1681-MSMA-20-04-00002

Seat No. ______

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 4 MARCH 2025 ( Regular ) MARCH - 2025

INTEGRATION THEORY

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-04-00002



**Time : 2:30 Hours]** **[Total Marks : 70**

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Define with example: Signed Measure on measurable space.

2 Prove or disprove ‘ A measure zero set is null set ’.

3 Define Positive and negative set with respect to a signed measure.

4 State Radon Nikodym theorem for Signed measures.

5 If $(A_i, b_i), i = 1, 2$ are Hahn-decomposition of $X$ with respect to signed measure $\mu$ then show that $A_1 \Delta A_2$ is null sets with respect to $\mu$.

6 Define with example: A measure absolutely continuous with respect to another measure.

7 Define with example: Complete measure.

8 Define with example: Semi Algebra of subsets of set.

9 State Tonelli’s theorem without proof.

10 Define Baire sets in a locally compact Hausdorff space.

Q.2 Answer the following (Any Two) 14

1 Let $\mathcal{A}$ be the $\sigma$ - algebra of all Lebesgue measurable sets in $\mathbb{R}$, $\mu$ be the Lebesgue measure on $(\mathbb{R}, \mathcal{A})$ and $f : \mathbb{R} \to [0, \infty]$ be a non negative Lebesgue measurable function then show that $\gamma : \mathcal{A} \to [0, \infty]$ defined by $\gamma(E) = \int_E f \, d\mu$, $\forall E \in \mathcal{A}$ is a measure on $(\mathbb{R}, \mathcal{A})$.

2 State and prove Hahn decomposition theorem.

3 Define $\sigma$ - algebra of subset of a set X. If X is any set then prove that $\mu : P(X) \to [0, \infty]$

$$\mu(A) = \begin{cases} \text{the number of elements} & ; \text{ if } A \text{ is finite} \\ \infty & ; \text{ if } A \text{ is infinite} \end{cases}$$

is measure on $(X, P(X))$

Q.3 Answer the following 14

1 State and prove Lebesgue decomposition theorem.

2 Describe by example that the hypothesis "$\mu$ is $\sigma$ -finite" in Radon-Nikodym theorem can not be dropped.

OR

Answer the following

1 State and prove Caratheodory extension theorem.

2 Let $\mu$ be a measure on an algebra $A$ of subset of a set $X$ and $\mu^*$ be the outer measure on $X$ induced by $\mu$ then prove that every element $E \in \mathcal{A}$ is $\mu^*$ measurable.



Q.4 Answer the following questions (Any Two)

Let $(X \times Y, \mathcal{F}, \mu \times \gamma)$ be the product measure space of two $\sigma$-finite complete measure spaces $(X, \mathcal{A}, \mu)$, $(Y, \mathcal{B}, \gamma)$ $E \in \mathcal{R}_{\sigma\delta}$ and $(\mu \times \gamma)(E) < \infty$ then show that, $g : X \to [0, \infty]$ defined by $g(x) = \gamma(E_x),\ \forall x \in X$ is measurable and $\int g\, d\mu = (\mu \times \gamma)(E)$.

Describe by example that the hypothesis " the non negativity of f" in Tonnelli's theorem can not be dropped.

Q.5 Answer the following (Any Two) 14

1 Let $(X \times Y, \mathcal{F}, \mu \times \gamma)$ be the product measure space of two $\sigma$-finite complete measure spaces. $\mathcal{R}$ be the semialgebra of measurable rectangles in $X \times Y$, $E \in \mathcal{R}_{\sigma\delta}$ and $x \in X$ then show that $E_x$ is a measurable subset of $Y$.

Let $X$ be a locally compact Hausdorff space. Then show that $C_c(x) = \{f : X \to \mathbb{R} / f \text{ is continuous and } supp\ f \text{ is compact in } X\}$ is vector space over $\mathbb{R}$ with respect to pointwise addition and scalar multiplication.

Prove that every compact Baire set K in a locally compact Housdorff space X is $G_\delta$.

Let $X$ be a locally compact Hausdorff space. $m$ be a $\sigma$ - algebra of subsets of $X$ such that $m \supset B_a(X)$ and $\mu$ be a finite measure on $(X, m)$ Then prove that if $\mu$ is inner regular then $\mu$ is regular.

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