Shree H.N. Shukla Group of Colleges

M.Sc. SEMESTER 4

Sub. Code: CMT-4004

Core Sub. 4: Graph Thoery

Question Bank

1 (a) Define following terms :

(i) Degree of a vertex in a graph

(ii) Simple graph

(iii) K-regular graph

(iv) Isomorphism of graphs

(v) Walk.

(b) Let $G=(V,E)$ be a graph with $|V|=m$ and $|E|=n$. Then prove that $\sum_{v\in V} d(v)=2n$. Using this deduce that the number of odd vertices (whose degree is odd) is always even.

(c) Draw a simple graph $G=(V,E)$ with $|V|=7, |E|=14$ and G has vertex V whose degree is less than or equal to 1.

2 Suppose $G=(V,E)$ be a finite graph. Then prove that $\exists\ g_1, g_2, \ldots g_k$ subgraphs of $G \ni g_1=(V_i, E_i),\ \forall i=1,2,..,k$ with following properties :

(i) Each g_i is maximal connected subgraph of G.

(ii) $V_i \cap V_j=\phi,\ \forall\, i, j\in\{1,\ldots,k\}$ and $i\neq j$.

(iii) $V=\bigcup_{i=1}^{n} V_i$ and $E=\bigcup_{i=1}^{k} E_i$.

(iv) If $g=(W,F)$ is any connected subgraph of G, then g must be a subgraph of g_i, for some $i\in\{1,\ldots,k\}$.

3 (a) For a simple graph $G(V,E)$, in standard notation prove that.

$$e \le \frac{(n-k)(n-k+1)}{2}$$

(b) Draw a connected graph $G=(V,E)$ with $|V|=8$, $|E|=9$, and $\exists\ v \in V \ni\ d_G(v) \ge 6$.

4 State and prove theorem of A. Dirac.

5 Define a tree. Let u, v be two distinet vertices of a tree T. Then prove that $\exists$ a unique path between u and v in T.

Suppose G is a self loop less graph and for any pair u, v of vertices in G $\exists$ a unique path between u and v in G. Then show that G must be a tree.

6 (a) Define minimally connected graph. Suppose G be a connected graph. Then prove that G is minimally connected iff G. is a tree.

(b) Let $G=(V,E)$ be a connected graph and S be a cut-set of G and Γ be a circuit of G. Then prove that $|S \cap E(\Gamma)| =$ even.

7 (a) Let $G=(V,E)$ be a non-complete connected graph with $|V| \ge 3$. Then show that the vertex connectivity of $G \le$ the edge connectivity of G.

(b) In standard notation prove that W_S and W_Γ both are orthogonal subspaces of W_G, where W_G is a vector space associate with a graph G.

8 (a) Suppose adjacent matrix for a graph G is given as follows :

$$X = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Then find $Y = X + X^2 + X^3 + X^4$. Also deduce from Y that G is a connected graph or not.

(b) Prove that any tree with at least two vertices is a 2-chromatic graph.

9 State and prove Eulerian theorem.

10 State and prove Max flow min cut theorem .

11 Prove that Kuratowski's first graph K_5 and second graph $K_{3,3}$ both are non-plannar graphs .

12 For a tree T, Prove that |E(T) | = |V(T)| -1 .