Shree H.N. Shukla College of Science
M.Sc. (Mathematics) Sem-1
IMP questions of Real Analysis

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1. Outer measure of any interval is its length.
2. Let E_1 and E_2 are measurable sets then show that $E_1 \cup E_2$ is measurable.
3. Lebesgue measure is translation invariant.
4. Let E ⊆ ℝ be measurable set than there exists a G_δ set G in ℝ with E ⊆ G Such that $m^*(G) = m^*(E)$.
5. If f be measurable functions define on measurable set E and f = g a.e on E then g be a measurable.
6. f and g are bounded measurable functions define on measurable set E of finite measure then $\int_E f + g = \int_E f + \int_E g$.
7. f and g are non negative measurable functions define on measurable set E then $\int_E f + g = \int_E f + \int_E g$.
8. f and g are bounded measurable functions define on measurable set E of finite measure then $\int_E f + g = \int_E f + \int_E g$.
9. f and g are measurable functions which are integrable over measurable set E then f+g is integrable over E and $\int_E f + g = \int_E f + \int_E g$.
10. f and g are measurable functions which are integrable over measurable set E $f \leq g$ a.e on E than $\int_E f \leq \int_E g$.
11. State and Prove bounded convergence theorem.
12. State and Prove monotone convergence theorem.
13. State and Prove Lebesgue convergence theorem.
14. State and prove Fatou's lemma
15. State Fatou's lemma. Show that we may have strict inequality in Fatou's lemma.
16. Let $< f_n >$ be a sequence of measurable function defined on E and f be A measurable real valued function define on E such that $f_n \to f$ in measure On E. Show that there is a subsequence $< f_{n_k} >$ which is converges to f a.e on E.
17. If f ∈ BV[a,b] than $P_b^a(f) + N_b^a(f) = T_b^a(f)$.
18. If f ∈ BV[a,b] than $P_b^a(f) - N_b^a(f) = f(b) - f(a)$.
19. f is function of bounded variation on [a,b] if and only if there are two monotonically increasing function g and h define on [a,b] such that f = g-h.
20. g : [0,1]→R define by

$$g(x) = x^2(\sin(1/x^2)) \quad \text{if } x \neq 0$$
$$= 0 \quad \text{if } x = 0$$

then show that g is not function of bounded variation on [0,1].

21. If f is integrable over on [a,b] and $\int_a^x f(t)dt$ f(t)dt = 0 for all x ∈ [a,b] then show that f = 0 a.e on [a,b].

22. f is measurable functions define on [a,b] such that f is integrable over [a,b]. Let F: [a,b]→ ℝ define by

F(x) = $\int_a^x f(t)dt$ than If F ∈ BV[a,b].

23. f is bounden and measurable functions define on [a,b]. Let F: [a,b]→ ℝ define by

F(x) = $\int_a^x f(t)dt$ than $F'(x) = f(x)$ a.e on [a,b].

24. f: [a,b]→ ℝ is absolutely continues than f ∈ BV[a,b].

25. If f is integrable over on [a,b] Let F: [a,b]→ ℝ define by

F(x) = $\int_a^x f(t)dt$ than F is absolutely continues.

26. State and prove Holder's inequality.

27. State and prove Minkowski's inequality.