

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 1 JANUARY 2025 (Regular) JANUARY - 2025

REAL ANALYSIS Faculty Code: 003 Subject Code: 16SI MSMA-CO-01-00002 Time: 230 Hours] Instruction All questions are compulsary

Answer Briefly any seven of the following (Out of ten) Q.1

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Define term: σ algebra on a non-empty set XLet $A, B \subseteq \mathbb{R}$ be any subsets and $m^*(B) \bigodot$. Prove that, $m^*(A \cup B) = m^*(A)$. Prove that, $m^*(A+y) = m^*(A)$, for any \mathbb{R} and

 $y \in \mathbb{R}$, where $A + y = \{x + y/x \in A\}$.

Prove or disprove that, the continuous function $\mathcal{F}^{\mathbb{R}} \to \mathbb{R}$ is a measurable function.

Define F_{σ} – set. Justify that, an open interval f_{σ} – set.

Write down any two from Littlewood's three principles without proof.

Define Lebesgue integral for a simple function on a measurable set E which vanishes outside of set of finite 7 measure.

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Define a characteristic function on a measurable set E

- 10 Let E be a measurable subset of \mathbb{R} and $f: E \to \mathbb{R}$ be a measurable function. Prove that, for any $f: E \to \mathbb{R}$
- number α , $\{x \in E/f(x) = \alpha\}$ is a measurable subset of E.
- Let $f, g: E \to R$ be two real valued functions on a measurable set E. Let f be a measurable function on E and f = g a.e. on E. Prove that, E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E and E is also a measurable function of E is Q.2 Answer the following (Any Two)

- Let $E_1, E_2, ..., E_n$ be a finite sequence of inputally disjoint measurable set. Let $A \subseteq \mathbb{R}$ be any pulset of \mathbb{R} . Prove that, $m^*(A \cap [U_{i=1}^n E_i]) = \sum_{i=1}^n m^*(A \cap E_i)$. RAJ01072657
 - Let D be a measurable subset of \mathbb{R} and $E \subseteq D$. Let \mathcal{X}_E is the characteristic function on D. Prove
- Q.3 Answer the following

 State and prove, Fate

 Let be a me that, x_E is a measurable function on $E \Leftrightarrow E$ is a measurable subset of \mathbb{R} .

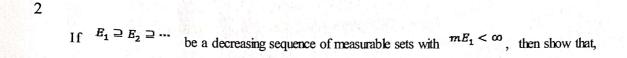
 Were the following State and prove, Fatou's lemma.

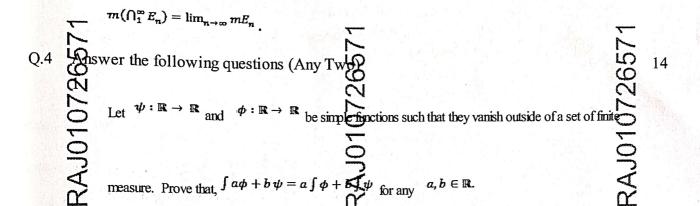
 Let E be a measurable set with E and E be bounded function. Prove the prove that E is a measurable function on E is a measurable subset of E.

 $\inf_{\psi \geq f} \int_{E} \psi \geq \sup_{\phi \leq f} \int_{E} \phi$. Where $\psi : E \to \mathbb{R}$ and $\phi : E \to \mathbb{R}$ are simple functions.

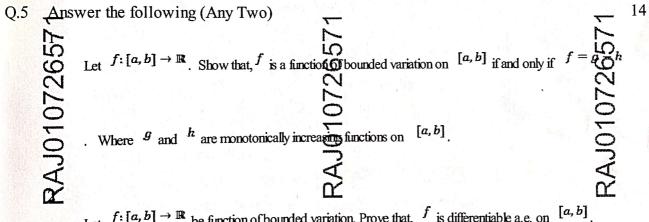
Answer the following

OR





2 Prove that, Lebesgue Convergence Theorem holds good if convergence a.e. is replaced by convergence in measure (convergence in sense of measure).



Let $f: [a,b] \to \mathbb{R}$ be function of bounded variation. Prove that, f is differentiable a.e. on [a,b]

