SHREE H. N. SHUKLA COLLEGE OF SCIENCE

M.Sc.(Mathematics) Sem-1

Question Bank

MATH.CMT-1004 : Theory of Ordinary Differential Equations

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1. Solve the IVP : $y_1' = -y_1,\ y_2' = y_1 + y_2,\quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

2. Let $p, q : I \to \mathbb{R}$ be a continuous, $t_0 \in I,\ y_0 \in \mathbb{R}$. Then prove that the IVP: $y' + p(t)y = q(t),\ y(t_0) = y_0$ has a unique solution $u(t) = y_0 e^{-P(t)} + e^{-P(t)} \int_{t_0}^{t} e^{p(r)} q(r) dr$, defined on the whole of I, where $P : I \to \mathbb{R}$ is defined by $P(t) = \int_{t_0}^{t} p(r) dr,\ \forall t \in I$.

3. Show that $\phi(t) = \begin{pmatrix} e^t & te^t \\ 0 & e^t \end{pmatrix}$ is a fundamental matrix of $y' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} y$ on $\mathbb{R}$.

4. State and prove Variation of constant formulae for scalar first order non-homogeneous differential equation.

5. State and prove Variation of constant formulae for scalar second order non-homogeneous differential equation.

6. Find general solution of $y^{(4)} + 16y = 0$.

7. Compute the first five terms of the series expansion at zero of the solution of the Legendre's equation $(1 - t^2)y'' - 2ty' + \alpha(\alpha + 1)y = 0$, where α is a constant.

8. Find the solution of the IVP : $y' = \begin{pmatrix} 3 & 3 \\ -5 & 3 \end{pmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}, \ y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

9. Find the solution of the IVP : $y' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}, \ y(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ on $(-\infty, \infty)$.

10. Prove that the eigen vector corresponding to the distinct eigen values of a $n \times n$ matrix A are linearly independent in $\mathbb{K}^n$.

11. Find fundamental matrix of $y' = A(t)y$ on $(-\infty, \infty)$, where $A(t) = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ on $(-\infty, \infty)$ and find $exp(tA)$.

12. Let $A(t)$ be a continuous $n \times n$ matrix on I, $\phi(t)$ be a fundamental matrix of $y' = A(t)y$ on I and C be a $n \times n$ non-singular constant matrix. Then prove that $\phi(t) \cdot C$ is a fundamental matrix of $y' = A(t)y$ on I.

13. Let $A(t)$ be a continuous $n \times n$ matrix on I and $\phi(t), \psi(t)$ be fundamental matrices of $y' = A(t)y$ on I. Then prove that there exists a non-singular $n \times n$ matrix C such that $\psi(t) = \phi(t) \cdot C, \ \forall t \in I$.

14. Let A be a constant 2×2 complex matrix then prove that there exists a constant 2×2 non-singular matrix B such that $B^{-1}AB$ has the following forms:

$$[a] \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}, \ \lambda \neq \mu \ [b] \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \text{ and } [c] \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

15. Let A be a constant 2×2 real matrix with eigen values $\alpha + i\beta$, where $\alpha, \beta \in \mathbb{R}$. Then prove that there exists a constant 2×2 non-singular real matrix T such that $T^{-1}AT = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$.

16. Prove that $a_0(t), \ a_1(t), \ a_2(t)$ which are analytic at t_0 and t_0 is a regular singular point of $a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$ then given equation can be written in the form $(t - t_0)^2 y'' + (t - t_0)\alpha(t)y' + \beta(t)y = 0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at t_0 and not all $\alpha(t_0), \ \beta(t_0)$ and $\beta'(t_0)$ are zero.

17. Prove that if $\alpha = 2m$, where $m \geqslant 0$ is an integer then a second linearly independent solution of the legendre equation $(1-t^2)y'' - 2ty' + \alpha(\alpha+1)y = 0$ valid in a nbhd of 0 can be expressed in the form of power series converges for $|t| < 1$.

18. Show that $ty'' + y' + y = 0$ has only one solution of the form $|t|^z \sum_{k=0}^{\infty} c_k t^k,\ c_0 = 1$ is an excluded nbhd of 0.

19. State and prove Gronwall's inequality.

20. Locate and classify all singular points of $(t-1)^3 y'' + 2(t-1)^2 y' - 7ty = 0$.

21. Solve $y'' - 3y' + 2y = 4e^{2t}$ with $y(0) = -3,\ y'(0) = 5$ using Laplace Transform.

22. Find $L^{-1}\left(\dfrac{n!}{(z-c)^{n+1}}\right)$

23. Solve $y'' + 25y = 10cos5t,\ y(0) = 2,\ y'(0) = 0$ using Laplace Transform.

24. Find $L^{-1}\left(\dfrac{3z+7}{z^2-2z-3}\right)$

25. State and prove Laplace Transform of Integral.

26. Find $L\left(t^n e^{ct}\right)$

☻ ***BEST OF LUCK*** ☻