SHREE H. N. SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics)
Semester-2

PRELIMS TEST

Subject : Complex Analysis | **Marks : 70**

Date: __ / __ / ____ | **Time : 2.5 Hours**

Q-1 Answer any seven questions. **(14)**

1) True or false ? Justify. S defined by $S_z = \overline{z}$ is a bilinear transformation.

2) If $\lambda \in \mathbb{C}, \lambda \neq 0, \lambda \neq 1$ then find the fixed points of the bilinear transformation S defined by $S_z = \lambda z$.

3) 0 is ________ of $e^{\frac{1}{z}}$

   (a) removable singularity

   (b) a simple pole

   (c) an essential singularity

   (d) not a singularity

4) 0 is a removable singularity of ________.

   (a) $\cos\left(e^{\frac{1}{z}}\right)$ (b) $\cos z$

   (c) $\frac{\sin z}{z}$ (d) $\frac{1}{z}$

5) Find the right side of the x-axis w.r.t. the orientation $(-1, 0, 1)$.

6) State the geometric meaning of winding number for the closed rectifiable curve.

(i) Define conformal mapping.

(ii) State Inverse function theorem.

7) State Riemann stieltje's theorem.

8) Evaluate $\int_{\sigma} \frac{z^2+1}{z^2+z+1} dz$; where $|z|=2$ is the circle with center 0 and radius 2.

9) Find bilinear transformation taking $i \to 1, 0 \to \infty, -i \to 0$.

Q-2 Answer any two question. **(14)**

1) State and prove minimum modulus theorem.

2) Define removable singularity of complex function "f" of a complex variable and given an example. Prove that $a \in ¢$ is a removable singularity of f iff $a_n = 0, \ \forall n \le -1$ in the Laurent's expansion $\sum_{n=-\infty}^{\infty} a_n (z-a)^n$ of f at a.

3) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.

Q-3 Answer any two questions. **(14)**

1) State and prove fundamental theorem of algebra.

2) State and prove Cauchy's integral formula.

3) State, without proof, Rouche's theorem. Prove that $3z^7 + 5z - 1$ has exactly one zero in $|z| < 1$ and is a real zero in $(0,1)$.

Q-4 Answer any two questions.

1) Define the right side, left side of a circle Γ in $\mathbb{C}_\infty$ w.r.t. an orientation of Γ. Find the right side of the imaginary. axis L w.r.t. the orientation $(-i, 0, i)$ and the left side of the unit circle Γ with centre at o w.r.t. the orientation $(-i, -1, i)$.

2) Find the bilinear transformation S taking $1 \to 0,\ 0 \to \infty,\ \infty \to 1$.

3) Given two circle Γ_1, Γ_2 in $\mathbb{C}_\infty$ and distinct $z_2, z_3, z_4 \in \Gamma_1$, distinct $w_2, w_3, w_4 \in \Gamma_2$ prove that $\ni$ a unique bilinear transformation S rt $S(\Gamma) = \Gamma'$ and $S(z_j) = w_j, \forall\, j = 2, 3, 4$

Q-5 Answer any two questions.

1) Every bilinear transformation can be written as composition of translation, dilation and inversion.

2) (i) State and prove Open Mapping Theorem.

(ii) Evaluate $\int_\sigma \frac{dz}{z^2 + i^2}$, where σ is given by $\sigma(t) = 2e^{it}\,|\cos 2t|$.

3) Define branch of logarithmic on a connected open set and prove that if $f : G \to \mathbb{C}$ be continuous, $g : H \to \mathbb{C}$ be differentiable with $g'(x) \neq 0; \forall x \in H$ and $f(G) \subset H$, $g(f(z)) = z; z \in G$ then f is differentiable and $f'(z) = \dfrac{1}{g^r(f(z))}; z \in G$.

-------------------------------------- ALL THE BEST --