Seat No.

Time : 2:30 Hours]

[Total Marks : 70

Instructions:

All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten)   14

1. Find the order and degree of PDE P+2q-r=0

2. Define Pfaffian Exact differential equation with an example

3. Find the integral curve of the partial differential equation
$$\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{z}$$

4. Derive PDE by eliminate the arbitrary function f from the equation $z = f(x^2 + y^2)$

5. Verify the equation $z = \sqrt{2y + b} + \sqrt{2x + a}$ is the solution of $z = \frac{1}{p} + \frac{1}{q}$

6. Define orthogonal trajectories of a system of curves on a surface

7. Derive partial differential equation for z=(x+a)(y+b)

8. Find the complete integral of $P^2 + q^2 = 1$

9. Solve $(3 + D')(2D + 5)^2 z = 0$

10. If $X = \left(\frac{a^2}{x^2}, \frac{b^2}{y^2}, \frac{c^2}{z^2}\right)$ Find Curl X

Q.2 Answer the following (Any Two)   14

1. Find orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ for some $\alpha \in R$, of its intersection with the family of the parallel planes $z = c$, where c is some parameter

2. Let X = (P, Q, R) where P, Q, R functions of x, y, z, then the Pfaffian differential equation said to be integrable if and only if $X \cdot \text{curl} X = 0$

3. Using nattani's method solve $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$

Q.3 Answer the following   14

1. Consider the relation f(u,v)=0, where u,v are functions of x, y, z then show that the partial differential equation of the given relation f(u,v)=0 is
$$\frac{\partial(u,v)}{\partial(x,y)} = p\frac{\partial(u,v)}{\partial(y,x)} + q\frac{\partial(u,v)}{\partial(x,z)}$$

2   Find the general solution of $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$

<div align="center">OR</div>

Answer the following

1   Find the integral surface of the $2y(z-3)p+(2x-z)q=y(2x-3)$ which is passes through $z=0$ and $x^2+y^2=2x$

   Find the surface which intersects the system of surface $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$ and $z=1$

Q.4   Answer the following questions (Any Two)

   Solve i) $p(1+q)=qz$ ii) $p^2+q^2=x+y$

   Discuss Jacobi's method to solve first order partial differential equation

Q.5   Answer the following (Any Two)

1   If $\beta D'+\gamma$ with $\beta\neq0$ is factor of $f(D,D')$ then show that the solution of the equation $f(D,D')z=0$ is $z=e^{-\frac{\gamma}{\beta}y}\phi(\beta x)$

   Solve $(D^3+3D^2D'-4D'^3)z=0$

   Solve $(D^2-D'^2)z=x^2-y$

   Find P.I. of $(3D^2-2D'+DD')z=\sin(x+2y)$