003-1163005

sem-3
2024

Seat No. ______

MASTER OF SCIENCE MATHEMATICS(W.E.F.-2016) MSC MATHS(2016) Semester - 3 Examination

October - 2024

DIFFERENTIAL GEOMETRY

Faculty Code : 003

Subject Code : 003-1163005

Time : 2.30Hours] **[Total Marks : 70**

Q.1 Answer the following : (Any seven out of ten, each of 02 marks) 14

1

1. Let $\propto : R \to R^3$ defined by $\propto(t) = (t, g(t), 0)$, where $g: R \to R$ be a differentiable function. Show that, $\propto$ is a regular curve and find the equation of tangent line.

2

2. Reparametrize the curve $\propto(u) = (a\,cosu, a\,sinu, cu)$, where $a, b, c \in R$ and $0 \leq u < \pi$ by $t = tan\left(\frac{u}{2}\right)$.

3

3. Find the arc length of a curve $\propto(t) = \left(t, \frac{t^2}{2}, 0\right)$.

4

4. What is curvature of a unit speed curve? Prove that, the curvature of a circle with radius r is $\frac{1}{r}$.

5 5. What does the following mean?

a. Osculating plane
b. Normal plane
c. Rectifying plane

6 6. With a relevant example, define the following:
Open set and Simple surface

7

7. Let $X: R^2 \to R^3$ defined by $X(u^1, u^2) = ((u^1)^2, (u^2)^2, u^1u^2)$ is simple? Describe your response in detail.

8

8. Prove that, $\kappa^2 = \kappa_n^2 + \kappa_g^2$, where notations are being usual.

9

9. Let $\propto(s)$ be a C^k curve in $XY-$Plane. Prove that, for non-zero curvature ($\kappa(s) \neq 0$) the values of torsion is must be zero ($\tau = 0$).

10

10. Consider the upper hemisphere $x(r,s) = \left(r, s, \sqrt{1-r^2-s^2}\right)$ and $\gamma(t) = (sint, 0, cost)$ then find κ_g and κ_n, where notations are being usual.

Q.2 Answer the following : (Any two out of three, each of 07 marks)

1 Show that, the right circular helix is a regular curve. Also find the equation of tangent line.

2

Let $\alpha: (a,b) \rightarrow R^3$ be a regular curve and $g: (c,d) \rightarrow (a,b)$ be a reparametrization in which set

$\beta = \alpha \circ g$. Show that, if $t_0 = g(r_0)$, the tangent vector field T of α at t_0 and the tangent vector field S of β at r_0 satisfy $S = \pm T$. Q.4

3 Show that, the arc length of the curve $\propto(t) = \left(2a\left(\sin^{-1}t + t\sqrt{1-t^2}\right),\ 2at^2, 4at\right)$ between the points $t = t_1$ to $t = t_2$ is $4a\sqrt{2}(t_2 - t_1)$.

Q.5

Q.3 Answer the following : (1 & 2 Both are compulsory, each of 07 marks)

1 1) Demonstrate that a necessary and sufficient condition for a curve to be straight line is that the curvature $\kappa = 0$ at all the points of the curve.

2

2) Let $\propto(s)$ be a unit speed curve. Show that, $\forall s$ and $\kappa(s) \neq 0$, the set $\{T(s), N(s), B(s)\}$ is an orthonormal set.

OR

Answer the following : (1 & 2 Both are compulsory, each of 07 marks) 14

1

(1) State Frenet – Serret appartus of the unit speed curve. Show that the curve

$\propto (t) = \left(r\cos\left(\frac{s}{r}\right), r\sin\left(\frac{s}{r}\right), 0\right)$ is a unit speed curve and obtain the Frenet – Serret appartus

for the unit speed curve $\propto (t)$.

2

(2) If $E = \{e_1, e_2, \dots, e_n\}$ is an orthonormal set of $n-$ dimensional inner product space V then

show that, a. E is basis for V and b. If $v \in V, v = \sum_{i=1}^{n} \langle e_i, v \rangle e_i$.

Q.4 Answer the following : 14

1 1. State and prove, the Frenet – Serret theorem.

2

2. Prove that, the unit speed curve $\alpha(s)$ with $\kappa(s) \neq 0$ is a helix is and only if there is a

constant c such that $\tau = c\kappa$.

Q.5 Answer the following : (Any two out of four, each of 07 marks) 14

1

1. Let $x: u \to R^3$ be a simple surface. Show that,

a. $x_{ij} = L_{ij} \cdot n + \sum \gamma_{ij}^{k} x_k$

b. For any unit speed curve $\gamma(s) = x(\gamma^1(s), \gamma^2(s))$ the values of

$\kappa_n = \sum_{i,j} L_{ij} (\gamma^i)'(\gamma^j)'$ and $\kappa_g \cdot S = \sum_k [(\gamma^k)'' + \sum_{i,j} \gamma_{ij}^{k} (\gamma^i)'(\gamma^j)'] x_k$

2

2. Let g be a matrix associated with proper coordinate patch $x: u \to R^3$ defined by

$g_{ij} = \langle x_i, x_j \rangle$, $1 \leq i,j \leq 2$ and elements of inverse of g, denoted by g^{ij}. Show that,

a. $|g| = |x_1 \times x_2|^2$

b. $g^{11} = \frac{g_{22}}{|g|}$, $g^{12} = g^{21} = -\frac{g_{12}}{|g|}$ and

c. $g^{22} = \frac{g_{11}}{|g|}$, $\Sigma_{k=1}^{2} g_{ik} g^{kj} = \delta_i^j$

3

3. If $X: u \to R^3$ is a simple surface and $f: v \to u$ is a co-ordinate transformation then show that, $Y = X \circ f: v \to R^3$ is also a simple surface.

4

4. State first fundamental form. If $u = \{(u^1, u^2) : (u^1)^2 + (u^2)^2 < 1\}$ and coordinate patch is $x(u^1, u^2) = (u^1, u^2, \sqrt{1 - (u^1)^2 - (u^2)^2})$ then find the first fundamental forms and $|g|$ of x.