Shree H.N.Shukla College of Science

MATHEMATICS

T.Y.B.Sc. (Sem.VI) (CBCS)

PAPER- 601

Graph Theory & Complex Analysis-II

QUESTION BANK

☑ Answer the following: [1 mark questions]

1) Write a degree of a pendent vertex.
2) Define: Simple Graph
3) Find the Nullity of connected graph with 4 vertices and 8 edges.
4) Write the number of internal vertices in a binary tree with 13 vertices.
5) Define: Regular Graph
6) How many edges are there in K_5 graph?
7) What is the degree of an isolated vertex in graph?
8) Write the number of vertices in a connected graph with 2 faces and 6 edges.
9) Write the edge connectivity of a tree.
10) How many edges are there in a tree with 5 vertices?
11) Write the degree of a pendant vertex.
12) Write the maximum number edges in a simple graph with 4 vertices and 2 components.
13) How many vertices are there in K_5 graph?
14) Write the formula for total number of edges in a complete graph with n vertices.
15) What is the number of pendant vertices in any binary tree with n vertices?
16) Define: Separable graph
17) Define: Self dual graph
18) Define: Acyclic Digraph
19) What is the chromatic number of a complete graph with 5 vertices?
20) Regions of a connected planar graph with 4 vertices and 6 edges are _____.
21) Define: Power series

22) Find radius of convergence of series $\sum \frac{z^n}{2^{n-1}}$.
23) Define: Complex Series
24) State Maclaurin series of an analytic function f(z).
25) Define: Residue of f(z) at pole Z_0.
26) Find $Res\left(\frac{\cos z}{z}, 0\right)$
27) Write an isolated singular point for $f(z) = \frac{1}{z-2}$.
28) Define: Singular point
29) Define: Linear mapping
30) Find fixed point of the bilinear transformation $w = \frac{3z-4}{z-1}$.
31) Define: Mobius mapping
32) Find the critical point of $w = \frac{1}{z-1}$.
33) Define: Circuit vector
34) Every edge in Incidence Matrix of exactly _____ vertices.
35) Define: Chromatic number of graph
36) Find singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.
37) Define: Bilinear mapping
38) Write expansion of coshz in maclaurian series.
39) Find radius of convergence for the series $\sum n!\ z^n$
40) Write the formula for finding the residue of f(z) at m^{th} order pole.

☑ Answer the following: [2 mark questions]

1) Prove that the number of vertices n in a binary tree is always odd.
2) Define: (i) Circuit (ii) Minimally connected graph
3) Define: Path Matrix
4) Define Cut-set vector with example.
5) Expand $\frac{1}{1+z}$ in Maclaurin's series.
6) Discuss about the convergence of series

$$\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{4^n (n+1)^3}.$$

7) Find the residue of $f(z) = \frac{z+2}{(z-1)(z-2)}$ at Simple pole.
8) Find Res(f(z), 1), where
$$f(z) = \frac{e^{2z}}{(z-1)^2}$$
9) Show that x+y=2 transform into the parabola u^2=-8(v-2) under the transformation $W=Z^2$.
10) Find critical point of $w = \frac{z-1}{z+1}$.
11) Define: (i) Path (ii) Hamiltonian circuit
12) Derive formula for finding residue of f(z) at simple pole Z_0.
13) Show that $W = \frac{az+b}{cz+}$ is conformal mapping.
14) Find radius of convergence of series $\sum_{n=1}^{\infty} \frac{z^n}{3^n-1}$
15) Define: (i) Diagraph (ii) Spanning tree
16) State and prove second theorem of Graph theory.
17) In a simple connected planner graph with f regions, n vertices and e edges (e>2). Prove that e ≤ 3n-6.
18) Expand sinz in Taylor's series for z_0=0.
19) Discuss the fixed point of bilinear transformation.
20) Define: (i) Null graph (ii) Pendant vertex

☑ Answer the following: [3 mark questions]

1) Prove that a graph is a tree iff it is minimally connected.
2) State and prove first theorem of Graph theory.
3) A connected simple planner graph G with n-vertices, e-edges and f-region then prove that (i) $e \geq \frac{3}{2} f$

(ii) $e \leq 3n - 6$

4) Prove that Complete graph K_4 is Self dual graph.
5) Expand $\frac{1}{z(z^2-3z+2)}$ in Laurent's series for (i) 1<|z|<2 (ii) 0<|z|<1
6) Expand e^z in term of (z-1).
7) Obtain the formula for finding the residue of f(z) at m^{th} order pole.

8) Find the value of integral $\int_C \frac{dz}{Z^3(Z+4)}$ $where\ C: |Z| = 2$
9) Prove that the transformation $w=2z+z^2$ maps the unit circle $|z|=1$ of z-plane into cardiod to w-plane.
10) Show that the composition of bilinear maps is again a bilinear.
11) Prove that every tree is two or more vertices is 2-chromatic.
12) Find a Mobius mapping which maps three point 1, 2, -1 in z-plane into 2, 1, -2 in w-plane.
13) Prove that

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$$

☑ Answer the following: [5 mark questions]

1) Prove that a Simple graph with n-vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
2) Explain Konigsberg Bridge Problem.
3) Prove that the complete graph of 5-vertices is non-planner graph.
4) Define: Incidence Matrix and its properties
5) State and prove Taylor's infinite series of an analytic function f(z).
6) Prove that

$$\cosh(z + z^{-1}) = a_0 + \sum_{n=1}^{\infty} a_n(z^n + z^{-n})$$

Where,

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} \cosh(2\cos\theta)\cos n\theta\, d\theta$$

7) Prove that

$$\int_0^{\infty} \frac{dx}{(x^2+a^2)^{n+1}} = \frac{\pi(2n)!}{(n!)^2(2a)^{2n+1}}\ , where\ a > 0.$$

8) State and prove Cauchy residue theorem.
9) Discuss the bilinear mapping $W=Z^2$.

10) Discuss the mapping of $W=e^{z}$ in Cartesian system.
11) Define: Adjacency Matrix and state its properties
12) Prove that a connected planner graph with n-vertices and e-edges has e-n+2 regions.
13) State and prove Euler theorem.
14) Using residue theorem prove that

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3} = \frac{3\pi}{8}$$

****BEST OF LUCK****