003-1163002

Seat No. ______

MASTER OF SCIENCE MATHEMATICS(W.E.F.-2016) MSC MATHS(2016) Semester - 3 Examination

October - 2024

FUNCTIONAL ANALYSIS

Faculty Code : 003

Subject Code : 003-1163002

Time : 2.30Hours] **[Total Marks : 70**

Q.1 Answer the following : (Any seven out of ten, each of 02 marks) 14

1 1. Define with example : Normed Linear Space.

2 2. Define with example : Equivalent norms.

3 3. Define with norm : Quotient Space.

4 4. State only : Open Mapping Theorem.

5 5. Define with example : Sub-linear functional.

6 6. State only : Zorn's lemma.

7 7. Define with example : Hilbert Space.

8 8. Define with example : Bounded Linear Transformation.

9 9. Define with example : Orthonormal Set.

10 10. Define with example : Nowhere dense set.

Q.2 Answer the following : (Any two out of three, each of 07 marks) 14

1 1. State and prove, Holder's Inequality.

2 2. State and Prove, Riesz lemma.

3 3. Prove that, Every finite dimensional subspace of a normed linear space X is complete.

Q.3 Answer the following : (1 & 2 Both are compulsory, each of 07 marks) 14

1 Let X and Y be Normed linear spaces and let $B(X,Y)$ be the space of all bounded linear transformations from X into Y. If Y is a Banach space then prove that, $B(X,Y)$ is also a Banach space.

2 State, Baire's Category theorem. Prove that, a Banachspace cannot have countably infinite Hamel Basis.

OR

Answer the following : (1 & 2 Both are compulsory, each of 07 marks) 14

1 State and prove, Closed Graph Theorem.

2 Let X be a normed linear space over $\mathbb{k}$ and Y be a closed vector subspace of X and $x_0 \in X \setminus Y$ then prove that, $F \in X'$ such that

(I) $\|F\| = 1$

(II) $F(y) = 0, \forall y \in Y$

(III) $F(x_0) = \inf_{y \in Y} \|x_0 - y\|$.

Q.4 Answer the following : 14

1 State and Prove, Projection Theorem.

2 State and prove, Polarization identity.

Q.5 Answer the following : (Any two out of four, each of 07 marks) 14

1 (a) Show that, $(l^1, \|\cdot\|_1)' \cong (l^\infty, \|\cdot\|_\infty)$.

2 (b) State and prove, Uniform Bounded Theorem.

3 (c) State and prove, Schwarz Inequality.

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(d) Let $X_1, \cdots, X_n$ be a norm linear space over K. Then Show that, $(X_1 \times \cdots \times X_n, \|\cdot\|)$ is a Banach space over K if and only if X_i is a Banach space over K, $\forall i = 1, \cdots, n$.

Where $\|(x_1, \cdots, x_n)\| = \max_{1 \le i \le n} \|x_i\|$, $\forall (x_1, \cdots, x_n) \in X_1 \times \cdots \times X_n$.