Shree H.N.Shukla College of Science

MATHEMATICS

S.Y.B.Sc. (Sem.IV) (CBCS)

PAPER- 401

Linear Algebra, Real Analysis & Differential Geometry

QUESTION BANK

❖ Answer the following: [1 mark questions]

1) Define: monotonic sequence
2) State Cauchy's General Principle of Convergence.
3) Define: Subsequence of a sequence
4) Give an example of a sequence which is lower bounded.
5) If the D'Alemert's ratio test fails, then what to do?
6) Narrate the Libnitz test for convergence of an alternating series.
7) Write the condition for convergence of $1 + r + r^2 + r^3 + \ ....$
8) Define: Oscillatory series
9) Define: Kernel of a linear transformation
10) Define: Linear transformation
11) Let T: U→V is a linear transformation. Let θ and θ' be zero vectors for U and V respectively, Prove that T(θ)=θ'
12) Define: Idempotent linear transformation
13) Define: Dual of a vector space
14) If dimU=m, dimV=n then what is the dimL (U, V)?
15) Define: Eigen value of a linear transformation
16) Define: Diagonalization of a linear transformation
17) Write the formula to find radius of curvature of the curve given by $r = f(\theta)$.
18) Define: Multiple point of a given curve
19) Find the radius of the curvature of the curve $$s = 4a \sin \psi$$
20) Define: Point of inflexion
21) Construct a subsequence of the sequence $\left\{2, \frac{1}{2}, 3, \frac{1}{3}, 4, \frac{1}{4}, . \ldots\right\}$ which is decreasing.
22) Sequence $\{S_n\}$ is defined as $S_1 = 1, 4S_n + 1 = 3S_n + 2$. Find its limit.

23) State the Bolzano-Weirsstrass theorem.

24) Define: Power series

25) Narrate P-series test

26) Define: Zero linear transformation

27) If dimU=4, dimV=3, then find the dimL (U, V).

28) Define: Eigen basis of a linear transformation

29) Define: Double point

30) Define: Node

31) Radius of curvature of $y^2=12x$ at the point (0,0) is ___________.

32) The curve $y=x^4$ is concave upwards at the point ___________.

33) A point on the curve through which pass two real branches of the curve is called ________.

34) What is the curvature at any point of the circle of radius r?

35) The curve $y=x^3$ has a point of inflexion at __________.

36) A possible double point on the curve $(y-x)^2+x^6=0$ is/are _______.

37) Define: Curvature and radius of curvature

38) The asymptote parallel to y-axis for the curve $x^{-2}+y^{-2}=1$ is _________.

❖ Answer the following: [2 mark questions]

1) Show that every convergent sequence is bounded.
2) Determine the sequence is convergent or not:

$$\left\{\frac{5n+(-1)^n}{2n+5}\right\}$$

3) Show that the series $\frac{1}{2}-\frac{2}{5}+\frac{3}{10}-\frac{4}{17}+\frac{5}{26}-$ ….. … … is absolutely convergent.
4) Test the convergence of ;

$$\sum_{n=0}^{\infty}\frac{3^{2n}}{2^{3n}}$$

5) Define: (i) Range of a linear transformation
   (ii) Non-singular linear transformation
6) Let T: $R^2 \to R^2$, T(x, y) = (x+1, y-2); ∀(x, y) ϵR^2, then show that T is not a linear transformation.
7) Let T: $R^2 \to R^2$, T(x,y)=(x, -y), ∀(x,y)ϵR^2 and B_1={(1, 1), (1, 0)} and

B_2= {(2, 3), (4, 5)}. Then find [T; B_1, B_2].

8) Define: Matrix associated with a linear transformation
9) For the curve

$$(x^2 + y^2)x - ay^2 = 0 ,$$

Prove that the origin is cusp.

10) Prove that $y = \log x$ is convex upward everywhere.
11) Find the radius of curvature of the curve $y = c \cosh\frac{x}{c}$.
12) Find asymptotes of the curve $(x^2 + y^2)x - ay^2 = 0$ parallel to co-ordinate axes.
13) Find points of inflexion of the curve $x^2y - 4x + 3y = 0$.
14) Determine the sequence is convergent or not: $\left\{1 + \frac{(-1)^n}{n}\right\}$
15) Prove that every convergent sequence is bounded.
16) Show that the series $\sum(-1)^n$ oscillates finitely.
17) Find a linear transformation T: $R^3 \to R^3$ such that R_T=SP{(1,5,0), (0,7,3)}
18) Find N_T and n(T) for the linear transformation T: $R^3 \to R^2$, T(x,y,z)=(x-y+z,x+y-z), ∀(x,y,z)ϵR^3

❖ Answer the following: [3 mark questions]

1) Show that the sequence {S_n} defined by $S_1 = \sqrt{2}$ & $S_{n+1} = \sqrt{2S_n}$ converges to 2.
2) Show that

$$\lim_{n\to\infty} \frac{1 + \frac{1}{3} + \frac{1}{5} + \;\dots\dots\dots\dots\dots + \frac{1}{2n-1}}{n} = 0$$

3) Test the convergence or divergence of the series:

$$\frac{1}{4*6} + \frac{\sqrt{3}}{6*8} + \frac{\sqrt{5}}{8*10} + \frac{\sqrt{7}}{10*12} + \;\dots\dots\dots\dots$$

4) Find the radius and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

5) Find the linear transformation T: $R^3 \to R^2$ such that $T(e_1)=(1, 1)$, $T(e_1+e_2)=(1, 0)$, $T(e_1+e_2+e_3)=(1, -1)$. Also find T (2, 5, 7), where $\{e_1, e_1+e_2, e_1+e_2+e_3\}$ is a basis of R^3.
6) Let T: $V \to V$ be any linear transformation such that $T^2-T+I=0$ then prove that T is non-singular.
7) Find Eigen values of the linear transformation $T:R^3 \to R^3$; T (a, b, c) = (a+b+c, a+b+c, a+b+c).
8) Linear transformation T: $P_2(R) \to P_3(R)$ is defined by $T\big(p(x)\big) = \int_o^x p(x)dx$. $B_1 = \{1, x, x^2\}$ and $B_2 = \{1, x, x^2, x^3\}$ are bases of $P_2(R)$ and $P_3(R)$ respectively. Find $[T: B_1, B_2]$.
9) Show that the parabola $y^2 = 4ax$ has no asymptotes.
10) Find the radius of curvature at origin for the curve $x^3 + y^3 = 3axy$ using Newton's method.
11) Find all asymptotes of the curve $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 = 1$.
12) Find double point of the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$.
13) Find oblique asymptote of the curve

$$y = \frac{x^2 + 2x - 1}{x}$$

14) Find Eigen values of the linear transformation T: $R^2 \to R^2$ defined as T(x, y) = (3y, 2x-y).
15) $A = \begin{bmatrix} 3 & 2 & 5 \\ -1 & 4 & -6 \end{bmatrix}$ is the corresponding matrix of linear transformation T: $R^3 \to R^2$ with standard bases of R^3 and R^2. For the new bases B_1={(1,1,1), (1,1,0), (1,0,0)} and B_2={(1,1), (1,0)} . Find $[T;B_1, B_2]$.
16) Show that $\{S_n\}$ is not convergent,
Where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots\ldots\ldots. \frac{1}{n}$
17) Find the radius of convergence for the series: $\sum_{n=0}^{\infty} \frac{x^n}{n+2}$

❖ Answer the following: [5 mark questions]

1) State and prove Cauchy's first theorem on limits.
2) Show that the sequence $\{S_n\}$ defined by

$$S_1 = 1 \;\&\; S_{n+1} = \frac{4+3S_n}{3+2S_n}, \forall n \in N$$ is convergent and find its limit.

3) Discuss the convergence of

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots\ldots\ldots\ldots\ldots$$

4) Discuss the convergence of

$$\sum \frac{1^2 * 4^2 * 7^2 * \ldots\ldots\ldots\ldots\ldots (3n-2)^2}{3^2 * 6^2 * 9^2 * \ldots\ldots\ldots\ldots (3n)^2}$$

5) Prove that L (U, V) is a vector space over F=R with respect to addition & scalar multiplication of linear transformation, where L (U, V)=the set of all linear transformations from U to V.
6) State and prove Rank-nullity theorem.
7) Find the Eigen value and Eigen vector for the linear transformation T: $R^3 \to R^3$, T(x, y, z) = (-2y-2z, -2x-3y-2z, 3x+6y+5z), ∀(x, y, z) ϵ R^3 by considering the standard basis of R^3.
8) Let T: V→V be a linear transformation and let B be any basis of V. Then T is singular if and only if det ([T; B]) =0.
9) Discuss double points of the curve

$$x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$$

10) Show that the radius of curvature of any point on the cardiod $r = a(1+\cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$. Hence prove that $\frac{\rho^2}{r}$ is constant.
11) Find the position and nature of double points of the curve

$$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0.$$

12) Find all asymptotes to the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy + 4x + 1 = 0.$$

13) Find asymptotes parallel to co-ordinate axis for the following curves:
    (i) $y(x^2-1) = x$
    (ii) $x^2y - 3x^2 - 5xy + 6y + 2 = 0$

******BEST OF LUCK******