disjoint sub graph, if $V_1 \cap V_2 = \phi$

  (ii) G_1 and G_2 are said to be edges disjoint sub graph, if $E_1 \cap E_2 = \phi$

DEFINITION: WALK

- Let G= (V, E) be a graph, alternating sequence of vertices and edges which is terminated by vertices such that every vertex and edges are incidence is called walk and it is denoted by W.
- E.g.; $v_1 e_1 v_2 e_2 v_3 \dots \dots \dots \dots . v_{n-1} e_n v_n$

REMARK:

- No edge appears more than once.
- A vertex may appear more than once.
- Walk is also known as edges train OR chain.
- Every walk of G is a sub graph of G.
- Initial and end vertices in walk are called terminal vertices.

DEFINITION: OPEN WALK

- Let G= (V, E) be a graph, initial vertex and end vertex are not same in walk, and then walk is called open walk.

DEFINITION: CLOSED WALK

- Let G= (V, E) be a graph, initial vertex and end vertex are same in walk, and then walk is called open walk.

Definition: Path

- Let G= (V, E) be a graph, in open walk any vertex is not repeat is called path and it is denoted by P_n.

REMARK

- Number of edges in path is called the length of path.
- Path does not contain loop but contain walk.
- Degree of all vertices other than terminal is 2.
- Degree of terminal vertices is 1.
- Every path is open walk but converse is not true.

DEFINITION: CIRCUIT

- Let G= (V, E) be a graph, in closed walk any vertex is not repeat except terminal vertices is called circuit and it is denoted by C_n.

REMARK

- Degree of any vertex in circuit is 2.
- Circuit is also known as cycle OR Polygon.
- Every loop is circuit but converse is not true.
- A circuit contain n-edges is called n-cycle.

DEFINITION: EULER LINE

- Let G= (V, E) be a graph, a closed walk contains all edges of G, which walk is called Euler line.

DEFINITION: EULER GRAPH

Let G= (V, E) be a graph, G contain Euler line then G is called Euler graph.

DEFINITION: UNICURSAL LINE

- Let G= (V, E) be a graph, an open walk contain all vertex of G then walk is called Unicursal line.

DEFINITION: UNICURSAL GRAPH

Let G= (V, E) be a graph, G contain Unicursal line then G is called Unicursal graph.

<u>DEFINITION:</u> HAMILTONIAN CIRCUIT

- Let G= (V, E) be a graph, a circuit contain all vertices of G, which circuit is called Hamiltonian circuit.

<u>DEFINITION:</u> HAMILTONIAN PATH

Let G= (V, E) be a graph, a path contains all vertices of G, which path is called Hamiltonian path.

DEFINITION: CONNECTED GRAPH

- Let G= (V, E) be a graph and v, u ϵ V there exist at least one path between u and v in G, then G is called connected graph.
- Otherwise G is called disconnected graph.

DEFINITION: UNION GRAPH

- Let G_1= (V_1, E_1) and G_2= (V_2, E_2) are two graph, V=V_1 ∪ V_2 and E=E_1 U E_2 then G= (V, E) is called union graph.

Definition: Intersection graph

- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are two graph, $V = V_1 \cap V_2$ and $E = E_1 \cap E_2$ then $G = (V, E)$ is called intersection graph of G_1 and G_2.

Definition: Ringsum graph

- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are two graph, $V = V_1 \cup V_2$ and $E = (E_1 \cup E_2) - E_1 \cap E_2 = E_1 \Delta E_2$ then $G = (V, E)$ is called Ringsum graph.

DEFINITION: COMPLEMENT OF GRAPH

- Let G= (V, E) be a simple graph, E^c is complement of E, E^c= (V*V) – E then G'= (V, E^c) is said to be complement of G.

DEFINITION: DECOMPOSITION OF GRAPH

Let G_1 and G_2 are two sub graph of G such that $G_1 \cap G_2 = \phi$ and $G_1 \cup G_2 = G$ then G is said to be decompose into G_1 and G_2.

DEFINITION: VERTEX DELETED GRAPH

- Let G= (V, E) be a graph, v_i ϵ V, we remove v_i from the graph G then G – {v_i} is said to be vertex deleted graph.

DEFINITION: EDGE DELETED GRAPH

Let G= (V, E) be a graph, e_i ϵ E, we remove e_i from the graph G then G – {e_i} is said to be edge deleted graph.

Definition: Fusion of graph

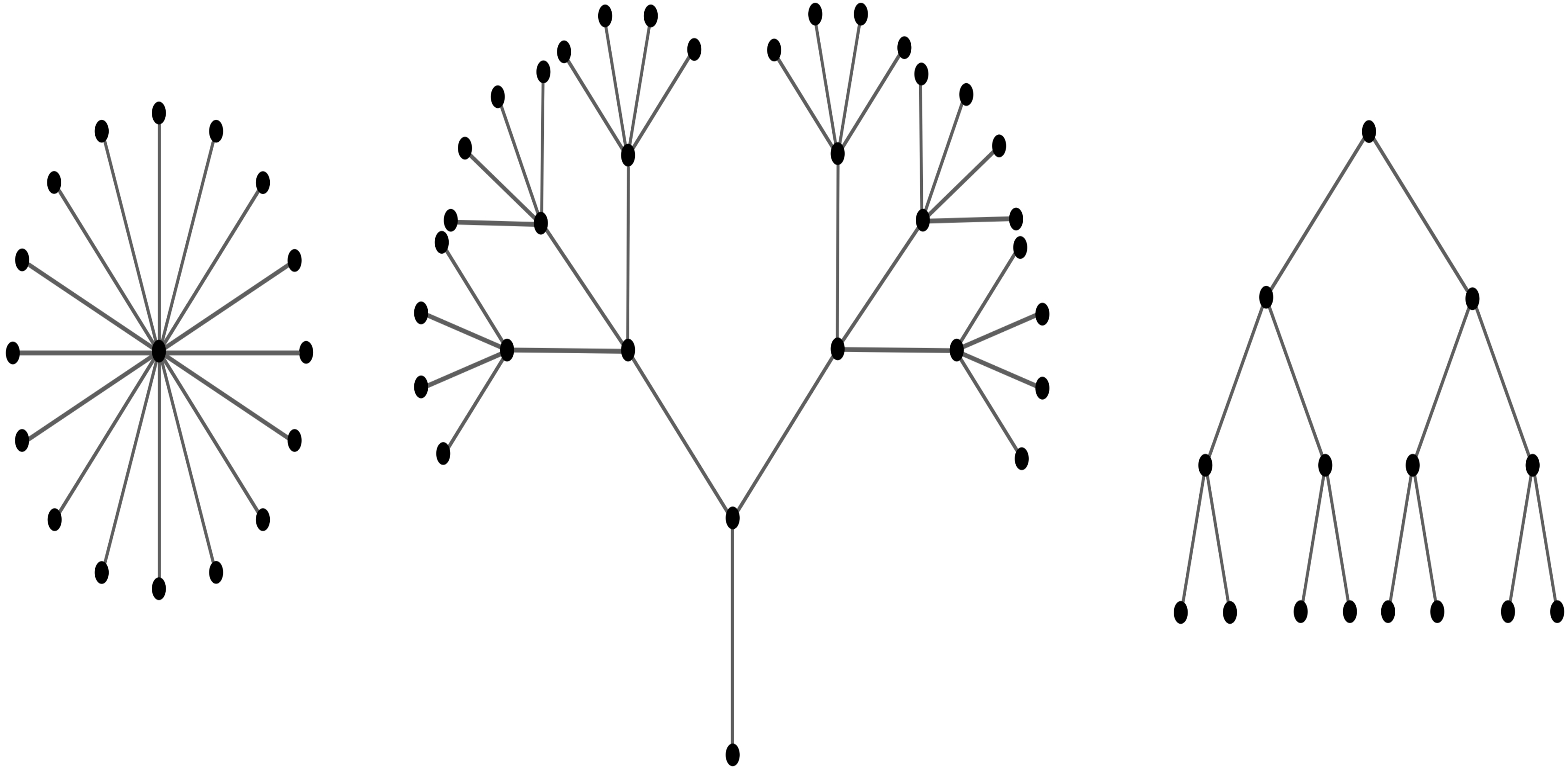
- Let G= (V, E) be a graph and u, v ϵ V, we apply vertex x in place of u and v are incidence on x and we get new graph G is called Fusion of graph.

Definition: Tree

- A connected graph G is said to be tree, if G has no circuit.

Remark

- Tree is connected and number of edges in tree with n-vertices is (n-1).
- Every pair of vertices has at least one path in G.
- Tree is minimal connected graph.
- The collection of tree is called Forest.



Examples of Tree

<u>Definition:</u> Minimal connected graph

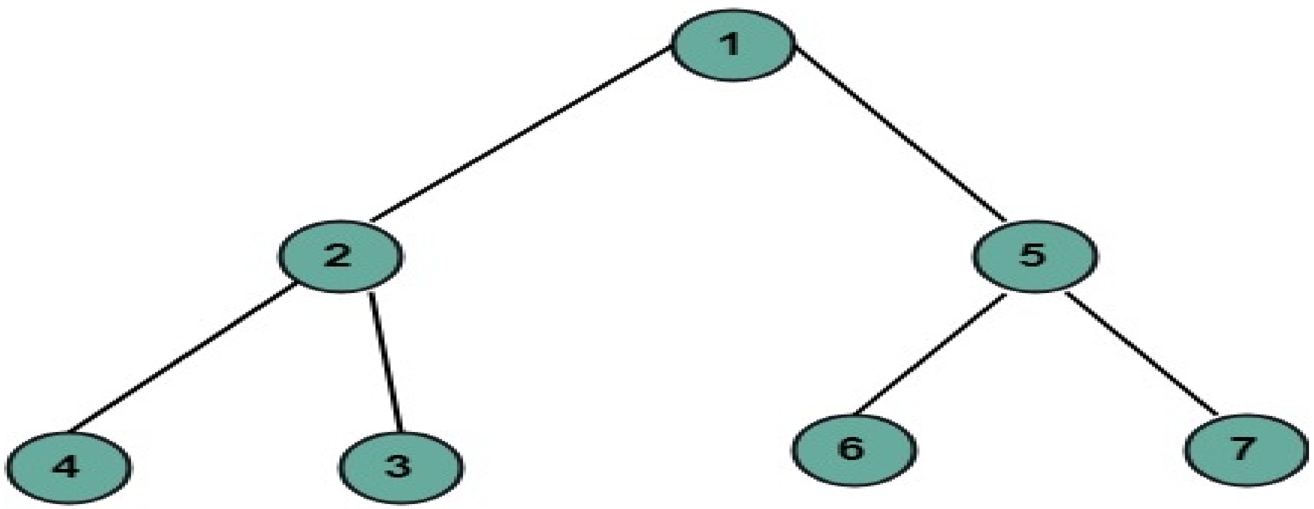
- G is connected graph, one edge remove from graph and graph is disconnected then G is called Minimal connected graph.

<u>Definition:</u> Rooted tree

- Any one vertex in T (tree) different to other vertices it is called root and tree containing rooted tree.

DEFINITION: BINARY TREE

- In tree degree of one vertex is 2 and degree of other vertices is one OR three which tree is said to be Binary tree.



Full Binary Tree

PROPERTIES OF BINARY TREE:

- All vertices of Binary tree except pendent vertex is called internal vertex.
- In Binary tree degree of vertex two is root of tree.
- Level: In a Binary tree a vertex v_i is said to be at level l_i, if v_i is at a distance of l_i from the root and the root is at level zero.
- The maximum level of any vertex in a Binary tree is called the height of the tree.

PROPERTIES OF BINARY TREE:

- Minimum possible height of n-vertex, Binary tree is minlmax;

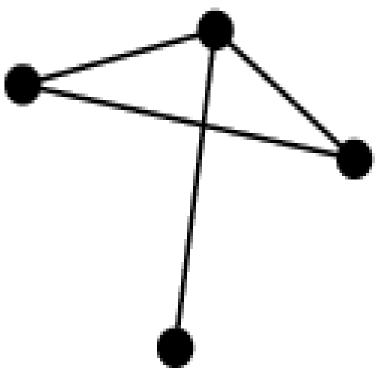
$$minlmax = [\log_2(n+1) - 1]$$

- Maximum possible height of n-vertex, Binary tree is maxlmax;

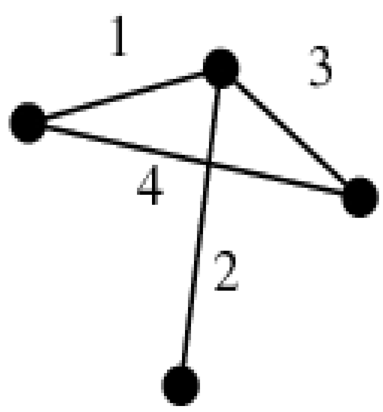
$$maxlmax = \frac{n-1}{2}$$

DEFINITION: LABEL GRAPH

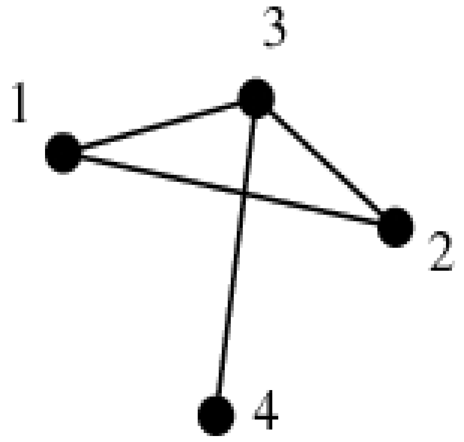
- A graph in which vertex T is a sign is a unique name of label is called Label graph.
- The number of Label tree with n-vertices is n^{n-2}.



unlabeled graph



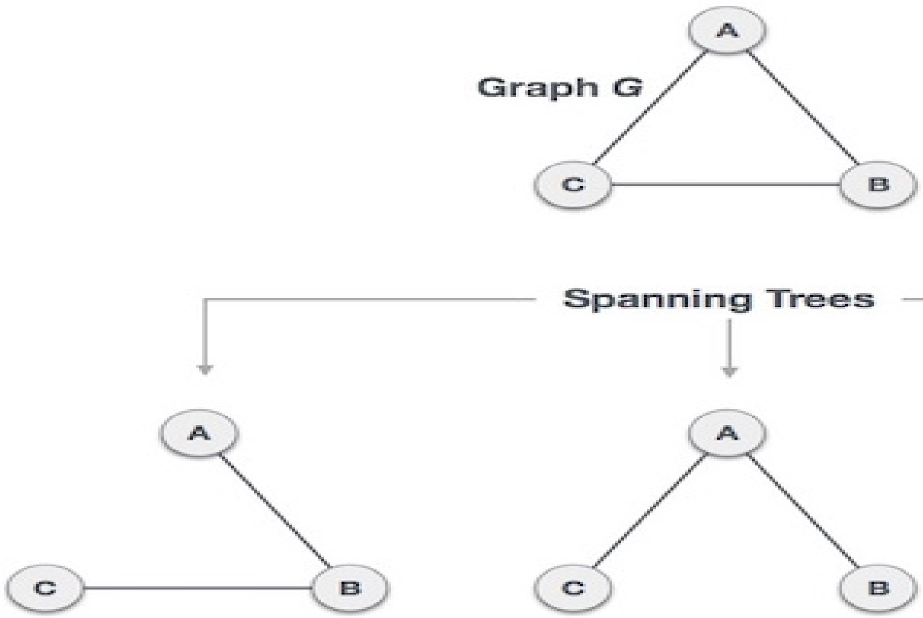
edge-labeled graph



vertex-labeled graph

DEFINITION: SPANNING TREE

- A tree T is said to be spanning tree of a connected graph G, if T is a sub graph of G and T contains all vertices of G.



NOTE:

- Edges in a spanning tree are called branches of tree.
- Edges of G it is not given spanning tree is called chord.
- Number of vertices is n, number of edges e in G then number of branches is spanning tree with respect to G is (n-1).
- Number of chord in spanning tree with respect to connected graph G is (e-n+1).

<u>Definition:</u> Fundamental number

- Number of vertices is n, number of edges is e and numbers of components k are called fundamental number in graph theory.

<u>Definition:</u> Rank of graph

- Fundamental number n, e, k in G then (n-k) is said to be Rank of G and it is denoted by r.
- **<u>i.e.</u>** r=n-k

DEFINITION: NULLITY OF GRAPH

- Fundamental number n, e, k in G then (e-n+k) is said to be Nullity of G and it is denoted by μ.
- **i.e.** $\mu = e - n + k$

DEFINITION: ISOMORPHISM OF TWO GRAPHS

- Two graphs G and G' are said to be isomorphic, if there is a one-one correspond between their vertices and between their edges such that the incidence relation with preserve.


THANK
YOU