Seat No. ________

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 1 JANUARY 2025 (Regular) JANUARY - 2025

REAL ANALYSIS

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-01-00002

Time : 2:30 Hours] **[Total Marks : 70**

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Define term: $\sigma-$ algebra on a non-empty set X.

2 Let $A, B \subseteq \mathbb{R}$ be any subsets and $m^*(B) = 0$. Prove that, $m^*(A \cup B) = m^*(A)$.

3 Prove that, $m^*(A+y) = m^*(A)$, for any $A \subseteq \mathbb{R}$ and $y \in \mathbb{R}$, where $A+y = \{x+y / x \in A\}$.

4 Prove or disprove that, the continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ is a measurable function.

5 Define $F_\sigma -$ set. Justify that, an open interval is an $F_\sigma -$ set.

6 Write down any two from **Littlewood's three principles** without proof.

7 Define Lebesgue integral for a simple function on a measurable set E which vanishes outside of set of finite measure.

8 Define a characteristic function on a measurable set E.

9 Write the statement of Monotone Convergence Theorem.

10 Let E be a measurable subset of $\mathbb{R}$ and $f: E \longrightarrow \mathbb{R}$ be a measurable function. Prove that, for any real number α, $\{x \in E / f(x) = \alpha\}$ is a measurable subset of E.

Q.2 Answer the following (Any Two) 14

Let $f, g: E \rightarrow R$ be two real valued functions on a measurable set E. Let f be a measurable function on E and $f = g$ a.e. on E. Prove that, g is also a measurable function on E.

Let $E_1, E_2, \ldots, E_n$ be a finite sequence of mutually disjoint measurable set. Let $A \subseteq \mathbb{R}$ be any subset of $\mathbb{R}$. Prove that, $m^*(A \cap [\cup_{i=1}^{n} E_i]) = \sum_{i=1}^{n} m^*(A \cap E_i)$.

3 Let D be a measurable subset of $\mathbb{R}$ and $E \subseteq D$. Let χ_E is the characteristic function on D. Prove that, χ_E is a measurable function on $E \Leftrightarrow E$ is a measurable subset of $\mathbb{R}$.

Q.3 Answer the following 14

State and prove, Fatou's lemma.

2 Let E be a measureable set with $mE < \infty$ and $f : E \rightarrow \mathbb{R}$ be bounded function. Prove that $\inf_{\psi \geq f} \int_E \psi \geq \sup_{\phi \leq f} \int_E \phi$. Where $\psi : E \rightarrow \mathbb{R}$ and $\phi : E \rightarrow \mathbb{R}$ are simple functions.

OR

Answer the following 14

1 State and prove, Lebesgue Convergence Theorem.

2

If $E_1 \supseteq E_2 \supseteq \cdots$ be a decreasing sequence of measurable sets with $mE_1 < \infty$, then show that,

$m(\cap_1^\infty E_n) = \lim_{n\to\infty} mE_n$.

Q.4 Answer the following questions (Any Two) 14

Let $\psi : \mathbb{R} \to \mathbb{R}$ and $\phi : \mathbb{R} \to \mathbb{R}$ be simple functions such that they vanish outside of a set of finite

measure. Prove that, $\int a\phi + b\psi = a\int \phi + b\int \psi$ for any $a, b \in \mathbb{R}$.

2 Prove that, Lebesgue Convergence Theorem holds good if convergence a.e. is replaced by convergence in measure (convergence in sense of measure).

Q.5 Answer the following (Any Two) 14

Let $f: [a, b] \to \mathbb{R}$. Show that, f is a function of bounded variation on $[a, b]$ if and only if $f = g - h$

. Where g and h are monotonically increasing functions on $[a, b]$.

Let $f: [a, b] \to \mathbb{R}$ be function of bounded variation. Prove that, f is differentiable a.e. on $[a, b]$.

3 State and prove, Bounded Convergence Theorem.

4 Show that, the outer measure is sub additive.