# Shree H.N.Shukla College of Science Rajkot

**MATHEMATICS**

**T.Y.B.Sc. (Sem. VI) (CBCS)**

**UNIT TEST**

**PAPER- 601**

**Complex Analysis-II**

**Time: 1 hour]** **[Total Marks: 30**

**Instruction: (i) All questions are compulsory.**

**(ii) Figures to the right indicate full marks of the question.**

**1. (A) Answer the following:** **[05]**

(1) Find singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.

(2) Find the critical point of $w = \frac{1}{z-1}$

(3) Define: Bilinear mapping

(4) Write expansion of coshz in maclaurian series.

(5) Define Residue of f (z) at pole $Z_0$.

**(B) Attempt any one:** **[02]**

(1) Derive formula for finding residue of f(z) at simple pole $Z_0$.

(2) Show that $W = \frac{az+b}{cz+d}$ is conformal mapping.

**(C) Attempt any one:** **[03]**

(1) Find a Mobius mapping which maps three point 1, 2, -1 in z-plane into 2, 1, -2 in w-plane.

(2) Find the value of integral $\int_C \frac{dz}{Z^3(Z+4)}$ $where\ C: |Z| = 2$

**(D) Attempt any one:** **[05]**

(1) State and prove Taylor's infinite series of an analytic function.

(2) State and prove Cauchy's Residue theorem.

**2. (A) Answer the following: [05]**

(1) Write the formula for finding the residue of f(z) at $m^{th}$ order pole.

(2) Define: Power series

(3) Find radius of convergence for the series $\sum n!\ z^n$

(4) Define: Complex series

(5) Find $Res\left(\frac{\cos z}{z}, 0\right)$

**(B) Attempt any one: [02]**

(1) Show that x+y=2 transform into the parabola $u^2=-8(v-2)$ under the transformation $W=Z^2$.

(2) Find radius of convergence of series $\sum_{n=1}^{\infty} \frac{z^n}{3^n-1}$

**(C) Attempt any one: [03]**

(1) Prove that the transformation $w=2z+z^2$ maps the unit circle |z|=1 of z-plane into cardiod to w-plane.

(2) Show that the composition of bilinear maps is again a bilinear.

**(D) Attempt any one: [05]**

(1) Prove that

$$\int_0^{\infty} \frac{dx}{(x^2+a^2)^{n+1}} = \frac{\pi(2n)!}{(n!)^2(2a)^{2n+1}}\ , where\ a > 0.$$

(2) Using residue theorem prove that

$$\int_{-\infty}^{\infty} \frac{\mathrm{dx}}{(1+\mathrm{x}^2)^3} = \frac{3\pi}{8}$$

# ****BEST OF LUCK****