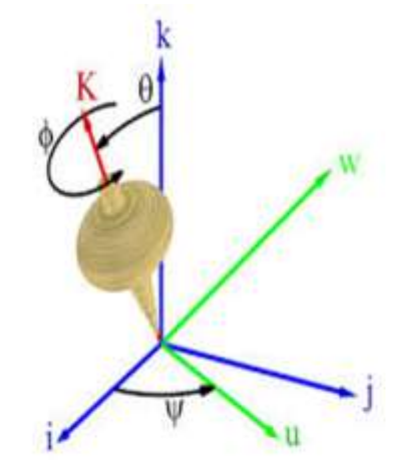
F.Y.B.Sc. SEM – I

Subject: Physics

Paper- 101

Unit -4

Rotational Mechanics & Gravitation



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INTRODUCTION:

According to Newton's first law of motion, "Every body continues to be in a state of rest or in a state of uniform motion along a straight line, unless it is compelled to change that state by some external force".

This law consists of two parts i.e., (1) A body at rest will not move on its own unless an external force acts on it and (2) A body which is in uniform linear motion will not change either its speed or direction of motion on its own, without the help of an external force.

The inability of a body to change its state on its own, without the help of external force is termed as inertia. Inertia is a fundamental property of the matter. The more is the mass of the body, the more will be inertia. Thus in translatory (linear) motion the mass of the body is a measure of coefficient of inertia.

Rigid body

If the distance between any two points in a body is not altered by applying a force, however large the force may be, the body is said to be a rigid body.

A rigid body may be defined as that body which does not undergo any change in its shape or size due to the application of force.

Actually, no body is a perfect rigid body. When the changes in the body are negligible, it can be considered as a rigid body.

Rotational motion

Each body is made of large number of tiny particles. In the case of linear motion, all the particles present in the body will have same linear velocity.

When the body rotates about a fixed line (axis of rotation), its motion is known as rotatory motion.

The axis of rotation may lie within the body or outside the body. If all the particles of a body move in a circular path about the axis of rotation, the rigid body is said to have pure rotational motion.

When a body is in rotational motion about an axis, all the particles present in the body will have same angular velocity, but different linear velocities. The values of the linear velocities of these particles depend on the distance of the particles from the axis of rotation, since $v = r\,\omega$

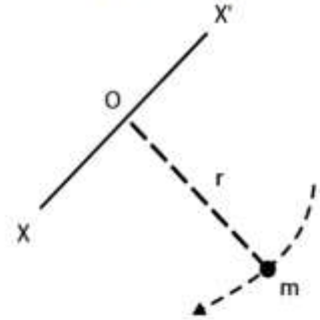
In rotational motion also, a rigid body which is free to rotate about an axis, opposes the change to θ be produced in its state. The measure of opposition not only depends on the mass of the body but also on the distances of the particles of the body from the axis of rotation.

Suppose a particle of mass 'm' is at a distance r from an axis, the product mr^2 is called the moment of inertia of the particle about that axis.

Thus in rotational motion the moment of inertia of the body about the given axis of rotation is a measure of coefficient of inertia.

Hence, the moment of inertia plays the same role in rotational motion as that of mass in translatory motion.

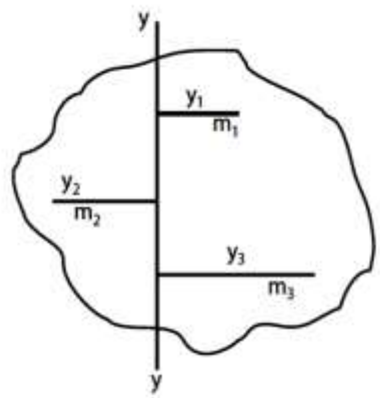
Consider a particle of mass 'm' is placed at a distance 'r' from the fixed axis. Then, the moment of inertia of the particle about the axis = mr^2. The S.I. unit for moment of inertia is kg m^2.



Moment of Inertia of a rigid body

Inertia of a body is its inability to change by itself its state of rest or of uniform motion in a straight line.

Similarly Moment of inertia of a body is its inability to change by itself its state of rest or of uniform rotatory motion about an axis.



An external torque (rotating effect of force) is necessary to change its state.

Let us consider the rotation of a rigid body about an axis. It consists of a large number of particles of masses m_1, m_2, m_3 etc., situated at distances r_1, r_2, r_3 etc., from the axis yy'. Then $m_1r_1^{\ 2}$ is known as the moment of inertia of the particle of mass m_1 about that fixed axis.

Then, the moment of inertia of the rigid body = Sum of moments of inertia of all the particles present in the body.

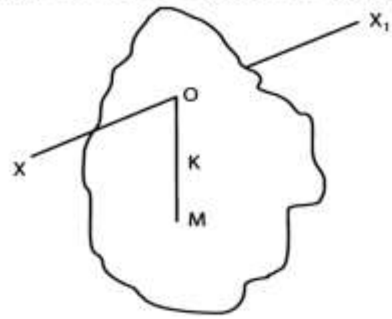
$I = m_1r_1^{\ 2} + m_2r_2^{\ 2} + m_3r_3^{\ 2} + ...$

$I = \Sigma mr^2$

Hence the moment of inertia of a rigid body about a fixed axis is the sum of the moment of inertia of all the particles of the rigid body.

Radius of gyration

Radius of gyration is the distance between the given axis and the centre of mass of the body. The centre of mass of a body is point where the entire mass of the body is supposed to be concentrated.



It is denoted by ' K '.

If M is mass of the body, then moment of inertia $I = MK^2$.

Hence, $mr^2 = MK^2$

Expression for Kinetic Energy of a Rigid body rotating about an axis:

Consider a rigid body rotating about a fixed axis XOX'. The rigid body consists of a large number of particles. Let $m_1, m_2, m_3, \ldots\ldots$ etc., be the masses of the particles situated at distances. $r_1, r_2, r_3, \ldots\ldots$ etc., from the fixed axis. All the particles rotate with the same angular velocity ω. But the linear velocities of the particles are different.

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$

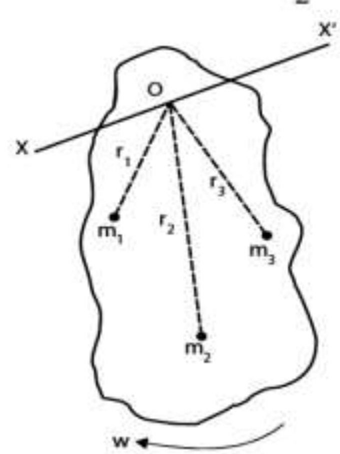
$$= \frac{1}{2}m(r\omega)^2 \qquad (\therefore v = r\omega)$$

$$= \frac{1}{2}m\,r^2\,\omega^2$$

The kinetic energy of the first particle $= \frac{1}{2}m_1 r_1^2\ \omega^2$

The kinetic energy of the second particle $= \frac{1}{2}m_2 r_2^2\ \omega^2$

The kinetic energy of the third particle and so on $= \frac{1}{2}m_3 r_3^2\,\omega^2$



The kinetic energy of the whole body is equal to the sum of the kinetic energy of all the particles present in the body.

The kinetic energy of the rigid body $= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + ...$

$$= \left(\frac{1}{2} m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ...\right)\omega^2$$

$$= \frac{1}{2} \Sigma m r^2 \omega^2$$

But, Σmr^2 = Moment of inertia of the rigid body about the fixed axis = I

Therefore, The kinetic energy of the rigid body $= \frac{1}{2} I \omega^2$

Angular momentum

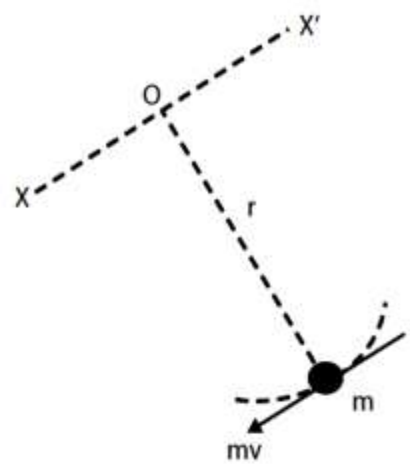
The moment of linear momentum is known as angular momentum.

Consider a particle of mass m is at a distance r from the axis of rotation. When a particle is in rotational motion about an axis, it has both linear velocity 'v' and angular velocity 'ω'.

Then, Angular momentum of the particle = linear momentum × perpendicular distance between the particle and the axis of rotation

$= m v \times r$

$= m r \omega \times r$ $(\because v = r\omega)$

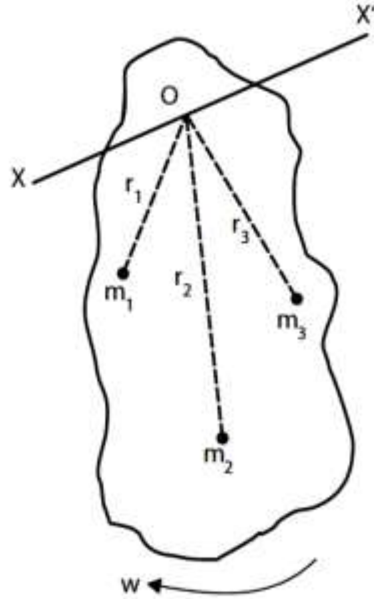


∴ Angular momentum $= mr^2\omega$

Where ω is the angular velocity of the particle.

The SI unit for angular momentum is $kg\ m^2 s^{-1}$

Expression for Angular momentum of a Rigid body rotating about an axis:



Consider a rigid body rotating about a fixed axis XOX'. The rigid body consists of a large number of particles. Let m_1, m_2, m_3... etc., be the masses of the particles situated at distances r_1, r_2, r_3, ... etc., from the fixed axis. All the particles rotate with the same angular velocity, but with different linear velocities depending on the values of 'r'.

Angular momentum = moment of linear momentum

= linear momentum × distance

= $mv \times r = mr\omega \times r \quad (\therefore v = r\omega)$

= $mr^2\omega$

$\therefore$ The angular momentum of the first particle $= m_1r_1^2\,\omega$

The angular momentum of the second particle $= m_2r_2^2\,\omega$

The angular momentum of the third particle $= m_3r_3^2\,\omega$

...and so on.

The angular momentum of the whole body is equal to the sum of the angular momenta of all the particles present in the body.

The angular momentum of the rigid body $\left.\right\}$ $L = m_1r_1^2\,\omega + m_2r_2^2\omega + m_3r_3^2\omega + ...$

$= (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ..)\,\omega$

$= \Sigma mr^2\,\omega$

But, Σmr^2 = moment of inertia of the rigid body = I

$\therefore$ The angular momentum of the rigid body = $I\omega$

Law of conservation of angular momentum

When there is no external torque acting on a rotating body, the angular momentum of that body remains a constant. This is the statement of law of conservation of angular momentum.

i.e., if I_1 and ω_1 are the initial moment of inertia and angular velocity of a rotating body and if I_2 and ω_2 are new moment of inertia and angular velocity of the body, without the help of any external torque, then according to this law,

$$I_1 \omega_1 = I_2 \omega_2$$

Examples

a) Consider a person standing on a turn-table with arms extended and a pair of weights, one in each hand. The table is made to rotate by a motor and then the motor is switched off. Now, if that person pulls his arms inwards, we can see a considerable increase in the speed of rotation. This is because, in the new position, his moment of inertia I about the axis of rotation decrease, due to the decrease in the value of r. Since the angular momentum is conserved, a decrease in the value
I results in an increase in the value of angular velocity ω. Therefore the person is found to rotate faster.

b) A circus acrobat, a diver or skater, all take advantage of this principle. consider a diver has just left a diving board with his arms and legs extended, with a particular angular momentum. If he now pulls his arms and legs in, his moment of inertia I decreases, leading to increase in angular velocity ω. By increasing ω, the diver can perform more number of somersaults, before entering the swimming pool.

Newton's laws of Gravitation

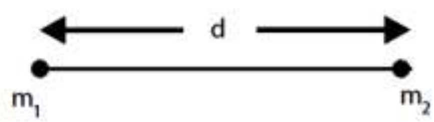
Law 1 : Any two particles of matter attract each other with a force

Law 2 : The force of attraction between any two objects is

i. directly proportional to the product of the masses

ii. inversely proportional to the square of the distance between them.

If m_1, m_2 are masses of two particles, separated by a distance 'd' then the force of attraction between the particles,



$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = G \frac{m_1 m_2}{d^2}$$

where G is known as "universal gravitational constant" and the value of $G = 6.6733 \times 10^{-11}\ N\ m^2 kg^{-2}$

Acceleration due to gravity

Galileo was the first to make a systematic study of the motion of a body under the gravity of Earth. He dropped various objects from the leaning tower of Pisa and made analysis of their motion under gravity. He came to the conclusion that "in the absence of air, all bodies will fall at the same rate". It is the air resistance that slows down a piece of paper or a parachute falling under gravity. If a heavy stone and a parachute are dropped where there is no air, both will fall together at the same rate.

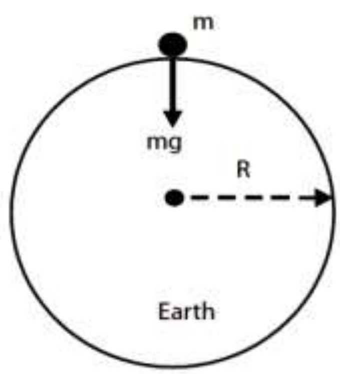
Experiments showed that the velocity of a freely falling body under gravity increases at a constant rate. i.e., with a constant acceleration. The acceleration produced in a body on account of the force of gravity is called acceleration due to gravity. It is denoted by 'g'. At a given place, the value of 'g' is the same for all bodies irrespective of their masses. It differs from place to place on the surface of the Earth. It also varies with altitude and depth.

The value of g at sea-level and at a latitude of 45° is taken as the standard (i.e) $g = 9.81 \text{ m s}^{-2}$.

Acceleration due to gravity at the surface of the Earth

Consider a body of mass m on the surface of the Earth as shown in the Figure. Its distance from the centre of the Earth is R (radius of the Earth).

The gravitational force experienced by the body, $F = \frac{GMm}{R^2}$
where M is the mass of the Earth.



From Newton's second law of motion,

Force F = mg.

Equating the above two forces, $\frac{GMm}{R^2} = mg$

$$\therefore g = \frac{GM}{R^2}$$

This equation shows that g is independent of the mass of the body m. But, it varies with the distance from the centre of the Earth. If the Earth is assumed to be a sphere of radius R, the value of g on the surface of the Earth is given by

Mass and Weight $g = \frac{GM}{R^2}$

WORKED PROBLEMS

1. If the radius of the earth is 6400km and the acceleration due to gravity is 9.8ms^{-2}. Calculate the escape velocity

Given: $R = 6400$ km and $g = 9.8$ ms^{-2}

Escape velocity $V_0 = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$

$V_0 = 11.2$ km per second

2. A satellite is revolving round the earth at a distance of 182 km from the surface of the earth. The radius of the earth is 6371 km and g is 9.81 ms^{-2}. Calculate the orbital velocity of the satellite.

Given: $g = 9.81$ ms^{-2}, $R = 6371 \times 10^3$ m, $h = 182 \times 10^3$ m

$(R+h) = (6371 + 182)\,10^3 = 6553 \times 10^3$ m

$$V_0 = \frac{gR^2}{(R+h)} = \sqrt{\frac{9.81 \times (6371 \times 10^3)^2}{6553 \times 10^3}}$$

$V_0 = 7795$ ms^{-1} (or) 7.795 kms^{-1}

QUESTIONS

Part – A and B

1. Define rigid body
2. Define moment of inertia of a particle
3. Define moment of inertia of a rigid body
4. Define radius of gyration
5. Define angular momentum
6. State the law of conservation of angular momentum
7. State Newton's I law of gravitation
8. State Newton's II Law of gravitation
9. What is a satellite?
10. Define escape velocity
11. Define orbital velocity
12. Give any two uses of artificial satellites
13. Derive an expression for the moment of inertia of a rigid body about an axis.
14. Explain Newton's Law of Gravitation
15. Explain escape velocity and orbital velocity
16. Write the uses of artificial satelites

Part – C

1. Derive an expression for the kinetic energy of a rigid body rotating about an axis.
2. Derive an expression for the angular momentum of a rigid body rotating about an axis.
3. Obtain an expression for the acceleration due to gravity on the surface of the earth.
4. Obtain an expression for the variation of acceleration due to gravity with altitude.
5. Derive an expression for the escape velocity from the surface of the earth
6. Derive an expression for the orbital velocity of a satellite.