RA-7

003-016201

M.Sc. (CBCS) Semester-II Examination
May-2013
Mathematics CMT – 2001 : Algebra – II

Faculty Code : 003
Subject Code : 016201

Time : 2 ½ Hours] **[Total Marks : 70**

Instructions : (1) Answer **all** the questions.
(2) Each question carries **14** marks.

1. Answer any **seven** : **7 × 2 = 14**

(a) State the lemma of Gauss.

(b) Determine the minimal polynomial of $\sqrt{2} + 5$ over Q.

(c) Define an algebraic closure of a field F.

(d) Construct splitting field over Q for the polynomial $x^3 - 2$.

(e) Let F be a finite field with characteristic of F equals p. Verify that $\psi : F \to F$ defined by $= \psi(\alpha) = \alpha^p$ is an automorphism of F.

(f) When is an extension field E of a field F called a simple extension ?

(g) Let E be a field. Let H be a subgroup of group of automorphisms of E. Define the fixed field of H.

(h) Let M be a module over a ring R. Let $m \in M$. Verify that $\{rm | r \in R\}$ is a submodule of M.

(i) When is an R-module called completely reducible ?

(j) Verify that Q $(2^{1/3}, 5^{1/2}, 7^{1/5})$ is a radical extension of Q.

2. Answer any **two** : **2 × 7 = 14**

(a) State and prove Eisenstein criterion.

(b) Prove the following :

(i) If a field E is a finite extension field of a field F, then E is an algebraic extension of F.

(ii) If a field $E = F(u_1, \ldots, u_r)$ is a finitely generated extension of a field F such that u_i is algebraic over F for I = 1,…., r, then E is finite over F.

(c) Prove that a field K admits no proper algebraic extensions if and only if any irreducible polynomial in $K[x]$ is of degree 1.

3. (a) Let an extension field E of a field F be finite and normal. Prove that E is the splitting field of a polynomial $f(x) \in F[x]$. **5**

(b) Let $K = F(x)$ be the field of rational functions in one variable over a field F of characteristic 3. Prove that the polynomial $y^3 - x \in K[y]$ is irreducible over K and has multiple roots. **5**

(c) Prove that any finite field F with exactly p^n elements is the splitting field of $x^{p^n} - x \in F_p[x]$. **4**

OR

(a) Let F and E be fields and let $\sigma_1, \sigma_2, \ldots, \sigma_n$ be distinct embeddings from F into E. Suppose that for $\alpha_1, \alpha_2 \ldots \alpha_n \in E$, $\sum_{i=1}^{n} a_i \sigma_i(a) = 0$ for all $a \in F$. Prove that $a_i = 0$ for all I = 1, 2, …, n. **5**

(b) Let n be a positive integer and let F be a field containing all the n^{th} roots of unity. Let K be the splitting field of $x^n - a \in F[x]$ over F. Prove that G(K/F) is abelian. **5**

(c) Express the symmetric function $x_1^3 + x_2^3 + x_3^3$ as a rational function of elementary symmetric functions. **4**

4. Answer any **two** : **2 × 7 = 14**

(a) Let f be an R-module homomorphism from an R-module M into an R-module N. Prove that M/Ker(f) ≅ f (M) as R-modules.

(b) Let R be a ring with unity and let M be an R-module. Prove that M is simple if and only if M ≅ R/I as R-modules where I is a maximal left ideal of R.

(c) Let F be a field and let $f(x) \in F[x]$ be of positive degree with no multiple roots. Prove that $f(x)$ is irreducible over F if and only if the Galois group of $f(x)$ is isomorphic to a transitive permutation group.

5. Answer any **two** : **2 × 7 = 14**

(a) Let F be a field and let $p(x)$ be an irreducible polynomial in $F[x]$. Let u be a root of $p(x)$ in an extension field E of F. Prove that F(u) = F[u].

(b) Let F be a finite field with exactly p^n elements and let m be a positive integer. Prove that there exists an extension field E of F with [E : F] = m.

(c) If a field E is a Galois extension of a field F, then prove that F is the fixed field of G(E/F).

(d) Let R be a ring with unity. If M is a free R-module with basis $\{m_1 \ldots, m_n\}$, then prove that $M \cong R^n$ as R-modules.

DGG-003-016201 Seat No. ________

M. Sc. (Sem. II) (Maths) Examination

May/June – 2015

Algebra - II (CMT-2001)

Faculty Code : 003
Subject Code : 016201

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.
(ii) Each question carries 14 marks.

1 Choose appropriate alternatives : (any seven) **7×2=14**

(1) Let $g(x) = x^2 - 5x + 6 \in Q[x]$. Which one of following is correct ?

(A) Q contains all the roots of g(x)

(B) g(x) is irreducible over Q

(C) $Q \cap \{\alpha \mid \alpha \text{ is a root of } g(x)\}$ is a single ton set.

(D) None of these

(2) Let $\frac{E}{F}$ and $\frac{K}{E}$ be finite extensions with $[K:E] = [E:F] = 5$.

What is degree of the field extension $\frac{K}{F}$?

(A) 5

(B) 25

(C) 10

(D) None of these

(3) Let F be a subfield of a field E and $\alpha, \beta \in F$. Which one of following is correct ?

(A) $\alpha + \beta \in F$, but $\alpha - \beta \notin F$

(B) $\alpha + \beta, \alpha - \beta \in F$, but $\alpha\beta \notin F$

(C) $\alpha + \beta, \alpha - \beta, \alpha\beta \in F$

(D) $F \cap \{\alpha + \beta, \alpha - \beta, \alpha\beta\} = \phi$

(4) Let F be a finite field. Which one of following is possible ?

(A) $|F| = 91$

(B) $|F| = 810$

(C) $|F| = 15$

(D) $|F| = 9$

(5) Let $f(x) = x^3 + x + 1 \in \mathbb{Z}_2[x]$. Which one of following is correct ?

(A) $f(x)$ is irreducible in $\mathbb{Z}_2[x]$

(B) $\mathbb{Z}_2$ contains a root of f(x)

(C) $\mathbb{Z}_2$ contains all the roots of f(x)

(D) None of these

(6) Let $\alpha = 2^{1/n}$ and $n \geq 3$. Which one of following is correct ?

(A) $\alpha \in Q$

(B) α is not algebraic over Q

(C) $[Q(\alpha):Q] = 2$

(D) $x^n - 2 \in Q[x]$ is the minimal polynomial of α over Q.

(7) Which of following fields is a prime field ?

(A) Q

(B) $\mathbb{Z}_p$ (p is a prime)

(C) (A) and (B)

(D) None of these

(8) Which of following field extensions is an infinite algebraic extension ?

(A) $\frac{\mathbb{R}}{Q}$

(B) $\frac{\bar{Q}}{Q}$

(C) $\frac{\mathbb{C}}{\mathbb{R}}$

(D) None of these

2 Attempt any two : **2×7=14**

(a) Let $f(x) = a_0 + a_1 x + \dots\dots\dots\dots + a_n x^n \in Z[x]$ and $n > 1$. Let p be a prime such that $p \mid a_0$, $p^2 \nmid a_0$, $p \mid a_1, \dots\dots\dots, p \mid a_{n-1}$ and $p \nmid a_n$. Then prove that f(x) is irreducible over Q[x].

(b) Let $\frac{E}{F}$ be a finite field extension. Prove that it is also an algebraic extension.

(c) Let $\frac{E}{F}$ and $\frac{K}{E}$ both are algebraic extensions. Then prove that $\frac{K}{F}$ is also an algebraic extension.

(d) Let F be a finite field. Then prove that $F^* = F - \{0\}$ is a cyclic group under multiplication.

3 Attempt any one : **1×14=14**

(a) Let F be a finite field and $|F| = p^n$, for some prime p, $n \in N$. Then prove that Aut(F), the group of all automorphisms of F is a cyclic group of order n.

(b) Let $\frac{E}{F}$, $\frac{K}{E}$ both are finite separable extensions. Prove that $\frac{K}{F}$ is also a finite separable extension.

(c) Let $\frac{E}{F}$ be a Galois extension and K be a subfield of E and it is super field for F. Prove that

(i) $\frac{K}{F}$ is normal $\Leftrightarrow \sigma(K) = K,\ \forall \sigma \in G\left(\frac{E}{F}\right)$

(ii) $\psi = \mathbb{C} \to \mathbb{D}$ defined by $\psi(K) = G\left(\frac{E}{K}\right)$ is bijection, where

$\mathbb{C} = \{K / K \text{ be a field and } F \subseteq K \subseteq E\}$

$\mathbb{D} = \left\{H / H \text{ is a subgroup of } G\left(\frac{E}{F}\right)\right\}$

(iii) $\frac{K}{F}$ is normall $\Leftrightarrow G\left(\frac{E}{K}\right) \Delta G\left(\frac{E}{F}\right)$

(iv) In third case $G\left(\frac{K}{F}\right) \simeq \frac{G\left(\frac{E}{F}\right)}{G\left(\frac{E}{K}\right)}$

4 Attempt any two : **2×7=14**

(a) Define prime field. Prove that for a field F, the prime subfield of F is either isomorphic to Q or it is isomorphic with $\mathbb{Z}_p$ (where p is a prime).

(b) Let $f : M \to N$ be an R-homomorphism on R-modules. Prove that Ker f and f(M) are submodules of M and N respectively.

(c) Show that $G\left(\frac{\not\subset}{\mathbb{R}}\right)$ is a group of order 2.

(d) State and prove Hilbert theorem 90.

(e) Let $\frac{E}{F}$ be an extension and $\alpha \in E$ be algebraic over F.

Let $p(x) \in F[x]$ be a polynomial of least degree whose one root is α. Then prove that

(1) p(x) is irreducible over F[x]

(2) p(x)/g(x) in F[x]

5 Attempt any seven : **7×2=14**

(1) Give definition of primitive polynomial and monic polynomial.

(2) Prove that $x^3 + 3x + 2$ is an irreducible polynomial in $\mathbb{Z}_7[x]$.

(3) Define cyclic extension with an example.

(4) Write down all the elements of the splitting field of $x^3 + x^2 + 1$ over $\mathbb{Z}_2$.

(5) Construct a field F with $|F| = 4$.

(6) Define R-module with an example.

(7) Prove or disprove $Q\left(2^{1/3}\right)/Q$ is a normal extension.

(8) Define separable polynomial, separable element and separable extension.

NCD-003-1162001 Seat No. ________

M. Sc. (Mathematics) (CBCS) (Sem. II) Examination

April / May – 2017

CMT-2001 : Algebra - II

(New Course)

Faculty Code : 003

Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Each question carries equal marks.

(2) All questions are compulsory.

1 Answer following short questions : **7×2=14**

(i) Define R-submodule of an R-module M. Let $m \in M$ and M be an R-module. Prove that $R_x = \{sx \mid s \in R\}$ is an R-submodule of M.

(ii) Prove following elementary properties of an R-module M : (a) $0m = 0, \ \forall\, m \in M$ and (b) $a0 = a, \forall\, a \in R$.

(iii) Prove that $x^3 + 3x + 2 \in \mathbb{Z}_7[x]$ is an irreducible polynomial.

(iv) Let $f(x) = x^3 + 6x^2 + 7x + 8 \in \mathbb{Z}[x]$. Find $f(x-2)$ and using Eisenstein criterian prove that $f(x)$ is irreducible polynomial over $\mathbb{Z}[x]$.

(v) Define transcendental element over Q and give three transcendental elements of $\mathbb{R}$ over Q.

(vi) Write down minimal polynomial of $2^{1/4}$ over Q.

(vii) Prove or disprove $Q\left(2^{1/4}, i\right)/Q$ is a Galois extension.

2 Attempt any two : **2×7=14**

(a) Prove that every finite extension is always an algebraic extension.

(b) Define finite extension. Let $E|_F$, $K|_E$ be two finite extensions. Prove that $K|_F$ is also a finite extension.

(c) Prove or disprove $Q\left(\sqrt{2}, \sqrt{3}, \ldots, \sqrt{p}, \ldots\right)|Q$ and $\bar{Q}|Q$ both are infinite algebraic extensions.

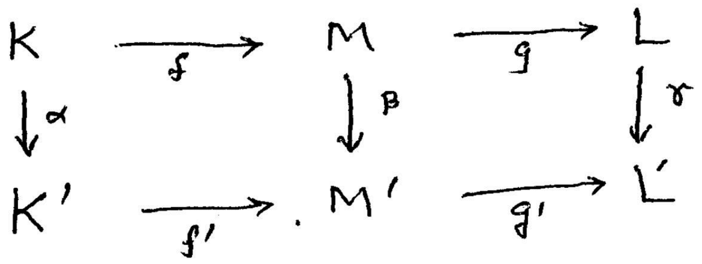
(d) Let F be a finite field. Prove that $F^* = F - \{0\}$ is a cyclic group under multiplication.

3 Attempt any one : **1×14=14**

(a) Let $E|_F$ and $K|_E$ be both finite seperable extensions. Prove that $K|_F$ is also a finite seperable extension.

(b) State and prove fundamental theorem of Galois theory.

(c) Define an exact infinite sequence of R-homomorphisms of R-modules. Suppose following diagram of R-modules and R-homomorphism is commutative and it has exact rows.



Prove that (i) β is one-one if α, r, f' all are one-one maps and (ii) β is onto if α, r, g all are onto maps.

4 Answer any two : **2×7=14**

(a) Let $(N_i)_{i \in \wedge}$ be a family of R-submodules of an R-module M. Prove that $\bigcap_{i \in \wedge} N_i$ is also an R-submodule of M.

(b) Let $f : M \to N$ be an R-homomorphism on R-modules. Prove that $\ker f$ and $f(M)$ are R-submodules of M and N respectively.

(c) Using Eisenstein criterian prove that $g(x) = 1 + x + x^2 + ... + x^{p-1}$ (p is prime) and $g(x+1)$ both are irreducible polynomials over $Q[x]$.

(d) Let F be a field. Prove that the prime subfield of F is either isomorphic to Q or it is isomorphic to $\mathbb{Z}_p$, for some prime p.

(e) Let R be a ring with unity. Prove that an R-module M is cyclic iff $M \simeq \frac{R}{I}$, for some left ideal I of R.

5 (a) Attempt any one of following : **1×8=8**

(1) For a field prove that following statements are equivalent :

$I \to K$ is algebraically closed.

$II \to$ if $p(x) \in K[x]$ and $p(x)$ is an irreducible polynomial then deg $p(x) = 1$.

$III \to$ For any $f(x) \in K[x]$ with deg $f(x) \geq 1$, $f(x)$ can be split into linear factors in $K[x]$.

$IV \to$ For any $f(x) \in K[x]$ with deg $f(x) \geq 1$, K contains all the roots of $f(x)$.

(2) State and prove primitive element theorem.

(b) Attempt any two of followings : **2×3=6**

(1) Define algebraically closed field. Which of followings is/are algebraically closed fields ?

Q, $\mathbb{R}$, $\mathbb{C}$, $\bar{Q}$.

(2) Prove that $F[x]$ is an F-module.

(3) Define F-automorphism for a field extension E/F and write down Galois group $G(\mathbb{C} \mid \mathbb{R})$ for the field extension $\mathbb{C} \mid \mathbb{R}$.

* M B Z - 0 0 3 - 1 1 6 2 0 0 1 - 1 7 0 - 4 *

MBZ-003-1162001 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

CMT - 2006 : Mathematics

(Algebra -II) (New Course)

Faculty Code : 003
Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Each question carries 14 marks.

1 Answer following short questions : **7×2=14**

(i) For a ring R, define R–sub module of an R-module M.

(ii) Let M be an R-module. In std. notation, prove that $(-a)m = a(-m), \forall\ a \in R \text{ and } \forall\ m \in M$.

(iii) Let $f(x) = x^3 + 6x^2 + 7x + 8$. Prove that $f(x-2)$ is an irreducible polynomail. Is $f(x)$ irreducible? (Y/N)

(iv) State Eisenstein Criterion.

(v) For the field extension $^{R}|_{Q}$, write down two elements of R–Q, which are algebraic over Q and write down four elements of R, which are not algebraic over Q (they are transcendental elements over Q).

(vi) Write down the minimal polynomial of –i over Q.

(vii) Define splitting field.

2 Attempt any **two** : **2×7=14**

(a) Let $^{E}|_{F}$ *and* $^{K}|_{E}$ both are algebraic extensions. Prove that $^{K}|_{F}$ is also an algebraic extension.

(b) Prove that $Q\left(\sqrt{2},\sqrt{3},\ldots\ldots,\sqrt{p},\ldots\right)|Q$ is an infinite algebraic extension.

(c) Let $p(x)\in F[x]$ be an irreducible polynomial and degree of $p(x)=n$. Let $^{E}|_{F}$ be an extension such that $\alpha\in E$ and α is a root of $p(x)$. Prove that $F[\alpha]=F(\alpha),\left[F(\alpha):F\right]=n$ and $\left\{1,\alpha,\alpha^{2},\ldots\ldots,\alpha^{n-1}\right\}$ is a basis of $F(\alpha)$ over F.

(d) Prove that every finite extension is an algebraic extension.

3 Attempt any **one** : **1×14=14**

(a) State and prove primitive element theorem.

(b) Let $^{E}|_{F}$ *and* $^{K}|_{E}$ both are finite separable extensions. Prove that $^{K}|_{F}$ is also a finite separable extension.

(c) Define algebraically closed field. For a field K, prove that following statements are equivalent :

(1) K is an algebraically closed field.

(2) If $p(x)\in K(x)$ and $p(x)$ is an irreducible polynomial, then degree of $p(x)$ is 1.

(3) Any $f(x)\in K[x]$, with degree of $f(x)\geq 1$, $f(x)$ can be split into linear factors in $K(x)$.

(4) Any $f(x)\in K[x]$, with degree of $f(x)\geq 1$, K contains all the roots of $f(x)$.

4 Attempt any **two** : **2×7=14**

(a) Let $f(x) \in F[x]$ be an irreducible polynomial. Prove that α is a multiple root of $f(x)$ if and only if $f'(x) = 0$ (All the coefficients of $f'(x)$ are multiple of char F.)

(b) Let char $k = p > 0$ and $f(x) \in k[x]$ be an irreducible polynomial. Prove that $f(x)$ has a multiple root if and only if $f(x) = g\left(x^{\rho}\right)$, for some $g(x) \in k[x]$.

(c) Let $f: M \dashrightarrow N$ be an R–homomorphism of R–modules. Prove that Ker f and $f(M)$ are R–sub modules of M and N respectively.

(d) Prove that $G\left({}^{C}|_{R}\right)$ is a group of order 2.

(e) Let F be a finite field. Prove that $F^{*} = F - \{0\}$ is a cyclic group under multiplication.

5 Attempt any **seven** : **7×2=14**

(1) Define F–automorphism.

(2) Give an example of a finite field F such that $|F|$ is not a prime.

(3) Write down all the roots of the polynomial $x^4 - 2 \in Q[x]$.

(4) Give definition of a finite field extension. Also give an example of a finite field extension.

(5) Write down the minimal polynomial of $\cos\left(\frac{2\pi}{p}\right) + i.\sin\left(\frac{2\pi}{p}\right)$ over Q.

(6) Write down definitions of a prime field and give an example of a prime field.

(7) Write down definitions of separable polynomial, separable element and separable extension.

(8) State first fundamental theorem of R–homomorphism.

(9) Define exact sequence of R–modules and R–homomorphisms.

(10) Define cyclic field extension and give an example of finite cyclic extension.

PAQ-003-1162001 Seat No. 025074

M. Sc. (Sem. II) Examination

August / September - 2020

Mathematics : CMT - 2001

(Algebra -II)

Faculty Code : 003

Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer Any **Seven** short questions : 7×2=14

(i) For a ring R, define R–module and give an example of an R–module.

(ii) Let M be an R–module. In standard notation, prove that $(-a)m = a(-m), \forall a \in R \ \& \ \forall m \in M$.

(iii) Let $f(x) = x^3 + 4x^2 - 11x + 13$. Prove that $f(x+1)$ is an irreducible polynomial over $Z[x]$.

(iv) Define finite field extension and give an example of finite extension.

(v) Write down all the roots of the polynomial $x^4 - 2 \in Q[x]$.

(vi) Write down the minimal polynomial of the number $\sqrt{2} + \sqrt{3}$ over Q.

(vii) Write down at least two irreducible polynomials of the ring $Z_2[x]$ whose degree is precisely two.

(viii) Give definition of an algebraic extension. Also give an example of an algebraic extension.

(ix) For a field extension $E|_F$, when we say E is finitely generated field over F? Also give definition of simple extension.

2 Attempt Any **Two** : 2×7=14

(a) Let $E|_F$ and $K|_E$ both are finite extensions. Prove that $K|_F$ is also a finite field extension.

(b) Let $p(x) \in F[x]$ be an irreducible polynomial and degree of $p(x) = n$. Let. $E|_F$ be an extension such that $\alpha \in E$ and α is a root of $p(x)$. Prove that $F[\alpha] = F(\alpha), [F(\alpha) : F] = n$ and $\{1, \alpha, \alpha^2, \dots\dots \alpha^{n-1}\}$ is a basis of $F(\alpha)$ over F.

(c) State and Prove Eisenstein Criterion.

3 Attempt Any **One** : 1×14=14

(a) Let $E|_F$ and $K|_E$ both are finite separable extensions. Prove that $K|_F$ is also a finite separable extension.

(b) (1) Let F be finite field. Prove that $F - \{0\}$ is a cyclic group under multiplication.

(2) Let F be a field and $F - \{0\}$ is a cyclic group under multiplication. Prove that F is a finite field.

(c) Let $E|_F$ be a finite extension. Prove that following statements are equivalent :

(1) $E = F(\propto)$, for some $\propto \in E$.

(2) There are only a finite number of sub fields of E containing F, as a sub field.

4 Attempt Any **two** : 2×7=14

(a) Let $f(x) \in F[x]$ be an irreducible polynomial. Prove that α is a multiple root of $f(x)$ if and only if $f'(x) = 0$ (All the coefficients of $f'(x)$ are multiple of char F).

(b) Let p be a prime. Prove that $f(x) = x^{p-1} + x^{p-2} + \dots + x + 1 \in \mathbb{Z}[x]$ is an irreducible polynomial over $\mathbb{Q}[x]$.

(c) Let $f : M \to N$ be an R–homomorphism of R–modules. Prove that Ker f and $f(M)$ are R–sub modules of M and N respectively.

5 Attempt Any **Two** : **2×7=14**

(1) Let R be a ring and M be an R–module. Prove that M is a cyclic R–Module if and only if $M \cong \frac{R}{I}$, for some ideal I of R.

(2) Let $f : M \to N$ be an onto R–homomorphism of R–modules. Prove that $\frac{M}{Ker\ f} \cong N$.

(3) Let A, B, be R–sub modules of two R–modules M and N respectively. In standard notation, prove that $\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$.

(4) Let K be a field and char $K = p > 0$. Prove that K is a perfect field if and only if $K = K^p$, where $K^p = \{\alpha^p / \alpha \in K\}$.

BBF-003-1162001 Seat No. 25118

M. Sc. (Sem. II) (CBCS) Examination

June / July - 2021

Mathematics : CMT - 2001

(Algebra - II)

Faculty Code : 003

Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1 Answer following seven questions : **7×2=14**

(1) Let $f(x)=x^3+x^2-16x+34\in Q[x]$. Using Eisenstein Criterion, prove that, $f(x+2)$ is an irreducible polynomial over $Q[x]$.

(2) Let $f(x)=x^3+4x^2-11x+13$. Prove that $f(x+1)$ is an irreducible polynomial over $Z[x]$.

(3) Give definition of a finite field extension. Also give an example of a finite field extension.

(4) For the field extension ${}^{R}|_{Q}$, write down two elements of $R-Q$, which are algebraic over Q and write down four elements of R, which are not algebraic over Q (they are transcendental elements over Q).

(5) For a field extension ${}^{E}|_{F}$, when we say E is finitely generated field over F? Also give definition of simple extension.

(6) Write down the minimal polynomial of the number $\sqrt{2}+\sqrt{3}$ over Q.

(7) Give an example of a finite field F such that $|F|$ is not a prime.

2 Answer following seven questions : **7×2=14**

(1) Write down all the roots of the polynomial $x^3-2 \in Q[x]$ and what is $\left[Q\left(2^{1/3}\right):Q\right]$? [Degree of the field extension $E|_Q$, where $E=Q\left(2^{1/3}\right)$].

(2) Define F–automorphism on E, where $E|_F$ is an extension.

(3) Write down definitions of separable polynomial, separable element and separable extension.

(4) Write number of elements of the field K and characteristic of K, where K is the splitting field of the polynomial $x^{p^n}-x$ over $Z_p[x]$.

(5) Define cyclic field extension and give an example of finite cyclic extension.

(6) For a ring R, define R–module and give an example of an R–module.

(7) For a ring R, define R–sub module of an R–module M. Also prove that, $N=\{rx / r \in R\}$ is an R–sub module of M, for some $x \in M$.

3 Answer following two questions : **2×7=14**

(a) Let p be a prime. Prove that, $f(x)=x^{p-1}+x^{p-2}+\ldots+x+1 \in \mathbb{Z}[x]$ is an irreducible polynomial over $\mathbb{Q}[x]$.

(b) State and prove, Eisenstein Criterion.

4 Answer following one question : 1×14=14

(1) Let $E|_F$ and $K|_E$ both are algebraic extensions. Prove that, $K|_F$ is also an algebraic extension.

(2) Let $p(x) \in F[x]$ be an irreducible polynomial and degree of $p(x) = n$. Let $E|_F$ be an extension such that $\alpha \in E$ and α is a root of $p(x)$. Prove that, $F[\alpha] = F(\alpha), [F(\alpha) : F] = n$ and $\{1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}\}$ is a basis of $F(\alpha)$ over F.

5 Answer following two questions : 2×7=14

(1) Find the splitting field of $x^3 + x^2 + 1 \in Z_2[x]$ over Z_2.

(2) Prove that, $Q(\sqrt{2}, \sqrt{3}, \ldots\ldots, \sqrt{p}, \ldots) | Q$ is an infinite algebraic extension.

6 Answer following two questions : 2×7=14

(1) Let R be a ring with unity and M is an R–module. Prove that, M is cyclic if and only if $M \simeq \frac{R}{I}$, for some left ideal I of R.

(2) Let A, B be R–sub modules of two R–modules M and N respectively. In standard notation, prove that, $\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$.

7 Answer following two questions : 2×7=14

(1) Let F be a finite field. Prove that, $F^* = F - \{0\}$ is a cyclic group under multiplication.

(2) Let F be a field and $F - \{0\}$ is a cyclic group under multiplication. Prove that, F is a finite field.

8 Answer following one question : 1×14=14

Define algebraically closed field. For a field K, prove that, following statements are equivalent :

(i) K is an algebraically closed field.

(ii) If $p(x) \in K[x]$ and $p(x)$ is an irreducible polynomial, then degree of $p(x)$ is 1.

(iii) Any $f(x) \in K[x]$, with degree of $f(x) \geq 1$, $f(x)$ can be split into linear factors in $K[x]$.

(iv) Any $f(x) \in K[x]$, with degree of $f(x) \geq 1$, K contains all the roots of $f(x)$.

9 Answer following one question : 1×14=14

For a field extension $E|_F$, when we say E is finitely generated field over F? Also give definition of simple extension and prove that, any finite separable extension is a simple extension.

10 Answer following two questions : 2×7=14

(1) Let $f(x) \in F[x]$ be an irreducible polynomial. Prove that, α is a multiple root of $f(x)$ if and only if $f'(x) = 0$ (All the coefficients of $f'(x)$ are multiple of char F).

(2) Let char $k = p > 0$ and $f(x) \in k[x]$ be an irreducible polynomial. Prove that, $f(x)$ has a multiple root if and only if $f(x) = g(x^p)$, for some $g(x) \in k[x]$.

DBW-003-1162001 Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2001

(Algebra-II)

Faculty Code : 003

Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Each question carries equal marks.
(3) Figure on the right indicate allotted marks.

1 Answer any **seven** short questions : 7×2=14

(i) Let $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n \in F[X]$ and $a_n \neq 0$. If F contains one root of $f(x)$, then prove that, $f(x)$ is reducible over F.

(ii) Let $f(x) = x^3 + 6x^2 + 7x + 8$. Prove that, $f(x-2)$ is an irreducible polynomial over $Z[x]$. Is $f(x)$ irreducible ? (Y/N).

(iii) $f(x) = x^3 + 4x^2 - 11x + 13$. Prove that $f(x+1)$ is an irreducible polynomial over $Z[x]$.

(iv) Define finite field extension and give an example of finite extension of degree 3.

(v) For the field extension $^{R}|_{Q}$, write down two elements of $R-Q$, which are algebraic over Q and write down four elements of R, which are not algebraic over Q (they are transcendental elements over Q).

(vi) Write down the minimal polynomial of the number $\sqrt{2}+\sqrt{3}$ over Q.

(vii) Give definition of algebraically closed field. Also give an example of an infinite algebraic extension.

(viii) Give an example of a finite field F such that $|F|=4$.

(ix) Let M be an R-module. In standard notation, prove that, $(-a)m = a(-m) = -(am), \forall\ a \in R \text{ and } \forall\ m \in M$

(x) For a ring R, define R-sub module of an R-module M. Also give an example of an R-sub module.

2 Attempt any two : **2×7=14**

(a) Let p be a prime. Prove that, $f(x) = x^{p-1} + x^{p-2} + x^{p-3} + ... + x + 1 \in \mathbb{Q}[x]$ is an irreducible polynomial over $\mathbb{Q}[x]$.

(b) Let $p(x) \in F[x]$ be an irreducible polynomial. Prove that, there is an extension $E|_F$ such that E contains one root of $p(x)$.

(c) Prove that, every finite extension is an algebraic extension.

3 Attempt any one : **1×14=14**

(1) Let $E|_F$ be a finite extension. Prove that, following statements are equivalent :

(i) E=F(α), for some $\alpha \in E$

(ii) There are only a finite number of sub fields of E containing F, as a subfield.

(2) State and prove, the Fundamental Theorem of Galois Theroy.

4 Attempt any **two** : **2×7=14**

(1) Let $E|_F$ be a field extension and $\alpha_1, \alpha_2, ..., \alpha_n$ be are algebraic over F. Prove that, $F(\alpha_1, \alpha_2, ... \alpha_n)|_F$ is a finite field extension.

(2) Let char k = p > 0 and $f(x) \in k[x]$ be an irreducible polynomial. Prove that, $f(x)$ has a multiple root if and only if $f(x) = g(X^p)$, for some g(x) $\in$ k[x].

(3) Let K be a field and char K = p>0. Prove that, K is a perfect field if and only if $K = K^p$, where $K^p = \{\alpha^p / \alpha \in K\}$.

5 Attempt any two : 7×2=14

(a) Let M be a free R-module and $\{e_1, e_2, ... e_n\}$ be a basis for M. Prove that, $M \cong R^n$

(b) Let F be a field and Char F = 0. Let n be a natural number and n^{th} root of unity $\in$F. Let $K|_F$ be a cyclic extension and [K : F] = n. Prove that, there exist an $\propto \in K$ such that $K = F(\propto)$ and $\propto$ satisfies the polynomial $f(x) = x^n - a \in f[x]$, where $a \in F$.

(c) State and prove, Hilbert Theorem 90.

(d) Let $f : M \to N$ be an R-homomorphism of R-modules. Prove that, Ker f and $f(M)$ are R-sub modules of M and N respectively.

Seat No. ______________

HN-003-1162001

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT-2001

(Algebra-II)

Faculty Code : 003
Subject Code : 1162001

Time : $2\frac{1}{2}$ / Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Each question carries equal marks.
(3) Figure on the right indicate allotted marks.

1 Answer any seven short questions. **7×2=14**

(i) Prove or disprove that, if $f(x)$ be a polynomial of degree 2 or 3 in a polynomial ring $F[x]$ over a field F, then $f(x)$ is reducible over $F \Leftrightarrow f(x)$ has a root in F.

(ii) Prove or disprove that, $x^3+3x+2 \in \mathbb{Z}_7[x]$ is irreducible over $\mathbb{Z}_7$.

(iii) Define term : Monic polynomial and justify that every monic polynomial is a primitive polynomial.

(iv) Define terms : Extension of fields and finite field extension (finite extension).

(v) Write down the minimal polynomial of the complex number $\sqrt{2}+\sqrt{3i}$ over Q.

(vi) Write down two definitions : Perfect field and Prime field.

(vii) Write down examples of two algebraic field extenstions, which are separable extensions.

(viii) Prove or disprove that, the set of generators of an R-module need not be unique.

(ix) Let $\{N_\lambda\}_{\lambda\in\Lambda}$ be a family of R-sub modules of an R-module M. Prove that $\bigcap_{\lambda\in\Lambda} N_\lambda$ is also an R-sub module of M.

(x) For a ring R, define R-sub module of an R-module M.

2 Attempt any two. $2\times7=14$

(a) Let $E|_F$ and $K|_F$ both are finite extensions. Prove that, $K|_F$ is also finite extension.

(b) Let $p(x) \in F[x]$ be an irreducible polynomial and degree of $p(x) = n$. Let $E|_F$ be an extension such that $\alpha \in E$ and α is a root of $p(x)$. Prove that, $F[\alpha] = F(\alpha)$, $[F(\alpha):F] = n$ and $\{1, \alpha, \alpha^2, \dots\dots, \alpha^{n-1}\}$ is a basis of $F(\alpha)$ over F.

(c) Prove that, every finite extension is an algebraic extension.

3 Attempt followings. $2\times7=14$

(1) Let $E|_F$ be a finite extension and $E = F(\alpha)$, for some $\alpha \in E$. Prove that, There are only a finite number of sub fields of E containing F, as a sub field.

(2) Let $E|_F$ be a finite extension and there are only a finite number of sub fields of E containing F, as a sub field. Prove that, $E = F(\alpha)$, for some $\alpha \in E$.

OR

3 Attempt followings. $2\times7=14$

(a) Let F be a finite field. Prove that, $F^* = F - \{0\}$ is a cyclic group under multiplication.

(b) Let F be a field and $F - \{0\}$ is a cyclic group under multiplication. Prove that, F is a finite field.

4 Attempt any two. $2\times7=14$

(1) Suppose following diagram of R-modules and R-homomorphisms.
$f: K \to M$, $g: M \to L$, $f': K' \to M'$, $g': M' \to L'$, $\alpha: K \to K'$, $\beta: M \to M'$ and $\gamma: L \to L'$ with $f(K) = \text{Ker } \beta = \ker g$ and $f'(K') = \text{Ker } g' = \beta(M)$. Prove that, if α, γ & f' are 1-1 then so is β.

(2) Let K be algebraically closed field and $p(x) \in K[x]$ be an irreducible polynomial over $K[x]$. Prove that degree of $p(x)$ is 1. Also deduce that, for any $f(x) \in K[x]$, with degree $f(x) \geq 1$, $f(x)$ can be split into linear factors in $K[x]$.

(3) Let F be a field and Char $F = 0$. Let n be a natural number and F contains n^{th} root of unity. Let $f(x) = x^n - a \in f[x]$ and E is the splitting field of $f(x)$ over F. Prove that, $E|_F$ is a cuclic extension.

5 Attempt any two. **7×2=14**

(1) Let A, B be R-sub modules of two R-modules M and N respectively. In standard notation, prove that,

$$\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}.$$

(2) Let R be a ring with unity and M is an R-module. Prove that, M is cyclic if and only if $M \cong \frac{R}{I}$, for some left ideal I or R.

(3) Let $E|_F$ be an extension and $[E:F]=2$. Prove that, $E|_F$ is a normal extension.

(4) Let $K|_k$ be a cyclic extension, $[K:k]=n$ and $G(K|_k) = <\sigma>$. Prove that, $\beta\, \sigma(\beta)\, \sigma^2(\beta) \ldots \sigma^{n-1}(\beta) = 1$, for some $\beta \in K$ if and only if $\beta = \alpha . (\sigma(\alpha)^{-1})$, where $\alpha \in K^*$.

RB-7

003-016202

M.Sc. (CBCS) Semester-II Examination
May-2013
Mathematics CMT – 2002 : Complex Analysis, Set – I

Faculty Code : 003
Subject Code : 016202

Time : 2 ½ Hours] **[Total Marks : 70**

Instructions : (1) Answer **all** the questions.
(2) Each question carries **14** marks.

1. Answer any **seven** questions : **7 × 2 = 14**

(i) $z_2, z_3, z_4 \in ⊄_\infty$ are distinct then $(z_2, z_2, z_3, z_4) =$ ______

(a) ∞ (b) 0

(c) 1 (d) –1

(ii) If T is a bilinear transformation and Γ is a circle in ⊄ then $T(\Gamma) =$

(a) Γ (b) a circle in $⊄_\infty$

(c) a straight line in ⊄ (d) $⊄_\infty$

(iii) The right side of the unit circle Γ with centre O w.r.t the orientation $(i, -1, -i) =$ _______

(a) Γ (b) $\{Z \in ⊄ \,\big|\, |Z| > 1\}$

(c) the upper half plane (d) the left half plane

(iv) ______ has atmost two distinct fixed points

(a) every bilinear transformation

(b) every bilinear transformation other than the identity

(c) Z^3

(d) $\sin z + z$

(v) _______ is not conformed at 0.

(a) cos z
(b) e^z
(c) sin z
(d) z – sin z

(vi) If $\gamma : [a, b] \to ⊄$ is piecewise smooth and $f : [\alpha, b\} \to ⊄$ is continuous then $\int_a^b f d\gamma =$ ______

(a) $\int_a^b (fo\gamma)(t)\, dt$
(b) $\int_a^b f'(t)\gamma'(t)dt$
(c) $\int_a^b (fo\gamma)(t)\, \gamma'(t)dt$
(d) $\int_a^b f(t)\gamma'(t)dt$

(vii) ______ is not a bilinear transformation

(a) $Tz = z^2$
(b) $Tz = \frac{1}{z}$
(c) $Tz = \frac{z+1}{z-1}$
(d) $\frac{z-1}{z+1}$

(viii) If γ is the circle $\gamma(t) = 2 + e^{it}, \forall t \in [0, 2\pi]$ then $V(\gamma) =$ ______

(a) 2π
(b) 4π
(c) $\frac{\pi}{2}$
(d) π

(ix) If G⊂⊄ is a region and f : G → ⊄ is analytic then

(a) u, v satisfy Cauchy-Riemann equations
(b) f : G → ⊄ is conformal
(c) f : G → ⊄ is onto
(d) f : G → ⊄ is one-one

(x) If $f : G \to \mathbb{C}$ is analytic then

(a) $f : G \to \mathbb{C}$ is a bijection

(b) $f : G \to \mathbb{C}$ is conformal

(c) $f : G \to \mathbb{C}$ does not vanish

(d) $f : G \to \mathbb{C}$ is infinitely differentiable.

2. Answer any **two** : **2 × 7 = 14**

(a) Define bilinear transformation. Prove that two bilinear transformation $Sz = \dfrac{az + b}{cz + d}$, $Tz = \dfrac{\alpha z + \beta}{\gamma z + \delta}$ are equal iff $\exists\ 0 \neq \lambda \in \mathbb{C}$ s-t $\alpha = \lambda a$, $\beta = \lambda b$, $\gamma = \lambda c$, $\delta = \lambda d$.

(b) Prove that every bilinear transformation is the composition of translation, dialation and inversion.

(c) If $Tz = \dfrac{az + b}{cz + d}$ is a bilinear transformation then prove that $T(\mathbb{R}_\infty) = \mathbb{R}_\infty$ iff a, b, c, d can be chosen to be reals.

3. (a) Prove that a bilinear transformation takes circles to circles. **7**

(b) Define orientation of a circle. State and prove orientation principle. **7**

OR

(c) Define branch of logarithm on a connected open set in $\mathbb{C}$. Prove that $\exists$ a branch of logarithm on $G = \mathbb{C} \setminus \{z \in \mathbb{C} | z \leq 0\}$ **7**

(d) Find an analytic function $f : G \to D$ s.t $f(G) = D$

where $G = \{z \in \mathbb{C} | \text{Re} z < 0\}$ and $D = \{z \in \mathbb{C} \,|\, |z| < 1\}$ **7**

4. Answer any **two** : **2 × 7 = 14**

(a) Prove that a continuous function $f : \mathbb{C} \to \mathbb{C}$ need not have a primitive.

(b) Define rectifiable path. Prove that $\gamma = [0, 1] \to \mathbb{C}$ defined by

$$\gamma(t) = \begin{cases} t + it \sin \frac{1}{t} \text{ if } t \neq 0 \\ 0 \text{ if } t = 0 \end{cases}$$

is a path but not rectifiable.

(c) Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi, \forall z \in \mathbb{C}$ st $|z| < 1$.

5. Answer any **two** : **2 × 7 = 14**

(a) Let $f : G \to \mathbb{C}$ be analytic and $\bar{B}(a, r) \subset G$ and $\gamma(t) = a + re^{it}$, $0 \leq t \leq 2\pi$. Prove that

$$f^{n}(a) = \frac{n!}{2\pi i} \int_\gamma \frac{f(w)}{(w - a)^{n+1}} dw, \forall n = 0, 1, 2, \ldots\ldots\ldots$$

(b) State and prove Cauchy's estimate.

(c) Prove that every non-constant polynomial with complex coefficients has a root.

(d) Find $\int_\sigma \frac{dz}{z^2 - 1}$, where $\sigma_{(t)} = 1 + e^{it}$, $\forall\, 0 \leq t \leq 2\pi$.

NCC-003-016202 Seat No. ________

M. Sc. (Mathematics) (CBCS) (Sem. II) Examination

April / May – 2017

Mathematics : CMT - 2002

(Complex Analysis) (Old Course)

Faculty Code : 003

Subject Code : 016202

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions.

(2) Each question carries 14 marks.

1 Answer any seven questions : **2×7=14**

(i) Define the chrodal metric on $\mathbb{C}_{\infty}$.

(ii) If $z_2, z_3, z_4 \in \mathbb{C}$ are distinct then (z, z_2, z_3, ∞) = ________ and (z, z_2, ∞, z_4) = ________.

(iii) True or false ? Justify. S defined by $S_z = \bar{z}$ is a bilinear transformation.

(iv) If $\lambda \in \mathbb{C}, \lambda \neq 0, \lambda \neq 1$ then find the fixed points of the bilinear transformation S defined by $S_z = \lambda z$.

(v) If $z, w \in \mathbb{C}$ and $\gamma : [0,1] \to \mathbb{C}$ is defined by $\gamma(t) = (1-t)z + tw$ $\forall\, t \in [0,1]$ then find $V(\gamma)$.

(vi) If $G \subset \mathbb{C}$ is a region, $f : G \to \mathbb{C}$ is analytic and $|f| : G \to \mathbb{R}$ is a constant function then prove that $f : G \to \mathbb{C}$ is a constant function.

(vii) 0 is ________ of $e^{\frac{1}{z}}$

(a) removable singularity

(b) a simple pole

(c) an essential singularity

(d) not a singularity

(viii) 0 is a removable singularity of ________.

(a) $\cos\left(e^{\frac{1}{z}}\right)$ (b) $\cos z$

(c) $\frac{\sin z}{z}$ (d) $\frac{1}{z}$

(ix) Find the right side of the x-axis w.r.t. the orientation $(-1, 0, 1)$.

(x) If $f : D \to D$ is analytic and $f(0) = 0$ then ________.

(a) $\left|f\left(\frac{3}{4}\right)\right| \le \frac{3}{4}$

(b) $f(z) = cz$, for none $C \in ¢$, $|C| = 1$

(c) $|f'(0)| = 2$

(d) $\left|f\left(\frac{1}{2}\right)\right| > \frac{1}{2}$

2 Answer any two questions : **2×7=14**

(a) Given two circle Γ_1, Γ_2 in $¢_\infty$ and distinct $z_2, z_3, z_4 \in \Gamma_1$, distinct $w_2, w_3, w_4 \in \Gamma_2$ prove that ∍ a unique bilinear transformation S rt $S(\Gamma) = \Gamma'$ and $S(z_j) = w_j, \forall\, j = 2, 3, 4$

(b) Define the right side, left side of a circle Γ in $\mathbb{C}_{\infty}$ w.r.t. an orientation of Γ. Find the right side of the imaginary. axis L w.r.t. the orientation $(-i, 0, i)$ and the left side of the unit circle Γ with centre at o w.r.t. the orientation $(-i, -1, i)$.

(c) Find the bilinear transformation S taking $1 \to 0, 0 \to \infty, \infty \to 1$.

3 (a) If γ is a rectifiable curve in $\mathbb{C}$, $f = \{\gamma\} \to \mathbb{C}$ is continuous and $C \in \mathbb{C}$ then prove that **7**

$$\int_{\gamma} f(z) dz = \int_{\gamma + c} f(z - c) dz.$$

(b) State, without proof, the fundamental theorem of calculus for line integrals. If $f : G \overset{\text{open}}{\subset} \mathbb{C} \to \mathbb{C}$ is continues with primitive and $\gamma, \sigma : [a, b] \to G$ are rectifiable s.t. $\gamma(a) = \sigma(a), \gamma(b) = \sigma(a)$ then prove that $\int_{\gamma} f = \int_{\sigma} f$. **7**

OR

(c) Prove that $\int_{0}^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi, \ \forall \ z \in \mathbb{C}, |z| < 1$.

(d) State, without proof, Cauchy's integral formula for higher derivatives of an analytic function $f : G \to \mathbb{C}$.

Evaluate $\int_{\gamma} \frac{\sin z}{z^3} dz$, where $\gamma(t) = \ell^{it}, \ \forall t \in [0, 2\pi]$

4 Answer any two questions : **2×7=14**

(a) State and prove minimum modulus theorem.

(b) Define removable singularity of complex function "f" of a complex variable and given an example. Prove that $a \in ¢$ is a removable singularity of f iff $a_n = 0, \ \forall \, n \leq -1$ in the Laurent's expansion $\sum_{n=-\infty}^{\infty} a_n (z-a)^n$ of f at a.

(c) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.

5 Answer any two questions : **2×7=14**

(a) State, without proof, Rouche's theorem. Prove that $3z^7 + 5z - 1$ has exactly one zero in $|z| < 1$ and is a real zero in $(0, 1)$.

(b) Find the Laurent's expansion of $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in

(i) $|z| < 1$

(ii) $|z| > 3$

(c) Let $f : G \overset{\text{open}}{\subset} ¢ \to ¢$ be analytic and one-one. Then prove that $f'(3) \neq 0, \forall \ 3 \in G$

(d) Find $\int_\gamma \frac{1}{z} dz$, where $\gamma = [1-i, 1+i, -1+i, -1-i, 1-i]$

* M C A - 0 0 3 - 1 1 6 2 0 0 2 - 1 7 0 - 3 *

MCA-003-1162002

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics : 2002

(Complex Analysis)

Faculty Code : 003

Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.

(2) All questions are compulsory.

(2) Each questions carries 14 marks.

1 Do as directed : (Attempt Any Seven) **14**

(a) Evaluate $\int_{\sigma} \frac{e^{iz}}{z^2} dz$ where $\sigma(t) = e^{it}; t \in [0, 2\pi]$.

(b) Define : (i) Pole. (ii) Essential singularity.

(c) State necessary and sufficient condition for an isolated singularity to be a removable singularity. Also mention the type of singularity for $f(z) = e^{\frac{1}{z}}$.

(d) State the geometric meaning of winding number for the closed rectifiable curve.

(e) (i) Define conformal mapping.

(ii) State Inverse function theorem.

(f) State Riemann stieltje's theorem.

(g) Evaluate $\int_{\sigma} \frac{z^2+1}{z^2+z+1} dz$; where $|z| = 2$ is the circle with center 0 and radius 2.

(h) Find bilinear transformation taking $i \to 1, 0 \to \infty, -i \to 0$.

2 Attempt any **two** of the following : **14**

(a) Define branch of logarithmic on a connected open set and prove that if $f : G \rightarrow \mathbb{C}$ be continuous, $g : H \rightarrow \mathbb{C}$ be differentiable with $g'(x) \neq 0; \forall x \in H$ and $f(G) \subset H, g(f(z)) = z; z \in G$ then f is differentiable and $f'(z) = \frac{1}{g^r(f(z))}; z \in G.$ **7**

(b) (i) Prove that $e^{z+w} = e^z.e^w; z, w \in \mathbb{C}.$ **4**

(ii) Justify with an example that $Log(z_1 z_2) \neq Log(z_1) + Log(z_2).$ **3**

(c) (i) Give an example which shows that Cauchy Riemann equations are merely necessary nut not sufficient. **2**

(ii) Prove that for an analytic function $f : G \rightarrow \mathbb{C};$ where G be an open connected subset of $\mathbb{C}$ and $G^* = \{\overline{z} / z \in G\}$ then $f^* : G^* \rightarrow \mathbb{C}$ defined by $f^*(z) = \overline{f(\overline{z})}; z \in G^*$ is analytic. **5**

3 All are compulsory : **14**

(a) Show that the set M = { S / S is a bilinear transformation} is a group under composition. **7**

(b) (i) State and prove Liouvilles Theorem. **3**

(ii) Prove that if $\gamma : [a, b] \rightarrow \mathbb{C}$ be a function of bounded variation with $a < c < b$ then $\gamma|[a, c] : [a, c] \rightarrow \mathbb{C}$ and $\gamma|[c, b] : [c, b] \rightarrow \mathbb{C}$ are function of bounded variation and $V(\gamma) = V(\gamma|[a, c]) + V(\gamma|[c, b])$ **4**

OR

3 All are compulsory : **14**

(a) Every bilinear transformation can be written as composition of translation, dilation and inversion. **7**

(b) (i) State and prove Open Mapping Theorem. **4**

(ii) Evaluate $\int_{\sigma} \frac{dz}{z^2 + i^2}$, where σ is given by $\sigma(t) = 2e^{it}\left|\cos 2t\right|$. **3**

4 Attempt any **two** of the following : **14**

(a) State and prove Fundamental theorem of algebra. **7**

(b) State and prove Minimum modulus theorem. Also give an example of a non-constant analytic function in $\mathbb{C}$ which may attains its minimum value but not maximum. **7**

(c) Prove that if $f : G - \{a\} \to \mathbb{C}$ be an analytic function and α is a pole of f then there exists $m \in \mathbb{N}$ and $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}; \forall \alpha = z.$ **7**

(d) State and prove Cauchy's Theorem. **7**

5 Attempt any **two** of the following : **14**

(a) State and prove Cauchy's Integral formula for second version. **7**

(b) Prove that every z^m of f has a finite order multiplicity. **7**

(c) Find Laurent's series expansion in the powers of z for $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in **7**

(i) $|z| < 1;$

(ii) $1 < |z| < 3;$

(iii) $|z| > 3.$

(d) State orientation Principle. Also show the concept of symmetric point with respect to a circle Γ is independent of choice of three points $z_2, z_3, z_4 \in \Gamma$. **7**

PAR-003-1162002 Seat No. 025074

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

CMT - 2002 : Mathematics

(Complex Analysis)

Faculty Code : 003
Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Figures on the right indicates marks.

1 Answer Any **Seven** questions : 2×7=14

(a) Write the statement of Cauchy's Integral Formula (1^{ST} Version).

(b) Define bounded variation and rectifiable path.

(c) Evaluate $\int_{\gamma} \frac{1}{z} dz$ where $\gamma(t) = e^{int}$ for $n \in \mathbb{N}$ and $t \in [0, 2\pi]$..

(d) Prove that $f = g$, if the set $\{z \in G : f(z) = g(z)\}$ has a limit point in G where $f, g : G \to \mathbb{C}$ is an analytic function and G is an open connected subsed of $\mathbb{C}$.

(e) Evaluate $\int_{|z|=3} \frac{f(z)'}{f(z)} dz$ if $f(z) = \frac{(z^2+1)^2}{(z^2+3z+2)^3}$.

(f) Write the statement of Taylors's Theorem.

(g) Define Meromorphic function and also write one example of Meromorphic Function.

(h) Show that $f : \mathbb{C} \to \mathbb{C}$ be defined as $f(z) = \sin z$ is not a bounded function.

(i) Define Pole and find the pole of $f(z) = \frac{e^{2z}}{z(z-1)}$ for $z \in \mathbb{C}$ also write the order of each pole.

(j) Write the statement of Open mapping Theorem.

2 Answer Any Two of the following : 7×2=14

(a) Let G be an open connected subset of $\mathbb{C}$ and $f:G\to\mathbb{C}$ is an analytic on G with $f'(z)=0$ for all $z\in G$ then show that f is constant function also show that one cannot drop the condition of connectedness in the above statement.

(b) Prove that $e^{z+w}=e^{z}\cdot e^{w}$ for all $z,w\in\mathbb{C}$ and $\overline{e^{z}}=e^{\overline{z}}$ for all $z\in\mathbb{C}$.

(c) Show that for an analytic function $f:G\to\mathbb{C}$ where G be an open connected subset of $\mathbb{C}$ and $G^{*}=\{\overline{z}:z\in G\}$ then $f^{*}:G^{*}\to\mathbb{C}$ defined by $f^{*}(z)=\overline{f(\overline{z})}$ for all $z\in G^{*}$ is analytic on G^{*}.

3 All are Compulsory. Attempt one pair of (a) and (b). 7×2=14

(a) Prove that the representation of Bilinear Transformation is not unique.

(b) Show that every Bilinear Transformation $S\neq I$ has atmost two fixed points and deduce that if S and T are Bilinear Transformation such that $S(z_i)=T(z_i)$, for $i=1,2,3$ for some distinct $z_1,z_2,z_3\in\mathbb{C}_\infty$ then $S=T$.

OR

(a) Prove that if $f\,g=o$ then $f=0$ or $g=0$, provided f and g are analytic function on an open connected set G.

(b) Prove that $\frac{1}{2\pi i}\int_{\gamma}\frac{dz}{z-a}$ is any integer, where $\gamma:[0,1]\to\mathbb{C}$ be any closed rectifiable curve and $a\notin\{\gamma\}$.

4 Answer Any Two questions : 7×2=14

(a) Let γ be a closed rectifiable curve in $\mathbb{C}$ then prove that $n(\gamma;a)$ is constant for "a" belonging to component of $G=\mathbb{C}-\{\gamma\}$. Also show that $n(\gamma;a)=0$ for "a" belonging to the unbounded component of G.

(b) Write the statement of Cauchy Integral Formula for Derivatives (2[nd] Version) and show that $\int_\gamma \frac{1}{(z-a)^n}\,dz = 0$ for every integer $n \geq 2$, provided γ is any closed rectifiable curve in $\mathbb{C}$ and $a \notin \{\gamma\}$.

(c) State and prove Liouville's Theorem.

(d) Write the statement of Identity Theorem and show that there does not exists an analytic function $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = z$, for all z such that $|z| = 1$ and $f(z) = z^2$, for all z such that $|z| = 2$.

5 Answer Any Two of the following questions : 7×2=14

(a) State and prove Cauchy's integral formula (1[st] version).

(b) Prove that if $f : G - \{a\} \to \mathbb{C}$ be an analytic function and α is pole of f then there exists $m \in \mathbb{N}$ and an analytic function $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}$ for all $z \in G - \{a\}$.

(c) Let A_1 and A_2 be two circles in $\mathbb{C}_\infty$ and S be a Bilinear Transformation such that $S(A_1) = A_2$ then prove that if z^* *and* z are symmetric with respect to A_1 then $S(z^*)$ and $S(z)$ are symmetric with respect to A_2.

(d) Write the standard form of Bilinear Transformation and prove that every Bilinear Transformation preserves cross ratio.

BBG-003-1162002 Seat No. 25118

M. Sc. (Mathematics) (Sem. II) (CBCS) Examination

July - 2021

CMT - 2002 : Complex Analysis

Faculty Code : 003
Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions: (1) Answer any five questions from followings.
(2) Each question carries 14 marks.
(3) There are 10 questions in total.

1 Answer the following *seven* questions: [7 X 2 = 14] 14

(1) Define: Stereographic projection, $T^{-1}: S \to \mathbb{C}_\infty$. What is its inverse?

(2) Prove that, the bilinear transformation $S \neq I$ has at most two distinct fixed points, where I is the identity transformation.

(3) Define: Path and Smooth path

(4) Define the following terms:
a) Translation
b) Dilation
c) Inversion

(5) Prove that, every differentiable function is continuous.

(6) Define: Analytic function. Also prove that, if $f, g: G \to \mathbb{C}$ are analytic then for $g \neq 0, \frac{f}{g}: G \to \mathbb{C}$ is analytic.

(7) Find the radius of converges of power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

2 Answer the following *seven* questions: [7 X 2 = 14] 14

(1) Define: Branch of logarithm

(2) Give the statement: Liouville's Theorem.

(3) Find the bilinear transformation taking $i \to 1, 0 \to \infty, -i \to 0$.

(4) Define: Conformal map. Is non – constant analytic function conformal? Justify.

(5) Define: Right side, left side of the circle Γ in $\mathbb{C}_\infty$ with respect to an orientation of Γ.

(6) If $z, w \in \mathbb{C}$ and $\gamma: [0,1] \to \mathbb{C}$ is defined by $\gamma(t) = (1-t)z + tw, \forall t \in [0,1]$ then find, $V(\gamma)$.

(7) Define the following terms:
a) Variation of function,
b) Function of bounded variation
c) Total variation of a function

3 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. Define the rectifiable path in $\mathbb{C}$. Give an example of a path which is not rectifiable.
2. State and prove, Cauchy's Estimate.

4 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. Let $\gamma: [0,2\pi] \to \mathbb{C}$ is a rectifiable path and $f(z) = z^m$ then find $\int_\gamma f$ for
   a) $\gamma(t) = tb + (1-t)a$, where a and b are the constants and $\forall t \in [0,2\pi]$.
   b) $\gamma(t) = e^{it}$, $\forall t \in [0,2\pi]$
2. a) State and prove, Taylor's Theorem.
   b) Evaluate: $\int_\sigma \frac{e^{iz}}{z^2} dz$; where $\sigma(t) = e^{it}, \forall t \in [0,2\pi]$.

5 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. Prove that, every bilinear transformation can be written as composition of translation, dilation and inversion.
2. Give an example of the complex function has no primitive. Justify.

6 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. Find, $\int_\sigma \frac{1}{z^2-1} dz$; where $\sigma(t) = 1 + e^{it}$, $\forall t \in [0,2\pi]$.
2. Show that, the set $M = \{S/S \text{ is a bilinear transformation}\}$ is a group under composition.

7 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. a. Show that: $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
   b. Show that: $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$
2. Prove that: If $\gamma[a,b] \to \mathbb{C}$ is a function of bounded variation with $a < c < b$ then $\gamma_{\backslash[a,c]}: [a,c] \to \mathbb{C}$ and $\gamma_{\backslash[c,d]}: [c,d] \to \mathbb{C}$ are functions of bounded variation and $V(\gamma) = V(\gamma_{\backslash[a,c]}) + V(\gamma_{\backslash[c,d]})$.

8 **Answer the following *two* questions: [2 X 7 = 14]** 14

1. Prove that, $\int_0^{2\pi} \frac{e^{iS}}{e^{iS}-z} ds = 2\pi, \forall z \in \mathbb{C}, |z| < 1$.
2. Let $\gamma: [a,b] \to \mathbb{C}$ be a piecewise smooth and $f: [a,b] \to \mathbb{C}$ be continuous. Prove that, $\int_a^b f\, d\gamma = \int_a^b f(t) \cdot \gamma'(t)\, dt$.

9 **Answer the following *one* questions: [1 X 14 = 14]** 14

1. State and prove, Libnitz's Rule

10 **Answer the following *one* questions: [1 X 14 = 14]** 14

1. Let $\gamma: [a,b] \to \mathbb{C}$ be a function of bounded variation and $f: [a,b] \to \mathbb{C}$ be a continuous function. Prove that, $\exists$ a unique $I \in \mathbb{C}$ such that given $\varepsilon > 0$, $\exists\, \delta > 0$ and the partition $P = \{a = t_0 < t_1 < \cdots < t_n = b\}$ of $[a,b]$ such that $||P|| = \max_{1\le k\le n}(t_k - t_{k-1}) < \delta$ and $\tau_k \in [t_{k-1}, t_k]$, $\forall k = 1,2,\ldots$
   $\Rightarrow \left|I - \sum_{k=1}^{n} f(\tau_k) \cdot (\gamma(t_k) - \gamma(t_{k-1}))\right| < \varepsilon$.

DBX-003-1162002 Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2002

(Complex Analysis)

Faculty Code : 003
Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Each questions carries equal marks.
(3) Figure on the right indicate allotted marks.

1 Answer any **Seven** short questions : 7×2=14

(1) State : Necessary and sufficient condition for an isolated singularity to be removable singularity.

(2) Let $a, b \in \mathbb{C}$ be fixed, $\gamma : [0,1] \to \mathbb{C}$ be defined as $\gamma(t) = (1-t)a + bt, \forall t \in [0,1], m \geq 0$ be an integer and $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = z^m, \forall z \in \mathbb{C}$, then find $\int_\gamma f$.

(3) Define : (i) Pole (ii) Essential singularity.

(4) Define : Diameter of a set in metric space.

(5) Find $(z, \infty\, 0, 1)$

(6) Define : Smooth and Piecewise smooth path.

(7) Define with example : Analytic function.

(8) State : Leibnitz's rule.

(9) If $(z) = \dfrac{az+b}{cz+d}$, then find $T^{-1}(z)$.

(10) Define : Rectifiable path and length of rectifiable path.

2 Attempt any two : $2 \times 7 = 14$

(a) State and prove : Taylor's theorem.

(b) State and prove : Fundamental theorem of calculus of line integral.

(c) State without proof : Cauchy's theorem for an open disc and find $\int_{\sigma} \frac{dz}{z^2-1}$; where $\sigma(t) = 1 + e^{it}, \forall t \in [0, 2\pi]$.

3 Attempt following both (a) and (b) : $2 \times 7 = 14$

(a) Find the bilinear transformations taking

(i) $i \to 1, 0 \to \infty, -1 \to 0$

(ii) $1 \to i, 0 \to -i, -1 \to 0$

(b) Find the following :

(i) $\int_{\alpha} \frac{1}{z} dz$; where $\alpha(t) = e^{int}$ and for all $t \in [0, 2\pi]$.

(ii) $\int_{\gamma} z^n \, dz, \forall n \in \mathbb{Z}$, where $\gamma(t) = e^{it}, \forall t \in [0, 2\pi]$

OR

3 Attempt following : $1 \times 14 = 14$

(a) State and prove : Necessary and sufficient condition for four distinct points in $\mathbb{C}_\infty$ to be on a circle in $\mathbb{C}_\infty$

4 Attempt any two $2 \times 7 = 14$

(a) Let $\gamma : [a,b] \to \mathbb{C}$ be a function of bounded variation, $f : [a,b] \to \mathbb{C}$ be a continuous function and $\{a = t_0 < t_1 < \ldots < t_n = b\}$ be the partition of [a, b]. Then prove that $\int_a^b f \, d\gamma = \sum_{k=1}^{n} \int_{t_{k-1}}^{t_k} f \, d\gamma$

(b) Give an example of the complex function has no primitive. Justify.

(c) Find Laurent's series expansion in the powers of z for $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in

(i) $|z| < 1$;

(ii) $1 < |z| < 3$;

(iii) $|z| > 3$.

5 Attempt any two : 7×2=14

(a) Prove that : Every bilinear transformation can be written as composition of translation, dilation and inversion.

(b) State and prove : Morera's theorem.

(c) It $\gamma : [a,b] \to \mathbb{C}$ is a rectifiable path and $f : \{\gamma\} \to \mathbb{C}$ is continuous then prove that

$$\left|\int_\gamma f\right| < \int_\gamma |f||dz| \le V(\gamma).\ \sup_{z\in\{\gamma\}}[f(z)].$$

(d) State and prove : Rouche's theorem.

Seat No. ____________

HO-003-1162002

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT-2002

(Complex Analysis)

00077

Faculty Code : 003

Subject Code : 1162002

Time : $2\frac{1}{2}$ / Total Marks : 70

Instructions :
(1) All questions are compulsory.
(2) Each question carries equal marks.
(3) Figure on the right indicate allotted marks.

1 Answer any seven short questions. $7 \times 2 = 14$

(1) Define : Stereographic projection, $T^{-1} : S \to \mathbb{C}_\infty$. What is its inverse?

(2) Define : Chordal metric on $\mathbb{C}_\infty$.

(3) Is the function $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = \overline{z}$ differentiable? Justify it.

(4) Define : Fixed point. If $\lambda \in \mathbb{C}, \lambda \neq 0, \lambda \neq 1$ then find the fixed points of the bilinear transformation S defined by $S_z = \lambda z$.

(5) Find the bilinear transformation taking, $i \to 1, 0 \to \infty, -i \to 0$.

(6) Define with example : Branch of multi-valued function.

(7) State and prove, triangular inequality for the two complex numbers.

(8) Find the radius of converges of power series, $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

(9) If $G \subset \mathbb{C}$ is region, $f : G \to \mathbb{C}$ is continuous with primitive and $\gamma, \sigma : [a, b] \to G$ are rectifiable such that $\gamma(a) = \sigma(a)$, and $\gamma(b) = \sigma(b)$ then prove that, $\int_\gamma f = \int_\sigma f$.

(10) Prove that : If $f : G \to \mathbb{C}$ is differentiable then it is continuous.

2 Attempt any two. $2 \times 7 = 14$

(a) State and prove : Fundamental theorem of calculus of line integral.

(b) State and prove : Maximum Modulus Theorem.

(c) State and prove : Cauchy Goursat's Theorem.

3 Attempt followings. $2 \times 7 = 14$

(1) Prove : Cauchy's theorem for an open disc.
(Let $a \in \mathbb{C}, R > 0, f : B(a,R) \to \mathbb{C}$ be analytic and γ be a closed rectifiable curve in $B(a,R)$. Then prove that, $\int_\gamma f = 0$).

(2) Derive, the formula of stereographic projection and Chordal metric on $\mathbb{C}_\infty$.

OR

3 Attempt followings. $2 \times 7 = 14$

(1) Let $G \subset \mathbb{C}$ be open, $f : G \to \mathbb{C}$ be analytic, $a \in G, r > 0, \overline{B}(a, r) \subseteq G$ and $\gamma : [0, 2\pi] \to \mathbb{C}$ defined by $\gamma(t) = a + r.e^{it}, \forall t \in [0, 2\pi]$. Then prove that,

$$f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(w)}{w-z}\,dw, \forall z \in \mathbb{C} \text{ and}$$

$|z-a| < r, \overline{B}(a, r) = \{z \in \mathbb{C} / |z-a| \le r\}$.

(2) Let $f(z) = 1 / \left(z - \frac{1}{2} - i\right)\left(z - 1 - \frac{3}{2} - i\right)\left(z - 1 - \frac{i}{2}\right)\left(z - \frac{3}{2} - i\right)$
and γ is the polygon $[0, 2, 2+2i, 2i, 0]$. Find, $\int_\gamma f$.

4 Attempt any two. $2 \times 7 = 14$

(1) Show that, the set $M = \{S / S \text{ is a bilinear transformation}\}$ is a group under composition.

(2) Prove that, if $G \subset \mathbb{C}$ and $H \subset \mathbb{C}$ are open $f : G \to \mathbb{C}$ be continuous, $g : H \to \mathbb{C}$ be differentiable with $g'(x) \ne 0; \forall x \in H$ and $f(G) \subset H, g(f(z)) = z; \forall z \in G$ then f is differentiable and $f'(z) = \dfrac{1}{g'(f(z))}; \forall z \in G$.

(3) Prove that, for an analytic function $f : G \to \mathbb{C}$; where G be an open connected subset of $\mathbb{C}$ and $G^* = \{\overline{z} / z \in G\}$ then $f^* : G^* \to \mathbb{C}$ defined by $f^*(z) = \overline{f(\overline{z})}, \forall z \in G^*$ is analytic.

5 Attempt any two. $7 \times 2 = 14$

(a) If γ is a rectifiable curve in $\mathbb{C}$, $f : \{\gamma\} \to \mathbb{C}$ is continuous and $c \in \mathbb{C}$ then prove that, $\int_{\gamma} f(z)dz = \int_{\gamma+c} f(z-c)\,dz$.

(b) Prove that, $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi, \forall z \in \mathbb{C}, |z| < 1$.

(c) If $\gamma : [a, b] \to \mathbb{C}$ is a rectifiable path and $f : \{\gamma\} \to \mathbb{C}$ is continuous then prove that,

$$\left|\int_{\gamma} f\right| \le \int_{\gamma} |f||dz| \le V(\gamma) \bullet \sup_{z \in \{\gamma\}} |f(z)|.$$

(d) Evaluate : $\int_{\gamma} \frac{e^z - e^{-z}}{z^n} dz$; where $n \in \mathbb{N}$ and

$\gamma(t) = e^{it}, \forall t \in [0, 2\pi]$.

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GQ-9463

Seat No.________

M. Sc. (Sem. II) Examination

March / April – 2009

Mathematics

(Topology - II)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) There are **four** questions.
(2) All questions are **compulsory**.

1 (a) Define a compact space. Give an example of a denumerable non-compact space. **6**

(b) If X and Y are compact then prove that $X \times Y$ is compact. **10**

(c) Prove that every closed subspace of a compact space is compact. **3**

OR

1 (a) Prove that every compact space which is Housdorff and has no isolated points is uncountable. **8**

(b) Prove that a subset of $\mathbb{R}$ is compact if and only if it is closed and bounded. **7**

(c) Give an example of a countable subset of $\mathbb{R}$ which is compact and has only one non-isolated point. **4**

2 (a) Prove that every closed subset of a locally compact space is locally compact. **6**

(b) Prove that $X \times Y$ is locally compact if and only if X and Y are locally compact. **8**

(c) Prove that the one point compactification of a non-compact locally compact T_2 space is compact. **3**

OR

2 (a) Define limit point compact space. Prove that every compact space is limit point compact. **6**

(b) Prove that every compact metric space has a Lebesque number. **8**

(c) Define the concept of uniformly continuous function. Give an example of a continuous function which is not uniformly continous. **3**

3 (a) Prove that $\mathbb{R}$ with lower limit topology is not second countable. **6**

(b) Define a separable space and prove that the continuous image of a separable space is separable. **8**

(c) Prove that every second countable space is separable. **3**

OR

3 (a) Prove that every metric space is first countable. **6**

(b) Prove that X and Y are first countable iff $X \times Y$ is first countable. **8**

(c) Give an example of a Lindelof space X such that $X \times X$ is not Lindelof. **3**

4 (a) Prove that every regular space is Housdorff and every metric space is regular. **8**

(b) Prove that every closed subspace of a normal space is normal. **6**

(c) Give an example of a normal space X such that $X \times X$ is not normal. **3**

OR

4 (a) Prove that Urysohn lemma is equivalent to normality among T_1 spaces. **11**

(b) Prove that every completely regular space is regular and product of two completely regular spaces is completely regular. **6**

RC-7

003-016203

M.Sc. (CBCS) (Sem.-II) Examination
May-2013
MATHEMATICS
Course No. 2003 (Topology – II)

Faculty Code : 003
Subject Code : 016203

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) There are **five** questions in this paper.
(2) Each question carries **14** marks.
(3) **All** questions are compulsory.

1. Fill in the blanks : (Each question carries **two** marks) **2 × 7 = 14**
   (a) A space X is compact if every open cover of X has ______ sub cover.
   (b) Every compact subspace of a Hausdorff space is ______.
   (c) The one point compactification of a locally compact, non-compact Hausdorff space is ______ and ______.
   (d) The subspace Q of rationals is not connected because Q is not ______.
   (e) If X and Y are not compact then X × Y is ______.
   (f) [0, 1] × [0, 1] with dictionary order topology is connected but not ______.
   (g) A space X is locally connected if and only if each ______ of each ______ subset of X is open in X.

2. Attempt any **two** of the following :

(a) Write the statement of Tube lemma without proof and using this prove that $X \times Y$ is compact if both X and Y are compact spaces. **7**

(b) Prove that **7**

(i) Every closed subspace of a compact space is compact.

(ii) Every compact Hausdorff space is regular.

(c) Prove that **7**

(i) Every closed subspace of a locally compact space is locally compact.

(ii) Every compact space is limit point compact.

3. **All** are compulsory.

(a) Define a filter on a set X. If X is a space, $E \subset X$ and $x \in X$ then prove that $x \in \bar{E}$ if and only if there is a filter on X which contains E and converges to x. **6**

(b) Prove that if X is compact then every filter on X has a cluster point in X. **4**

(c) Let X be a space and $x \in X$. Prove that the collection of all neighbourhoods of x is a filter on X. **4**

OR

All are compulsory.

(a) Prove that $X \times Y$ is locally compact if and only if both X and Y are locally compact. **7**

(b) Suppose $f : X \to Y$ is open, continuous and onto. Prove that if X is locally compact then Y is locally compact. **4**

(c) Give an example of a locally compact space which is not compact. **3**

4. Attempt any **two** of the following :

(a) Suppose $f : X \to Y$ is continuous and onto. Prove that if X is connected then Y is also connected. **7**

(b) Define a path connected space. Prove that every path connected space is connected. **7**

(c) Prove that a component of a space X is **7**

(i) connected

(ii) maximal connected

(iii) closed

5. Do as directed : (Each question carries **two** marks) **$2 \times 7 = 14$**

(a) Give an open cover of the set $\mathbb{N}$ of natural numbers with discrete topology which has no finite sub cover.

(b) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(c) Let $A = \{0\} \cup \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots, \frac{1}{n} \ldots\ldots\right\}$. Is A a compact subset of $\mathbb{R}$? give reasons for your answer.

(d) Give an example of a denumerable disconnected space.

(e) Give an example of a compact, Hausdorff space which is uncountable.

(f) Give an example of a finite connected space.

(g) Give the definition of a separation of a space X.

NCE-003-1162003 **Seat No.** ______

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics - 2003

(Topology - II)

(New Course)

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper.
(2) Each question carries 14 marks.
(3) All questions are compulsory.

1 Fill in the blanks: (Each question carries two marks)

(a) In a T_1 - space every singleton subset is ________.

(b) Every compact subspace of a Hausdorff space is ________.

(c) Every Locally compact Hausdorff space is ________.

(d) A complete subspace of any metric space is ________.

(e) Urysohn's Lemma is equivalent to the separation axion ________.

(f) A closed and bounded subset of $\mathbb{R}^n$ is ________.

(g) The one point compactification of a non compact Locally compact Hausdorff space is ________ and ________.

2 Attempt any two of the following :

(a) Prove that any open subspace of a Locally compact Hausdorff space is Locally compact. 7

(b) Prove that 7

(i) Every subspace of a Hausdorff is Hausdorff.

(ii) Suppose X is a T_1 space which satisfies Uryson's Lemma. Prove that X must be normal.

(c) Prove that every compact space is limit point compact. 7

3 All are compulsory :

(a) Prove that any compact subset of a Hausdorff space is closed. **6**

(b) Prove that any closed subspace of a complete metric space is complete. **4**

(c) Prove that a T_1 space X is regular if and only if for every open set U and $x \in U$ there is an open set V such that $x \in V \subset \bar{V} \subset U$. **4**

OR

3 All are compulsory :

(a) State and Prove Lesbesgue's Covering Lemma. **7**

(b) Prove that a T_1 space X is normal if and only if for each closed set A and an open set U with $A \subset U$ there is a closed set V such that $A \subset V \subset \bar{V} \subset U$ **4**

(c) Give an example of an infinite topological space which is not compact. **3**

4 Attempt any two of the following :

(a) State Tube Lemma and then prove that $X \times Y$ is compact if both X and Y are compact. **7**

(b) Prove that Y^X with uniform metric is a complete metric space if Y is a complete metric space. **7**

(c) Prove that $\mathbb{R}^n$ is a complete metric space. **7**

5 Do as directed : (Each question carries two marks)

(a) Give the definition of a family of subsets having Finite intersection property.

(b) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(c) Let $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \dots\dots\dots, \frac{1}{n}\dots\dots\right\} \cup \{0\}$. Is A a compact subset of $\mathbb{R}$? Give reasons for your answer.

(d) Give an example of an Uncountable complete metric space which is a proper subspace of $\mathbb{R}$.

(e) Define : (i) T_1 space and (ii) Completely regular space.

(f) Give an example of a countable subset of $\mathbb{R}$ which is not Locally compact.

(g) Determine if the set [0, 1) is a complete subspace of $\mathbb{R}$ or not ?

MCB-003-1162003 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics : CMT - 2003

(Topology – II)

Faculty Code : 003

Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper.

(2) Each question carries 14 marks.

(3) All questions are compulsory.

1 Fill in the blanks : (Each question carries two marks) **14**

(a) In a Hausdorff space every singleton subset is ________

(b) Every closed subspace of a compact space is ________

(c) The one point compactification of a locally compact, non-compact hausdorff space is ________ and ________

(d) A closed subspace of any complete metric space is ________

(e) Tietz's extension theorem is equivalent to the separation axiom ________

(f) A closed and bounded subset of $\mathbb{Q}$ need not be ________

(g) An infinite set with co-finite topology satisfies ________ separation axiom but does not satisfy ________ separation axiom.

2 Attempt any **two** of the following : **14**

(a) Prove that any open subspace of a locally compact Hausdorff space is locally compact.

(b) State and prove Lebesgue's covering lemma.

(c) Prove that

(i) Every subspace of a $T_1 - space$ is $T_1 - space$.

(ii) Suppose $X \times Y$ is hausdorff. Prove that X and Y both are hausdorff.

3 All are compulsory : **14**

(a) Prove that any compact hausdorff space is regular. **6**

(b) Prove that a T_1 space X is regular if and only if for every open set U and $x \in U$ there is an open set V such that $x \in V \subset \overline{V} \subset U$. **4**

(c) Prove that $(\mathbb{R}, d)$ is a complete metric space. **4**

OR

3 All are compulsory : **14**

(a) Prove that every sequentially metric space is compact. **6**

(b) Give an example of an infinite topological space which is not compact. **4**

(c) Prove that a T_1 space X is normal if and only if for each closed set A and an open set U with $A \subset U$ there is a closed set V that $A \subset V \subset \overline{V} \subset U$. **4**

4 Attempt any **two** of the following : **14**

(a) State Tube Lemma and then prove that $X \times Y$ is compact if both X and Y are compact.

(b) Prove that $C(X,Y)$ and $B(X,Y)$ are closed subspaces of the space Y^X (with the topology induced from uniform metric).

(c) Prove that $\mathbb{R}^n$ is a complete metric space.

5 Do as directed : (Each question carries two marks) **14**

(a) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(b) Give the definition of a limit point compact space.

(c) State (i) Urysohn's lemma. (ii) Tietz's extension theorem.

(d) Give an example of uncountable subset of $\mathbb{R}$ which is not locally compact.

(e) Determine if the set $\mathbb{R}-\{0\}$ is a complete subspace of $\mathbb{R}$ or not?

(f) $A=\left\{1,\frac{1}{2},\frac{1}{3},........,\frac{1}{n}\right\} U \{0\}$. Is A a locally compact subset of $\mathbb{R}$? Give reasons for your answer.

(g) Give an example of compact metric space which is uncountable.

* P A S - 0 0 3 - 1 1 6 2 0 0 3 *

PAS-003-1162003 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

Mathematics

Topology - II : CMT - 2003

Faculty Code : 003

Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer any **seven** of the following : **7x2=14**

(1) Define : Hausdorff space and Normal space.

(2) Show that (X, T) is T_1 if the cofinite topology is weaker than T.

(3) Prove that, homeomorphic image of a $T_{3\sim}$ space is T_3.

(4) State : Tietze's extension theorem.

(5) Define : Completely regular space. Also show that a completely regular space is regular.

(6) Is $\mathbb{N}$ compact with cofinite topology ? Justify your answer.

(7) Define finite intersection property with an example.

(8) Define : Uniform continuous function with an example.

(9) Show that every convergent sequence is Cauchy sequence.

(10) Prove that $\mathbb{R}$ with standard topology is regular.

2 Attempt any **two** : **2x7=14**

(1) Prove that X is T if and only if every single subset of X is closed.

(2) Prove that every compact subset of a T_2 space is closed.

(3) Show that a metric space is complete if every Cauchy sequence in X has a convergent subsequence.

3 Attempt any one : **1x14=14**

(1) (i) Define : Regular space. Prove that a subspace of a regular space is regular.

(ii) Prove that a T_1-space is normal if and only if given a closed set A and an open set U containing A, there exists an open set V such that $A \subseteq V \subseteq \overline{V} \subseteq U$.

(2) Every regular space with a countable basis is normal.

4 Answer any **two** of the following : **2x7=14**

(1) State and prove Tube Lemma.

(2) State and prove Lebesgue's Covering Lemma.

(3) Prove that X is compact if and only if whenever C is a collection of closed sets having finite intersection property, the intersection of all elements of C is non-empty.

5 Attempt any **two** : **2x7=14**

(1) Let X be a T_1-space and $A \subseteq X$. The prove that the point x is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

(2) Prove that every regular space is Hausdoff. Is the converse true ? Justify.

(3) Prove that every compact space is limit point compact.

(4) A subspace of a complete metric space is complete if and only if it is closed.

BBH-003-1162003 Seat No. 25118.

M. Sc. (Sem. II) Examination

July - 2021

CMT - 2003 : Mathematics

(Topology - II)

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions:

1) Attempt any five questions from the following.
2) There are total ten questions.
3) Each question carries equal marks.

1) Answer the following:

1. Define with example: T_1-Space.
2. Whether the co-finite topological space $(\mathbb{R}, \tau_f)$ is a T_1-Space? Justify your answer.
3. Whether the K-topological space $(\mathbb{R}, \tau_k)$ is a Hausdorff space? Justify your answer
4. Define with example: Regular Space.
5. Define with example: Uniform metric.
6. Whether the discrete topological space $(\mathbb{R}, \tau_d)$ is a normal space? Justify your answer.
7. Whether a completely regular space is regular? Justify your answer.

2) Answer the following:

1. Whether the co-finite topological space $(\mathbb{R}, \tau_f)$ is compact? Justify your answer.
2. Define with example: Locally compact space.

3. State Urysohn's lemma.
4. Whether every convergent sequence in a metric space is Cauchy? Justify your answer.
5. Whether the indiscrete topological space $(\mathbb{R}, \tau)$ is sequentially compact? Justify your answer.
6. Let $X \times Y$ be a compact space. Whether X is compact? Justify your answer
7. Define with example: Complete metric space.

3) Answer the following:

a) Prove that:

i. Homeomorphic image of a T_1-space is a T_1-space.

ii. Homeomorphic image of a Hausdorff space is Hausdorff.

b) Prove that every metrizable space is Hausdorff.

4) Answer the following:

a) Let X be aT_1-space and $A \subseteq X$. Prove that $x \in X$ is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

b) Prove that a topological space X is Hausdorff if and only if the set $\Delta = \{(x, x): x \in X\}$ is a closed subset of $X \times X$.

5) Answer the following:

a) Let X be a T_1-space. Prove that X is normal if and only if for any closed subset A of X and open set U containing A, there is an open set V containing A such that $\bar{V} \subseteq U$.

b) Prove that every metrizable space is normal.

6) Answer the following:

a) State and prove Lebesgue covering lemma.

b) Let Y be a subspace of a topological space X. Prove that Y is compact if and only if every covering of Y by open sets in X contains a finite sub collection covering Y.

7) Answer the following:

a) Prove that every compact subspace of a Hausdorff space is closed.

b) State and prove Extreme value theorem.

8) Answer the following:

a) Let (X, d) be a complete metric space and $Y \subseteq X$. Prove that Y is complete if and only if Y is closed.

b) Prove that $X \times Y$ is locally compact if and only if X and Y are locally compact.

9) Answer the following:

a) State and prove Heine-Borel theorem.

b) Let X be a compact space and Y be a Hausdorff space. Let $f: X \to Y$ be a bijective continuous function. Prove that f is a homeomorphism.

10) Answer the following:

a) Prove that every compact space is a limit point compact space.

b) Prove that continuous image of a compact space is compact.

DBY-003-1162003 Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2003

(Topology-II)

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries 14 marks.

1 Answer any seven of the following :

(1) Define with example : T_3-Space.

(2) Define with example : Normal Space.

(3) Define with example : Completely Regular Space.

(4) Define with example : Limit point Compact Space.

(5) State only Tietze Extension Theorem.

(6) Justify whether the real line $\mathbb{R}$ is compact or not ?

(7) Define with example : Complete metric space.

(8) Give Statement of the finite intersections property.

2 Answer any two of the following :

(1) Let X and Y be two topological spaces. Prove that $X \times Y$ is a T_2-*space* if and only if X and Y are T_2-spaces.

(2) Prove that, a topological space X is Hausdorff if and only if the set $\Delta = \{(x, x) : x \in X\}$ is a closed subset of $X \times X$.

(3) Let X be a T_1-space. Prove that, X is regular if and only if, for any point $x \in X$ and open set U containing x, there is an open set V containing x such that $\overline{V} \subseteq U$.

3 Answer any one of the following :

(1) (a) Prove that, homeomorphic image of a normal space is normal.

(b) Let X_1 and X_2 be two topological spaces. Prove that, $X_1 \times X_2$ is a completely regular space if and only if X_1 and X_2 are completely regular spaces.

(2) (a) Prove that, a regular space with countable basis is normal.

(b) State and prove, Tube Lemma.

4 Answer the following :

(1) State and prove, Extreme Value Theorem.

(2) Let X be a topological space. Prove that, X is compact if and only if for every collection C of closed sets $\{C_\alpha : \alpha \in I\}$ in X having the finite intersection property.

$$\bigcap_{\alpha \in I} C_\alpha \neq \varnothing.$$

5 Answer any two of the following :

(1) Prove that, a sequence in $\mathbb{R}^n$ is convergent relative to the Euclidean metric d if and only if it is convergent relative to the square metric ρ.

(2) Let X and Y be two topological spaces. Prove that X × Y is a compact space if and only if X and Y are compact spaces.

(3) Prove that, every compact subspaces of a metric space is closed and bounded.

(4) State and prove, Heine-Borel Theorem.

Seat No. ______________

HP-003-1162003

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT - 2003

(Topology - II)

Faculty Code : 003

Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :
(1) All questions are compulsory.
(2) Each question carries equal marks.
(3) Figure on the right indicate alloted marks.

1 Answer any **seven** short questions : **7×2=14**

(1) Define term : T_2 - Space and give an example of T_2-Space.

(2) Let $f: X \to Y$ is onto, continuous, closed and X is a T_1-Space. Prove that, Y is a T_1-Space.

(3) Give an example of a space which is not compact.

(4) Prove that, $\mathbb{R}$ with standard topology is a T_2-Space.

(5) Prove that, if Y is an open subspace of a locally compact, Hausdorff space X then Y is locally compact.

(6) Prove that, every compact Hausdorff space is regular.

(7) Prove that, every subspace of a completely regular space is completely regular.

(8) Define terms : Cover and Subcover.

(9) Prove that, Every Cauchy sequence (x_n) in $(\mathbb{R}, d)$ is a bounded sequence.

(10) Define term : Completion in a metric space and give an example of it.

2 Attempt any **two** : 2×7=14

(1) If X is a T_2-Space and (x_n) is a sequence in X and suppose $(x_n) \to x$ as well as $(x_n) \to y$ in X. Prove that, $x = y$.

(2) Prove that, a space X is a T_1-Space if and only if for every $x \in X, \{x\}$ is a closed subspace of X.

(3) Prove that, a space X is a Hausdorff if and only if the set $\Delta = \{(x, x) / x \in X\}$ is a closed subset of $X \times X$.

3 Attempt followings : 2×7=14

(1) Prove that, $X \times Y$ is compact if and only if both X and Y are compacts.

(2) Prove that, if X is compact Hausdorff space without isolated points then X must be uncountable.

OR

3 Attempt followings : 2×7=14

(1) Prove that, a subspace Y of $\mathbb{R}$ is compact if and only if Y is closed and bounded subset of $\mathbb{R}$.

(2) State and prove, Lebesgue Covering Lemma.

4 Attempt any **two** : 2×7=14

(1) Let $f : X \to Y$ is onto, open and continuous and X is locally compact. Prove that, Y is also locally compact.

(2) Prove that, $X \times Y$ is locally compact if and only if X and Y are locally compact spaces.

(3) If X and Y are homeomorphic and X is regular then prove that Y is also regular.

5 Attempt any **two** : 7×2=14

(1) Prove that, $X \times Y$ is completely regular if and only if X and Y are completely regular.

(2) Prove that, a T_1-Space X is normal iff whenever $A \subset X$ is closed, $U \subset X$ is open and $A \subset U$, there is an open set V such that $A \subset V \subset \overline{V} \subset U$.

(3) Prove that, a compact, Hausdorff space is normal.

(4) Let (X, d) be a metric space and (x_n) be a sequence in X. Prove that (x_n) is Cauchy in (X, d) if and only if (x_n) is Cauchy in $(X, \overline{d})$.

RD-10

003-016204

M.Sc. (CBCS) (Sem.II) Examination

May-2013

MATHEMATICS CMT-2004 : METHODS IN PARTIAL DIFFERENTIAL EQUATIONS

Faculty Code : 003

Subject Code : 016204

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) Answer **all** the questions.

(2) Each question carries **14** marks.

1. Answer any **seven** : **7 × 2 = 14**

(a) Find the integral curves of the equations $\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{z}$.

(b) Find the direction cosines of the normal to the surface $2x + 3y + 5z = 7$ at the point (1, 0, 1).

(c) Verify that the equation $y\,dx - x\,dy + 2y^2\,dz = 0$ is integrable.

(d) Prove that the differential form $\frac{dx}{x^2} + \frac{dy}{y^4} + \frac{dz}{z^3}$ is an exact differential.

(e) Eliminate the constants a and b from the equation

$(x - a)^2 + (y - b)^2 + z^2 = 1$.

(f) If the equations $\frac{dx}{(x - y)y^2} = \frac{dy}{(y - x)x^2} = \frac{dz}{(x^2 + y^2)z}$ have integrals

$x^3 + y^3 = c_1$, $\frac{x - y}{z} = c_2$, then determine the particular solution to $(x - y)y^2\,p + (y - x)x^2 q = (x^2 + y^2)z$ through the curve $xz = 27$, $y = 0$.

(g) Find a complete integral of the equation $pq = 9$.

(h) If $z = f(x + 5iy) + g(x - 5iy)$ where the functions f and g are arbitrary, then prove that $25\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

(i) Find a particular solution to $(D - 1)^3 z = e^{3x + 2y}$.

(j) Find the solution to the Pfaffian differential equation

$zy^4 dx + zx^4 dy - x^4 y^4 dz = 0$.

2. Answer any **two** : **2 × 7 = 14**

(a) Find the integral curves of the equations $\frac{dx}{x + y} = \frac{dy}{y + z^2} = \frac{dz}{z}$.

(b) Show that the orthogonal trajectories on the hyperboloid $x^2 + y^2 - z^2 = 1$ of the conics in which it is cut by the system of planes $x + y = c$ are its curves of intersection with the surface $(x - y) z = k$ where k is a parameter.

(c) Let P, Q, R be functions of x, y and z. If X = (P, Q, R) is such that the dot product of X and curl (X) is zero, then for any arbitrary function μ of x, y and z, prove that the dot product of μX and curl (μX) is also zero.

3. (a) Prove that given one integrating factor of the Pfaffian differential equation $X_1 dx_1 + X_2 dx_2 + \dots + X_n dx_n = 0$, we can find infinitely many of them. **5**

(b) Verify that the equation $y(1 + z^2)\, dx - x(1 + z^2)\, dy + (x^2 + y^2)\, dz = 0$ is integrable and find its primitive. **5**

(c) Solve the equations : **4**

(i) $(y - z)\, dx + dy - dz = 0$

(ii) $(y^2 + z^2)\, dx + xydy + xzdz = 0$

OR

(a) Find the general integral of the partial differential equation

$z(xp - yq) = y^2 - x^2$. **5**

(b) Find the equation of the system of surfaces which cut orthogonally the system of surfaces $x^2 + y^2 + z^2 = cxy$. **5**

(c) Find the envelope of the one-parameter system of surfaces

$(x - a)^2 + (y - 5a - 3)^2 + z^2 = 1$. **4**

4. Answer any **two** : **2 × 7 = 14**

(a) Find the complete integral of the equation $(p^2 + q^2)\, y = qz$.

(b) (i) Find the complete integral of the equation $p^3q^3z = p^4(xq^3 + p^2) + q^4(yp^3 + q^2)$.

(ii) Solve the equation $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right) = 1$ using Jacobi's method.

(c) Find the general solution of the equation $\frac{\partial^2 z}{dx^2} - \frac{\partial^2 z}{dy^2} = e^{2x + 3y}$.

5. Answer any **two** : **2 × 7 = 14**

(a) Find the equation of the integral surface of the differential equation $x^3p + y(3x^2 + y)\, q = z\,(2x^2 + y)$ which passes through the parabola $x = 1, y^2 = z - y$.

(b) Prove that the equation $yz(y + z)\, dx + zx\,(z + x)\, dy + xy\,(x + y)\, dz = 0$ is integrable and find its solution.

(c) Reduce the equation $(n - 1)^2 \frac{\partial^2 z}{dx^2} - y^{2n}\frac{\partial^2 z}{dy^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ to canonical form and find its general solution.

(d) Solve the equations :

(i) $r + s - 2t = e^{x-y}$

(ii) $r + s - 12t + 12p - q + 35 = 0.$

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DJJ-003-016204

DJJ-003-016204

Seat No. ________

M. Sc. (Mathematics) (Sem. II) (CBCS) Examination

May / June – 2015

MATHS. CMT - 2004 : Methods in Partial Differential Equations

Faculty Code : 003
Subject Code : 016204

Time : Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1. Answer any **Seven** $7 \times 2 = 14$
   - (a) Find a primitive of $ydx + xdy + 3z^2dz = 0$.
   - (b) Find the integral curves of the equations $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$.
   - (c) Find $curl(7y^2z^2, 5z^2x^2, 9x^2y^2)$.
   - (d) Eliminate the arbitary function f from the equation $z = f(x - y)$.
   - (e) Find a complete integral of $z = p^3 + q^3$.
   - (f) If $F(D, D')$ is reducible and if $(2D' + 3)$ is a factor of $F(D, D')$, then verify that $z = e^{\frac{-3y}{2}}\phi(2x)$ is a solution of $F(D, D')z = 0$, where $\phi(\xi)$ is an arbitrary function of the single variable ξ.
   - (g) Find a partial differential equation for which a complete integral is $2z = ay^2 + bx^2 - \frac{1}{b}$, where a and b are arbitrary constants.
   - (h) Find a particular integral of $(16D'^2 - 25D^2)z = e^{2x+3y}$.
   - (i) Verify that the equation $r + 2s + t = 0$ is of type parabolic.
   - (j) Find a complete integral of $yp - xq = 0$.

2. Answer any **Two** $2 \times 7 = 14$
   - (a) Find the integral curves of the sets of equations:
     - (i) $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$
     - (ii) $\frac{dx}{y(x+y)+2z} = \frac{dy}{x(x+y)-2z} = \frac{dz}{z(x+y)}$.
   - (b) Find the orthogonal trajectories on the cylinder $y^2 = 2z$ of the curves in which it is cut by the system of planes $x + z = c$, where c is a parameter.
   - (c) Verify that the equation $y(1 + z^2)dx - x(1 + z^2)dy + (x^2 + y^2)dz = 0$ is integrable and find its primitive.

3. (a) Find the general integral of the linear partial differential equation $(x + z)p + (y + z)q + z = 0$. 5
   - (b) Find the equation of the integral surface of the partial differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0, x^2 + y^2 = 2x$. 5
   - (c) (i) Find the envelope of the one-parameter system of surfaces $x^2 + y^2 + (z - a)^2 = 1$.
   - (ii) Determine the envelope of the two-parameter system of surfaces $(x - a)^2 + (y - b)^2 + z^2 = 1$. 4

OR

3. (*a*) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2,\ z = 0$. 5

(*b*) Find a complete integral of the partial differential equation $xpq + yq^2 = 1$. 5

(*c*) Show that an equation of the form $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = f(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})$ is always soluble by Jacobi's method. 4

4. Answer any **Two** $2 \times 7 = 14$

(*a*) Reduce the equation $r - x^2 t = 0$ to canonical form.

(*b*) Verify that the equation $yz(y+z)dx + zx(z+x)dy + xy(x+y)dz = 0$ is integrable and find its solution.

(*c*) Prove that $yz(z^2+yz-2y) = x^2$ is a solution of $2x(y+z^2)p + y(2y+z^2)q = z^3$.

5. Answer any **Two** $2 \times 7 = 14$

(*a*) Suppose that the equation $Pdx + Qdy + Rdz = 0$ is integrable. Prove that the dot product of $X = (P, Q, R)$ and $curl X$ is equal to 0.

(*b*) Solve the equation $D^3 - 2D^2D' - DD'^2 + 2D'^3 = e^{x+y}$.

(*c*) Find a complete integral of $(p^2 + q^2)y = qz$.

(*d*) Let $F(D, D') = \sum_r \sum_s c_{rs} D^r D'^s$, where c_{rs} are constants. Prove that $F(D, D')(e^{ax+by}\phi(x, y)) = e^{ax+by} F(D + a, D' + b)\phi(x, y)$.

NCF-003-016204 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May – 2017

Course - 2004 : Mathematics

(Methods in Partial Differential Equation)

[Old Course]

Faculty Code : 003

Subject Code : 016204

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) There are **five** questions.

(ii) **All** questions are **compulsory**.

(iii) **Each** question carries **14** marks.

1 Do as directed : (Each question carries **two** marks) **14**

(a) Find the complementary function of the equation $\left(D^3+3D^2D'-4D'^3\right)z=0$.

(b) Eliminate the arbitrary constants a and b from $2z=(ax+y)^2+b$.

(c) Define Pfaffian differential equation with an example and write down the condition for the pfaffian differential to be integrable.

(d) Find the complete integral of $yp-xq=0$.

(e) Find the P.d.e. of $z=f\left(\frac{xy}{z}\right)$ where f is an arbitrary function.

(f) Find curl $\left(7y^2z^2,\ 5x^2z^2,\ 9x^2y^2\right)$.

(g) Classify the equation $4y^2r+x^2t=0$.

2 Answer any two of the following : **2×7=14**

(a) Find the general solution to

$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that u contains in the combination of $x+y+z$ and $x^2+y^2+z^2$.

(b) Find the orthogonal trajectories on the conicoid $(x+y)z=1$ of the conics in which it is cut by the system of planes $x-y+2=k$ where k is the parameter.

(c) Prove that a pfaffian differential equation

$$\left(xy^2-zx\right)dy+yzdx=-\left(xz^2-yx\right)dz$$

is integrable. Also find the complete primitive.

3 All are compulsory : **14**

(a) (i) Determine the envelope of one parameter of system of the surfaces $x^2+y^2+(z-a)^2=1$. **4**

(ii) Determine the envelope of two parameter of system of the surfaces $(x-a)^2+z^2+(y-a)^2=1$.

(b) Find the integral curves of the equation **5**

$$\frac{dx}{(x+z)}=\frac{dy}{y}=\frac{dz}{\left(z+y^2\right)}.$$

(c) Prove that **5**

$$F(D,D')\left[e^{ax+by}\cdot h(x,y)\right]=e^{ax+by}F(D+a,D'+b)\left[h(x,y)\right].$$

OR

3 All are compulsory : **14**

(a) Solve the equation : **4**

(i) $z(z+y)dx+z(z+x)dy-2xydz=0$.

(ii) $(1+x)yzdx+(1+y)xzdy+(1+z)xydz=0$.

(b) Prove necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $f(u,v)=0$ not involving x and y explicility is that $\frac{\partial(u,v)}{\partial(x,y)}=0$. **5**

(c) Using Natani's method **5**

$x(y^2-1)dx+y(x^2-z^2)dy-z(y^2-1)dz=0$.

4 Answer any two of the following : **2×7=14**

(a) Solve $p=(z+qy)^2$.

(b) Classify the equation and convert it into canonical form $x^2r+4t=xy$. $(x\neq 0)$.

(c) If $(\beta D'+\gamma)^2$ with $\beta\neq 0$ is a factor of $F(D,D')$, then a solution of the equation $F(D,D')$ is,

$$z=e^{\frac{-\gamma}{\beta}y}\left(\varnothing_1(\beta x)+y\varnothing_2(\beta x)\right)$$

Where $\varnothing_i=\varnothing_i(\varepsilon)$ is an arbitrary function of a single variable $(i=1,2)$.

5 Answer any two of the following : **2×7=14**

(a) Find the G.S. of $(2D+3D')(D-D')z=\frac{(x^2+xy)}{2}$.

(b) (i) Find the solution of $(x^2D^2-y^2D'^2+xD-yD')z=0$.

(ii) Using Jacobi's method solve $xp^2+yq^2=z$.

(c) Find the equation of integral surface of the differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ passes through $y=0$ and $x=1$.

MCC-003-1162004 Seat No. ______

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics - 2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) All questions are compulsory.
(3) Each questions carries 14 marks.

1 Do as directed : (Each question carries two marks) **14**

(a) Find the complete integral of $zpq = p + q$.

(b) Find the integral curves of the equation
$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

(c) Solve the equation $r + s - 2t - p - 2q = 0$.

(d) Check whether the p.d.e. $3y(a-z)\,dy = \left(y - z^2 + \left(a^2 + y\right)\right) dz$ is integrable or not ?

(e) If $z = f(x+ky) - g(x-ky)$ where f and g are arbitrary functions and k is a constant then show that $z_{yyy} = k^2 z_{xx}$.

(f) Find the equation of a tangent plane to the surface $y^3 + x^2 + 3yz - z^3 = 6$ at point $(-2, -1, -3)$.

(g) Verify the equation is exact or not
$$(yz)\,dx + \left(x^2 z - xy\right) dz + \left(x^2 y - xz\right) dy = 0.$$

2 Answer any **two** of the following : **2×7=14**

(a) Solve using Nattani's method $\left(y^2 + z^2\right) dx + xydy + xzdz = 0$.

(b) Classify the equation and convert it into canonical form
$y^2 r - p = x^2 t - q$.

(c) If $(\beta D'+\gamma)^2$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is,

$$z = e^{\frac{-\gamma}{\beta} y} \left(\phi_1(\beta x) + y\,\phi_2(\beta x)\right)$$

Where $\phi_i = \phi_i(\varepsilon)$ is an arbitrary function of a single variable $(i = 1, 2)$.

3 All are compulsory : **14**

(a) Find the integral curves of the equation **4**

$$\frac{dx}{\left(x^3 + 3xy^2\right)} = \frac{dy}{\left(y^3 + 3yx^2\right)} = \frac{dz}{(2z)\left(x^2 + y^2\right)}.$$

(b) Solve using homogenous method **5**

$$yz(y+z)\,dx + xy(x+y)\,dz + xz(x+z)\,dy = 0$$

(c) Find the orthogonal trajectory on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with family of plane parallel to $z = c$. **5**

OR

3 All are compulsory : **14**

(a) Using Charpit's method solve $(z + qy)^2 = p$. **5**

(b) Solve $\left(3D^2 + 8DD' + 4D'^2\right) z = e^{y-2x} + e^{x-y}$. **4**

(c) Find the surface which intersects the surface of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. **5**

4 Answer any two of the following : **2×7=14**

(a) Prove that for any non-zero functions $\mu = \mu(x, y, z)$ and $X = (P, Q, R)$ where P, Q, R are the functions of x, y, z then

$$X \cdot Curl\, X = 0 \text{ iff } (\mu X) \cdot Curl\,(\mu X) = 0.$$

(b) Prove that a pfaffian differential equation

$$\left(y^2 + yz\right) dx + \left(xz + z^2\right) dy = -\left(-yx + y^2\right) dz$$ is integrable.

Also find the complete primitive.

(c) Show that the complete integral of the equation $f\left(u_x, u_y, u_z\right)=0$ is $u=ax+by+g\left(a,b\right)z+c$ where a,b,c are constants. Also find the complete integral of the equation $u_x \cdot u_x \cdot u_z = u_x + u_y + u_z$.

5 Answer any **two** of the following : **2×7=14**

(a) (i) Solve $f\left(x+y, x-\sqrt{z}\right)$.

(ii) Using Jacobi's Method Solve $xyp=q$.

(b) Find the System of orthogonal trajectories on the sphere $x^2+y^2+z^2=a^2$ of its intersection with the paraboloids $xy=cz$.

(c) Find the general solution of

$\left(-8D'^2+2DD'+D^2\right)z=\left(2x+3y\right)^{\frac{1}{2}}$ using general method.

RBM-003-1162004 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

May / June - 2019

Mathematics : Paper - 2004

(Methods in Partial Differential Equations)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) There are **five** questions.

(2) All questions are **compulsory**.

(3) Each question carries 14 marks.

1 Answer any **seven** of the following : 14

1) Write down the set of parametric equations of sphere.

2) Define : a) Equation of Normal Line and b) Complete Integral.

3) Solve $f\left(x+y, x-\sqrt{z}\right)$.

4) Define integrating factor of a pfaffian differential equation with an example.

5) Determine the direction ratios at point P to the curve of intersection of $ax^2+by^2+cz^2=6$ and $x+y+z=2$.

6) Determine the envelope of two parameter systems of the surfaces $z^2+(y-b)^2+(x-a)^2=3$.

7) State the general form of Wave equation and Laplacian equation.

8) Solve $\left(D^3-7DD'^2+6D'^3\right)z=0$.

9) Justify whether the equation $2uvdu + uv^2dv = 0$ is integrable or not ?

10) Find complete integral of $pqz = p^2\left(xq + p^2\right) + q^2\left(yp + q^2\right)$.

2 Answer any **two** of the following :

a) State and prove the necessary and sufficient condition under which the pfaffian differential equation (for three variables) is integrable. 7

b) Solve the partial differential equation $zq = (p^2 + q^2)y$ using Charpits method. 7

c) Find the general form of the complete integral of $f\left(u_x, u_y, u_z\right) = 0$ and illustrate for the method for the equation $u_x + u_y + u_z = u_x u_y u_z$. 7

3 Answer the following :

a) Find the integral surface of $2x\left(y + z^2\right)p + y\left(2y + z^2\right)q = z^3$ and deduce the solution to the form $yz\left(yz + z^2 - 2y\right) = x^2$ provided $\frac{c_2 + 1}{c_1^2} = 1$. 7

b) i) Solve $\cos(x + y)\, p + \sin(x + y)\, q = z$. 4

ii) Find the particular integral of $(3D^2 - 2D' + DD')z = \sin(x + 2y)$. 3

OR

3 Answer the following :

a) Prove $F(D, D')\left[e^{ax+by}\right] = e^{ax+by}F(a, b)$. 4

b) Solve the partial differential equation $z^2 = pqxy$ using Jacobi's method. 5

c) Prove necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$, a relation $f(u, v) = 0$ not involving x and y explicitly is that $\frac{\partial(u, v)}{\partial(x, y)} = 0$. 5

4 Answer the following :

a) Find the integral surface of $(x - y)p + (y - x - z)q = z$ which passes through circle $x^2 + y^2 = 1$ and line $z = 1$. 7

b) Using Nattani's method solve the partial differential equation $z(z + y^2)\,dx + z(z + x^2)\,dy = xy(x + y)\,dz$. 7

5 Answer any **two** of the following :

a) Classify the equation and convert it into canonical form $2r - 5s + 3t = x$. 7

b) Find the general solution. 7

$$\left(x^2D^2 - y^2D'^2 + xD - yD'\right)z = x - y.$$

c) i) Find particular integral of $\left(D^2 - D'^2\right)z = x^2 - y$. 3

ii) If $(\alpha D + \beta D' + \gamma)$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is, 4

$$z = e^{\frac{-\gamma}{\alpha}x}\left(\phi(\beta x - \alpha y)\right).$$

Where $\phi = \phi(\varepsilon)$ is an arbitrary function of a single variable.

PAT-003-1162004 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

Mathematics : Course No. - 2004

(Methods in Partial Differential Equation)

(New Course)

Faculty Code : 003

Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries **14** marks.

1 Answer any **seven** : (Each question carries **two** marks) **14**

(a) Define : (i) Complementary function and (ii) Singular solution.

(b) Solve $\left(D^3 - 7D'^2 + 6D'^3\right)Z = 0$.

(c) Verify the equation $z = \sqrt{zy+b} + \sqrt{2x+a}$ is the solution of $z = \frac{1}{p} + \frac{1}{q}$.

(d) State Lipchitz condition for the functions of three variables (x, y, z) from the point (a, b, c).

(e) Find the complete integral of $p^3 + q^3 = 3$.

(f) Find the direction cosines of the normal to the surface $4x - 6y - 10z = 7$ at the point (2, 1, 1).

(g) Verify the equation is exact or not $\left(y^2\right)dx + \left(x^2\right)dy + \left(3x^2\right)dz = 0$.

(h) Solve $f\left(x^2 + y^2 + z^2,\ z^2 - 2xy\right) = 0$.

(i) Determine the envelope of the two parameters system of surfaces $(x-a)^2 + (y-b)^2 + z^2 = 1$.

(j) Define : (i) Tangent plane and (ii) Pffafian Differential form.

2 Answer any **two** of the following : **2x7=14**

(a) Find the general solution of

$$(D-D')(D+D')z=e^{2x-y}(x+2y).$$

(b) Find the primitive solution of

$$2y(a-x)\,dx+\left[\left(z-y^2\right)+(a-x)^2\right]dy-ydz=0.$$

(c) If $(\alpha D+\beta D'+\gamma)^n$ with $\alpha\neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')Z = 0$ is,

$$z=e^{\frac{-\gamma}{\alpha}x}\left(\phi_1(\beta x-\alpha y)+y\phi_2(\beta x-\alpha y)+.......+y^{n-1}\phi_n(\beta x-\alpha y)\right).$$

Where $\phi_i=\phi_i(\varepsilon)$ is an arbitrary function of a single variable $(i=1,2...,n)$.

3 All are compulsory : **14**

(a) Solve using Nattani's method **5**

$$2yzdx-2xzdy-\left(x^2-y^2\right)(z-1)\,dz=0.$$

(b) Find the particular integral of **5**

$$\left(D^2+2D'^2-2DD'\right)z=\cos(x+y).$$

(c) Solve the equation $\dfrac{dx}{(x+z)}=\dfrac{dy}{(y)}=\dfrac{dz}{\left(z+y^2\right)}$. **4**

OR

3 All are compulsory : **14**

(a) Prove that for any non-zero functions $\mu=\mu(x,y,z)$ and $X=(P,Q,R)$ where P, Q, R are the functions of x,y,z then $X\cdot Curl\,X=0\,Iff\,(\mu X)\cdot curl(\mu X)=0$. **5**

(b) Find the equation of the system of curves on the cylinder $2y=x^2$ orthogonal to its intersection with the hyperbola of one-parameter system $xy=z+c$. **5**

(c) Find the integral curves of the equation **4**

$\frac{dx}{(cy-bz)}=\frac{dy}{(az-cx)}=\frac{dz}{(bx-ay)}$ and show that they are circles.

4 Answer the following both : **2x7=14**

(a) Solve the partial differential equation

$$px\left(z-2y^2\right)=(z-qy)\left(z-y^2-2x^3\right).$$

(b) Determine the partial differential equation from the relation $F(u,v)=0$, where u and v are functions of x, y and z, with z is dependent of x and y.

5 Answer any **two** of the following : **2x7=14**

(a) Describe Jacobi's method.

(b) Classify the equation and convert it in canonical form $2r-5s+3t=x$.

(c) Solve $2(z+xp+yp)=yp^2$ using Charpits's method.

(d) Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curve of intersection with the plane parallel to plane $X-Y$.

BBI-003-1162004 Seat No. 25118

M. Sc. (Sem. II) Examination

July - 2021

CMT - 2004 : Mathematics

(Methods in Partial Differential Equations)

Faculty Code : 003

Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions:

(1) Attempt any five questions from the followings.

(2) There are total ten questions.

(3) Each question carries equal marks.

1) Answer the following: **14**

a) Solve, $(2D^2 - 5DD' + 2D'^2)z = 0$.

b) If $z = f(x + ky) - g(x - ky)$ where f and g are arbitrary functions and k is a constant then show that, $z_{yy} = k^2 z_{xx}$.

c) Find the envelope of two parameter system of surface $(x-2)^2 + (y-3)^2 = 1 - z^2$.

d) Solve, $pqz = p^2(xq + p^2) + q^2(yp + q^2)$.

e) Find the particular integral of $(D^2 - D'^2)z = x^2 - y$.

f) Write the canonical form of hyperbola and parabola form.

g) Find the direction cosines of the normal to the surface $x^2 + 3z - y^2 = 7$ at point $p(1,2,-1)$.

2) Answer the following: **14**

1) Let $F(D, D') = \sum_r \sum_r c_{rs} D^r D'^s$, where the values c_{rs} are constants. Prove that, $F(D, D')\left(e^{ax+by} g(x,y)\right) = e^{ax+by} F(D + a, D' + b) g(x,y)$.

2) i) Prove that, if $u_i (i = 1,2, \dots, n)$ is a solution of $F(D, D')z = 0$ then its linear combination is also a solution of $F(D, D')z = 0$.

ii) Eliminate arbitrary function from $z = f(x^2 - y) + g(x^2 + y)$ and obtain the partial differential equation.

3) Answer the following: **14**

a) Find the envelope of one-parameter system of surface $x^2 + y^2 + (z - c)^2 = 1$.

b) Define: (i) Tangent Plane (ii) Normal Line.

c) Verify the equation $z^2 + \mu = 2(1 + \gamma^{-1})(x + \gamma y)$ is the complete integral of $z = \frac{1}{p} + \frac{1}{q}$.

d) Solve, $f\left(\frac{xy}{z}\right) = z$.

e) Determine the direction ratios of the curve of intersection of the surfaces $2x^2 - 5y^2 + 3z^2 = 0$ and $z - y - x = 1$.

f) Is the equation $yzdx + (x^2y - zx)dy + (x^2z - xy)dz = 0$ is exact? Justify your answer.

g) Prove that sum of Complementary Function and Particular Integral is a solution of $F(D, D')z = f(x, y)$.

4) Answer the following: **14**

1) Solve the partial differential equation:

$$(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z).$$

2) Solve $x(y^2 - 1)dx + y(x^2 - z^2)dy - z(y^2 - 1)dz = 0$, using Nattani's method.

5) Answer the following: **14**

1) Classify the equation and convert it in canonical form $r - 2s + 4t = 0$.

2) If $(\beta D' + \gamma)^n$ with $\beta \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is ,

$$z = e^{\frac{-\gamma}{\beta}y}(\emptyset_1(\beta x) + y\emptyset_2(\beta x) + \cdots \ldots + y^{n-1}\phi_n(\beta x))$$

6) Answer the following: **14**

1) Prove that, an equation of Clairaut form $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$ is always solvable by Jacobi's method. Hence, solve the equation

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right) = 1.$$

2) Using Charpits's method, solve, $qz = (p^2 + q^2)y$.

7) Answer the following: **14**

1) i) Find the particular integral of $(D - D')(5 - D')^2 z = e^{5y-x}\sin(5y - x)$

ii) Solve, the equation $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2-(x^2+y^2)}$.

2) Find the complete integral of $xpq + yq^2 = 1$.

8) Answer the following: **14**

1) Let P, Q, R be functions of x, y, z. If $X = (P, Q, R)$ is such that the dot product of X and curl(X) is zero then for any arbitrary function μ of x, y, z. Prove that, the dot product of μX and $curl(\mu X)$ is zero and vice versa.
2) Find the orthogonal trajectories on the surface $y^2 = 2z$ of its curve of intersection with the system of planes $x + y = k$ where k is a parameter.

9) Answer the following: **14**

1) Find General solution of $(D^2 + D - D'^2 - D')z = 2e^{x+y}(x - 3y + 5)$.
2) Prove that, $yz(z^2 + yz2y) = x^2$ is a solution of $2x(y + z^2)p + y(2y + z^2)q = z^3$.

10) Answer the following: **14**

1) Find the integral surface of the given partial differential equation $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$, which passes through the lines $y = 0$ and $xz = a^3$.
2) Find the primitive solution of

$$(z^2 + 2xy)dx + (x^2 + 2yz)dy + (y^2 + 2xz)dz = 0.$$

DBZ-003-1162004

Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer any seven from the following : **14**

(a) Define (i) Complete solution (ii) Pffafian form with each of its example.

(b) Solve, $\left(\frac{\partial^3 z}{\partial z^3} - 3\frac{\partial^3 z}{\partial^2 x \partial y} + 2\frac{\partial^3 z}{\partial y^3}\right) = 0.$

(c) Find the differential equation of all the spheres whose centre lie on the z-axis.

(d) Find the complete integral of $p^2 = q + x$.

(e) Is the equation $6ydx - 7y^2dx = (4x^2)dy$ is exact ? Justify your answer.

(e) Is the equation $6ydx - 7y^2dx = \left(4x^2\right)dy$ is exact ? Justify your answer.

(f) Verify the equation $z^2 + \mu = 2(1+\gamma^{-1})(x+\gamma y)$ is the complete integral of $z = \frac{1}{p} + \frac{1}{q}$.

(g) Find the particular integral $(D^2 - D')z = 2y - x^2$.

(h) Write down two sets of parametric equations for sphere.

(i) Find the direction cosines of the normal to the surface $x+y+z=10$ at point $p(1,1,1)$.

(j) Determine the envelope for the equation $(x-a)^2+(y-b)^2+z^2=1$.

2 Answer any two of the following : **14**

(1) State and prove, the necessary and sufficient condition under which the Pfaffian differential equation (for three variables) is integrable.

(2) If $(\alpha D+\beta D'+\gamma)^2$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is,

$$z = e^{-\frac{\gamma}{\alpha}x}(\varnothing_1(\beta x-\alpha y)+y\varnothing_2(\beta x-\alpha y))$$

(3) Solve, $(2D^2-5DD'+2D'^2)z = \sin\ (2x+y)$

3 Answer the following : **14**

(1) (i) Solve, the partial differential equation : $x(x^2+3y^2)p-y(y^2+3x^2)q-2z(y^2-x^2)=0$.

(ii) Find the solution of $y^2p-xyq=x(z-2y)$.

(2) Solve by Nattani's method $yzdx-(x^2y-zx)dy+(x^2z-xy)dz=0$.

OR

3 Answer the following : **14**

(1) Solve by Charpits method $2(z+xp+yq)=yp^2$

(2) Write down, how to solve a partial differential equation $f(x,u_1,u_3)=g(y,u_2,u_3)$ using Jacobi's method and illustrate the method by finding a complete integral of $2x^2u_1^{\,2}yu_3 = x^2u_2+2yu_1^{\,2}$?

4 Answer the following : 5+5+4 = 14

(1) Solve by Jacobi's method $(p^2+q^2)y=qz$.

(2) Prove that, the relation $F(U, V)=0$ where U and V are the functions of x, y and z obtain the partial differential equation of the type $Pp+Qq=R$.

(3) Solve, $4r+12s+9t=e^{3x-2y}$.

5 Answer any two of the following : 14

(1) Classify the equation and convert it in canonical form $y^2r+4x^2t=xy(x \neq 0 \neq y)$.

(2) (i) Solve the equation

$$\frac{dx}{xy+y^2+2z}=\frac{dy}{x^2+xy-2z}=\frac{dz}{z(x+y)}$$

(ii) Form the P.D.E. from $z = xf(y) + yg(x)$

(3) Find the integral surface of the given partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$, which passes through the lines $y=0$ and $x=1$

(4) Find the orthogonal trajectories on the surface $(x+y)z=1$ of its curve of intersection with the system of planes $x-y+z = k$ where k is a parameter.

HQ-003-1162004

Seat No. ____________

HQ-003-1162004

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT-2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ / Total Marks : 70

Instructions : (1) All the questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Answer any seven from the followings : **2×7=14**

(a) If $z = f(x+ky) - g(x-ky)$, where f and g are arbitrary functions then, show that, $z_{yy} - k^2 z_{xx} = 0$.

(b) Define Pfaffian form and complete solution with an example.

(c) Solve, $\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} = p^2(xq+p^2) + q^2(yp+q^2)$.

(d) Solve, $(D^3 - 4D^2D' + 4DD'^2)z = 0$.

(e) Find the partial differential equation for the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$.

(f) Find the direction cosine of the normal to the surface $4x - 6y - 10z = 7$ at the point $p(2, 1, 1)$.

(g) Define terms : (i) Tangent plane (ii) Normal Line.

(h) Prove that, sum of complementary function and particular integral is a solution of $F(D, D')z = f(x, y)$.

(i) Find an integral curve for $yzdx - zxdy = xydz$.

(j) Find the complete integral of $\frac{\partial z}{\partial y} = e^{\frac{\partial z}{\partial x}}$.

2 Answer any two of the followings : **7×2=14**

(1) Find the integral curves of $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

(2) Solve : $xp - yq = y^2 - x^2$.

(3) If $(\beta D' + \gamma)$ with $\beta \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is, $z = e^{\frac{-\gamma}{\beta} y} (\phi(\beta x))$, where $\phi = \phi(\varepsilon)$ is an arbitary function of a single variable.

3 Answer the following : **7×2=14**

(1) Find the surface, that intersect the system of surface $z(x+y) = C(3z+1)$ orthogonally and it passes through the circle with centre 0 and radius 1 and another curve is $z = 1$.

(2) Solve, using Charpits's Method $p^2 - y^2 q = y^2 - x^2$.

OR

3 Answer the following : **7×2=14**

(1) Solve, using Jacobi's Method $xyp = q$.

(2) Find the primitive solution of :

$$2y(a-x)dx + [(z-y^2) + (a-x)^2]dy - ydz = 0.$$

4 Answer any two of the following : **7×2=14**

(1) Solve, the partial differential equation using Nattani's Method :

$$x(y^2-1)dx + y(x^2-z^2)dy - z(y^2-1)dz = 0.$$

(2) Find the orthogonal trajectories on the cylinder $y^2 = 2z$ of the curve of which is cut by the system of planes $x + y = c$, where c is a parameter.

(3) Write down, how to solve a partial differential equation $f(x, u_1, u_3) = g(y, u_2, u_3)$, using Jacobi's method and illustrate the method by finding a complete integral of $2x^2 u_1^2 y u_3 = x^2 u_2 + 2y u_1^2$.

5 Answer any two of the following : $7 \times 2 = 14$

(1) Classify the equation and convert it in the canonical form $4r - s + t = 0$.

(2) (i) State : Wave equation and Diffusion equation and classify its nature.

(ii) Verify that, $u = f(x - vt + iy\alpha) + g(x - vt - iy\alpha)$ is a solution of the equation $u_{xx} + u_{yy} = \frac{1}{c^2} u_{tt}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$.

(3) Let $F(D, D')z = \sum_{i,j=finite} C_{rs} D^i D'^j$, where the value of C_{rs} are constants. Prove that,

$$F(D, D')e^{ax+by}.g(x,y) = e^{ax+by} F(D+a, D'+b) g(x, y).$$

(4) State, Laplacian equation and solve it after transforming it into its canonical transformation.

RE-2

003-016206

M. Sc. (Sem.-II) Examination
May-2013
EMT – 2001 : Classical Mechanics – II

Faculty Code : 003
Subject Code : 016206

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) There are **5** questions.
(2) Attempt **all** the questions.
(3) Each question carries equal marks.

1. Choose appropriate alternative/alternatives (any **seven**) :

(1) Which of the following is/are true ?

(a) $F_1 (q, Q, t)$ (b) $F_2 (q, P, t)$

(c) $F_3 (q, P, p)$ (d) $F_4 (p, P, t)$

(2) For any orthogonal transformation matrix

(a) $A^{-1} = \tilde{A}$ (b) A^{-1} is singular

(c) A^{-1} can not be defined (d) $A^{-1} = A$

(3) [4, 4] equals to

(a) 0 (b) – 1

(c) + 1 (d) 2

(4) If $\bar{L}$ and $\bar{w}$ are angular momentum and angular velocity respectively then

(a) $\bar{L} = \bar{I} / \bar{w}$ (b) $\bar{L} = \bar{w} / \bar{I}$

(c) $\bar{L} = \bar{w}$ (d) $\bar{L} = I\bar{w}$

(5) Routh's procedure is a combination of

(a) Lagrangian and Hamiltonian formulations

(b) Lagrangian and Newtonian formulations

(c) Principle of least action and D'Alembert's principle

(d) All of (a), (b), (c)

(6) In the sequence of rotations in the discussion of Eulerian angles 'line of noses' is the intersection of

(a) xy and yz planes (b) xy and $\xi'\eta'$ planes

(c) xy and $x'y'$ planes (d) None of these

(7) The determinant of any proper orthogonal matrix is always

(a) negative (b) – 1

(c) + 1 (d) 0

(8) In Routh's procedure non-cyclic co-ordinates satisfy

(a) Lagrange's equations (b) Euler's equations

(c) Cayley-Klein equations (d) Hamilton's equations

(9) Which of the following is/are Hamilton's canonical equations ?

(a) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = 0$ (b) $\dot{q}_i = \frac{\partial H}{\partial p_i}$

(c) $-\dot{p}_i = \frac{\partial H}{\partial q_i}$ (d) $\delta \int_{t_1}^{t_2} L\, dt = 0$

(10) For F_3 type generating functions

(a) $q_i = -\frac{\partial F_3}{\partial p_i}$ (b) $q_i = -\frac{\partial F_3}{\partial q_i}$

(c) $\dot{q}_i = \frac{\partial F_3}{\partial p_i}$ (d) $\dot{q}_i = -\frac{\partial F_3}{\partial p_i}$

2. Attempt any **two :**

(a) Derive the matrix of 2×2 orthogonal transformation.

(b) Derive Euler's equations for rigid bodies.

(c) Discuss in detail the use of direction cosines as a tool to describe orientation of any rigid body.

3. Attempt the following :

(a) State and prove Euler's theorem for rigid bodies.

(b) Discuss in detail the finite rotations and obtain the corresponding matrix of rotation.

OR

Attempt the following :

(a) Define coriolis force and discuss in brief any two effects of it.

(b) Discuss in detail the infinitesimal rotations and derive the formula $dr = r \times dR$

4. Attempt any **two :**

(a) Define Poisson brackets of two functions. Also state and prove Jacobi's identity for Poisson brackets.

(b) Discuss in detail the Routh's procedure.

(c) Discuss in detail the principle of least action.

5. Attempt any **two :**

(a) Discuss in detail the analytic solution of a heavy symmetrical top.

(b) State Hamilton's variational principle and using it derive canonical equations.

(c) Show that Hamilton's principle function differs at most by the indefinite time integral of the Lagrangian only by a constant.

(d) For the problem of Harmonic oscillator derive that $q = \sqrt{\frac{2a}{m\, w^2}}\, \sin \omega(t + \beta)$ where notations are being usual.

NCG-003-1162006 Seat No. ________

M. Sc. (Mathematics) (Sem. II) (CBCS) Examination

April / May - 2017

EMT - 2001 : Classical Mechanics - II

(New Course)

Faculty Code : 003

Subject Code : 1162006

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.

(2) There are 5 questions.

(3) Figures to the right indicate full marks.

1 Attempt the following : (Any **Seven**) **14**

(1) (i) Define cyclic co-ordinate.

(ii) State Hamilton's variational principle.

(2) Define Poisson bracket of the functions u and v.

(3) State the postulates of special theory relativity.

(4) State only the transformation equations when the generating function is of the type $F_4(p,P,t)$.

(5) State only the Euler's equations of motion for a rigid body with one point fixed.

(6) State only the Hamilton - Jacobi equation.

(7) State minimum two differences each between Lagrange's procedure and Hamilton's procedure.

(8) A body has the dimensions represented by $7i + 6j$ *mt.* in reference frame S. What will be these dimension will be represented in the system S' moving with velocity $0.6\ c$ along positive X-axis?

(9) The half life of a radioactive particle is 10^{-7} sec when it is at rest. What will be the half life when it is traveling with the speed of $0.99\ c$?

(10) State only the Jacobi's identity for the Poisson bracket.

2 Attempt the following : **14**

(a) Show that the angular velocity vector is same in both the co-ordinate systems.

OR

(a) Derive Lorentz transformation equations.

(b) Define moment of inertia of a rigid body about some axis. Prove that the moment of inertia about a parallel axis through the C.M. plus the moment of inertia of the body as if concentrated at the C.M. with respect to the original axis.

3 Attempt the following : **14**

(a) Derive Hamilton's canonical equations.

(b) Discuss in detail the principle of least action.

OR

(b) For the problem of simple harmonic oscillator prove that

$$q = \sqrt{\frac{2E}{m\omega^2}} \sin(\omega t + \alpha)$$

4 Attempt the following : **14**

(a) Obtain Hamilton's principal function for the problem of one dimensional simple harmonic oscillator.

(b) (i) Discuss in detail the phenomenon of length contraction.

(ii) A rod has proper length 100 cm. is in a satellite which is moving with velocity. *0.6 c*. What will be the difference of lengths measured by an observer situated in the (a) laboratory (b) satellite

(c) (i) State all the four types of generating functions and derive the transformation equations if the generating function is $F_2(q, P, t)$.

(ii) Show that the transformations $Q = \log\left(1 + q^{\frac{1}{2}} \cos p\right)$,

$p = 2\left(1 + q^{\frac{1}{2}} \cos p\right) q^{\frac{1}{2}} \sin p$ are canonical and find

the suitable generating function.

5 Attempt the following : (Any **Two**) **14**

(a) Discuss in detail the Routh's procedure.

(b) Find the analytic solution of a torque free motion

(c) Prove in the usual notation the relation $E = mc^2$.

(d) For the Poisson bracket of two function prove that

(i) $[au + bv, w] = a[u, w] + b[v, w]$

(ii) $[uv, w] = [u, w]v + u[v, w]$

MCD-003-1162006 Seat No. ______

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

EMT - 2001 : Mathematics

(Classical Mechanics - II) (New Course)

Faculty Code : 003
Subject Code : 1162006

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.
(2) Figures to the right indicate full marks.
(3) There are 5 questions.

1 Attempt the following : (any **seven**) **14**

(1) State only the Galilean Transformation Equations when the reference frame S' is moving in the direction of positive X-axis.
(2) State only the Lorentz transformation equations when the reference frame S' is moving in the direction of positive X-axis.
(3) State the postulates of special theory relativity.
(4) State only the transformation equations when the generating function is of the type $F_2(q, P, t)$.
(5) State only the Euler's equations of motion for a rigid body with one point fixed.
(6) State only the Hamilton - Jacobi equation.
(7) State only the Hamilton's canonical equations.
(8) Which equations are satisfied by cyclic coordinates in Routh's procedure ?
(9) Which equations are satisfied by non- cyclic coordinates in Routh's procedure ?
(10) Define Poisson bracket of two functions u and v.

2 Attempt the following : **14**

(a) Show that the angular velocity vector is same in both the co-ordinate systems.

OR

(a) Derive Galilean transformation equations.
(b) Derive Euler's equations of motion for a rigid body with one point fixed.

3 Attempt the following : **14**

(a) Express the components of angular velocity ω of a rigid body along the space set of axes in terms of Euler angles.

(b) Explain in detail the phenomenon of time dilation.

OR

(b) State and prove Jacobi's identity for the Poisson bracket of two functions.

4 Attempt the following : **14**

(a) Derive Hamilton's canonical equations.

(b) (i) Discuss in detail the phenomenon of length contraction.

(ii) A rod has proper length 1000 cm. is in a satellite which is moving with velocity 0.6c. What will be the difference of lengths measured by an observer situated in the

(a) laboratory

(b) Satellite

(c) (i) State all the four types of generating functions and derive the transformation equations if the generating function is $F_1(q, Q, t)$.

(ii) Show that the transformations $Q = \log\left(\frac{1}{q}\sin p\right)$, $p = q\cot p$ are canonical and find the suitable generating function.

5 Attempt the following : (any **two**) **14**

(a) Prove in the usual notation the relation $E = mc^2$.

(b) Find the analytic solution of a torque free motion.

(c) Discuss in detail the Routh's procedure.

(d) Obtain Hamilton's principal function for the problem of one dimensional simple harmonic oscillator.

(e) Establish the relation $m = \dfrac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}$ where notations are being usual.

PAU-003-1162006 Seat No. 025074

M. Sc. (Sem. II) Examination

August / September - 2020

EMT - 2001 : Mathematics

(Classical Mechanics - II)

(New Course)

Faculty Code : 003

Subject Code : 1162006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) There are 5 questions.

1 Attempt the following : (Any **Seven**) 14

(1) Define inertial frame of reference.

(2) State only the Transformation Equations of Newtonian Relativity when the reference frame S is moving in the direction of positive X–axis.

(3) State the postulates of special theory relativity.

(4) State minimum two differences between Newtonian Relativity and Einstein's Relativity.

(5) State only the Euler's equations of motion for a rigid body with one point fixed.

(6) State minimum four differences between Lagrangian formulation and Hamiltonian formulation.

(7) Are Poisson brackets commutative ? Justify your answer.

(8) Which equations are satisfied by cyclic coordinates in Routh's procedure ?

(9) State only the transformation equations when the generating function is of the type $F_4(p_i, P_i, t)$.

(10) State only the Hamilton – Jacobi equation.

2 Attempt the following : (Any Two) 14

(a) Prove in the usual notations the relation $\bar{L} = I\bar{\omega}$.

(b) Derive Galilean transformation equations.

(c) Derive Euler's equations of motion for a rigid body with one point fixed.

3 Attempt the following : 14

(a) State and prove Jacobi's identity for the Poisson bracket of two functions.

(b) Explain in detail the phenomenon of time dilation.

OR

(a) Express the components of angular velocity ω of a rigid body along the space set of axes in terms of Euler angles.

(b) Prove that the moment of inertia about a given axis is equal to the moment of inertia about a parallel axis through the C. M. plus the moment of inertia of the body as if concentrated at the center of mass with respect to the original axis.

4 Attempt the following : 14

(a) (i) Discuss in detail the phenomenon of length contraction.

(ii) A rod has proper length 100 cm. is in a frame which is moving with velocity 0.6c. What will be the difference of lengths measured by the observers situated in the (a) laboratory (b) the moving frame.

(b) (i) State all the four types of generating functions and derive the transformation equations if the generating function is $F_1(q_i, Q_i, t)$.

(ii) Show that the transformations,

$$Q = \log\left(1 + q^{\frac{1}{2}} \cos p\right), P = 2\left(1 + q^{\frac{1}{2}} \cos p\right) \sin p$$ are

canonical and find the suitable generating function.

5 Attempt the following : (Any **Two**) 14

(a) Discuss in detail the Routh's procedure.

(b) Discuss in detail the variation of mass with velocity in the context of special theory of relativity.

(c) Prove in the usual notation the relation $E = mc^2$.

(d) Discuss in detail the motion of a heavy symmetrical top.

BBJ-003-1162006 Seat No. 29118

M. Sc. (Sem. II) Examination

July - 2021

EMT-2001 : Mathematics

(Classical Mechanics-II)

Faculty Code : 003
Subject Code : 1162006

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Attempt the following **14**

(1) State both the postulates of special theory relativity.
(2) Define cyclic co-ordinates.
(3) Define: Proper length and proper time.
(4) State Lorentz- Fitzgerald contraction hypothesis.
(5) State only the transformation equations when the generating function is of the type $F_3(p_i,Q_i,t)$.
(6) Define: Poisson brackets of two functions u and v.
(7) State minimum three differences between Newtonian theory and Theory of relativity.

2 Attempt the following. **14**

(1) How Δ-variation differs from δ-variation?
(2) Does c + c = 2c? Justify your answer.
(3) Does [u, v] = - [v, u]? Justify your answer.
(4) State only the Euler's equations of motion for a rigid body with one point fixed.
(5) Define: Action in mechanics.
(6) Define: Generalized momentum.
(7) Define: Generating function.

3 Attempt the following 14

(1) Obtain the expression for angular momentum for discrete rigid body and continuous rigid body.

(2) Derive Galilean transformation equations of motion which connects a stationary reference frame S and a moving reference frame S'.

4 Attempt the following. 14

(1) Discuss in detail the variation of time with velocity in relativistic mechanics.

(2) Prove in the usual notations: [u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0.

5 Attempt the following. 14

(1) Prove that the moment of inertia about a given axis is equal to the moment of inertia about a parallel axis through the C. M. plus the moment of inertia of the body as if concentrated at the center of mass with respect to the original axis.

(2) An electron is moving with a speed of 0.85c in a direction opposite to that of moving photon. Calculate the relative velocity of electron and photon.

6 Attempt the following 14

(1) Discuss in detail the variation of mass with velocity in the context of special theory of relativity.

(2) Obtain Hamilton's principal function for the motion of one dimensional simple Harmonic oscillator and show that the of Hamilton's principal function differs from indefinite time integral of Lagrangian only by a constant.

7 Attempt the following 14

(1) Derive Euler's equations of motion for a rigid body with one point fixed.

(2) Express the components of angular velocity ω of a rigid body along the space set of axes in terms of Euler angles.

8 Attempt the following 14

(1) (i) Discuss in detail the phenomenon of length contraction.

(ii) A rod has proper length 200 cm. is moving in a space shuttle with velocity 0.8c.What will be the difference of lengths measured by the observers situated in the (a)laboratory (b)In space shuttle.

(2) Prove in the usual notation the relation $E = mc^2$.

9 Attempt the following 14

(1) Discuss in detail the motion of a heavy symmetrical top.

(2) Derive Lorentz transformation equations.

10 Attempt the following 14

(1) Discuss in detail the principle of least action.

(2) Find the analytic solution of a torque free motion of a rigid body.

DCA-003-1162006 Seat No. ______

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : EMT-2001

(Classical Mechanics - II)

Faculty Code : 003
Subject Code : 1162006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Attempt the following : (any **seven**) 14

(1) State both the postulates of special theory relativity.

(2) Define: Proper length and proper time.

(3) Does $c + c = 2c$? Justify your answer.

(4) State Lorentz- Fitzgerald contraction hypothesis.

(5) State only the transformation equations when the generating function is of the type $F_3(p_i, Q_i, t)$.

(6) Define: Poisson brackets of two functions u and v.

(7) State minimum three differences between Newtonian theory and Theory of relativity.

(8) Define: Action in mechanics.

(9) State only transformation equations of Newtonian relativity.

(10) Define : Generalized momentum.

2 Attempt the following : 14

(a) Obtain the expression for angular momentum for discrete rigid body and continuous rigid body.

(b) Derive Lorentz transformation equations of motion which connects a stationary reference frame S and a moving reference frame S'.

OR

(b) Discuss in detail the variation of time with velocity in relativistic mechanics.

3 Attempt the following : 14

(a) Prove that the moment of inertia about a given axis is equal to the moment of inertia about a parallel axis through the *C. M.* plus the moment of inertia of the body as if concentrated at the center of mass with respect to the original axis.

OR

(a) Express the components of angular velocity ω of a rigid body along the space set of axes in terms of Euler angles.

(b) Prove in the usual notations :

$$[u,[v,w]]+[v,[w,u]]+[w,[u,v]]=0$$

OR

(b) An electron is moving with a speed of 0.85 c in a direction opposite to that of moving photon. Calculate the relative velocity of electron and photon.

4 Attempt the following : 14

(a) Explain in detail the variation of mass with velocity and establish the relation $m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}$

(b) Derive Hamilton's canonical equations.

5 Attempt the following : (any **two**) 14

(a) Prove in the usual notation the relation $E = mc^2$.

(b) Obtain Hamilton's principal function for the motion of one dimensional simple Harmonic oscillator and show that the Hamilton's principal function differs from indefinite time integral of Lagrangian only by a constant.

(c) Discuss in detail the motion of a heavy symmetrical top.

(d) Discuss in detail the principle of least action.

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Seat No. ________________

HR-003-1162006

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : EMT-2001

(Classical Mechanics-II)

Faculty Code : 003

Subject Code : 1162006

Time : $2\frac{1}{2}$ / Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Attempt the following : (any seven) **14**

(1) State Lorentz – Fitzgerald contraction hypothesis.

(2) State minimum three differences between Newtonian theory and theory of relativity.

(3) Define : Poisson brackets of two functions u and v.

(4) Define : Cyclic co-ordinates.

(5) Define : Proper length and proper time.

(6) State minimum four differences between Lagrangian formulation and Hamiltonian formulation.

(7) State both the postulates of special theory relativity.

(8) Which equations are satisfied by cyclic coordinates in Routh's procedure ?

(9) State only the transformation equations when the generating function is of the type $F_3 (p_i, Q_i, t)$

(10) State only the Euler's equations for a torque free motion.

2 Attempt the following : (any two) 14

(a) Prove in the usual notations the relation $\overline{L} = I\overline{\omega}$.

(b) Derive Galilean transformation equations of motion which connects a stationary reference frame S and a moving reference frame S'.

(c) Prove in the usual notations:

$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0$

3 Attempt the following : 14

(a) Derive Hamilton's canonical equations of motion.

(b) Express the components of angular velocity ω of a rigid body along what space set of axes in terms of Euler angles.

OR

(a) Derive Euler's equations of motion for a rigid body with one point fixed.

(b) An electron is moving with a speed of 0.85c in a direction opposite to that of moving photon. Calculate the relative velocity of electron and photon.

4 Attempt the following : 14

(a) Prove that the moment of inertia about a given axis is equal to the moment of inertia about a parallel axis through the *C.M.* plus the moment of inertia of the body as if concentrated at the center of mass with respect to the original axis.

(b) (i) Discuss in detail the phenomenon of length contraction.

(ii) A rod has proper length 2000 m is moving in a space shuttle with velocity 0.8 c. What will be the difference of lengths measured by the observers situated in the (a) laboratory (b) in space shuttle.

5 Attempt the following : (any two) 14

(a) Explain detail the variation of mass with velocity and establish the relation $m = \dfrac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}}$.

(b) Obtain Hamilton's principal function for the motion of one dimensional simple Harmonic oscillator and show that the of Hamilton's principal function differs from indefinite time integral of Lagrangian only by a constant.

(c) Prove in the usual notation the relation $E = mc^2$.

(d) Derive the transformation equations, if the generating function is $F_1 (q_i, Q_i, t)$

(e) Discuss in detail the principle of least action.
