Shree H.N.Shukla College of Science Rajkot

MATHEMATICS

S.Y.B.Sc. (Sem. IV) (CBCS)

UNIT TEST

PAPER- 401

Linear Transformation

Time: 1 hour] **[Total Marks: 30**

Instruction: (i) All questions are compulsory.

(ii) Figures to the right indicate full marks of the question.

1. (A) Answer the following: **[05]**

1) Define: Zero linear transformation
2) Define: Kernal of a linear transformation
3) Let T: $U \to V$ be a linear transformation and let θ and θ' be zero vectors of U and V respectively. Prove that $T(\theta) = \theta'$
4) Define: Idempotent linear transformation

(B) Attempt any one: **[02]**

1) Find a linear transformation T: $R^3 \to R^3$ such that $R_T = SP\{(1,5,0), (0,7,3)\}$
2) Find N_T and n(T) for the linear transformation T: $R^3 \to R^2$, $T(x,y,z)=(x-y+z,x+y-z)$, $\forall (x,y,z) \in R^3$

(C) Attempt any one: **[03]**

1) Prove that composition of two linear transformations is again a linear transformation.
2) Find the linear transformation T: $R^3 \to R^2$ such that $T(e_1)=(1,1)$, $T(e_1+e_2)=(1,0)$, $T(e_1+e_2+e_3)=(1,-1)$. Also find T (2, 5, 7), where $\{e_1, e_1+e_2, e_1+e_2+e_3\}$ is a basis of R^3.

(D) Attempt any one: [05]

1) State and prove Rank-Nullity theorem.
2) Prove that L (U, V) is a vector space over R with respect to addition and scalar multiplication of linear transformation.

2. (A) Answer the following: [05]

1) Define: Dual of a vector space
2) Define: Eigen value of a linear transformation
3) If dimU=4, dimV=3, then find the dimL (U, V).
4) Define: Eigen basis of a linear transformation

(B) Attempt any one: [02]

1) Let T: $R^2 \to R^2$, T(x,y)=(x, -y), $\forall$(x,y)ϵR^2 and B_1={(1,1), (1,0)} and B_2={(2,3), (4,5)}. Then find [T;B_1, B_2].
2) Define: Matrix associated with a linear transformation

(C) Attempt any one: [03]

1) Find Eigen values of the linear transformation T: $R^2 \to R^2$ defined as T(x, y) = (3y, 2x-y).
2) $A = \begin{bmatrix} 3 & 2 & 5 \\ -1 & 4 & -6 \end{bmatrix}$ is the corresponding matrix of linear transformation T: $R^3 \to R^2$ with standard bases of R^3 and R^2. For the new bases B_1={(1,1,1), (1,1,0), (1,0,0)} and B_2={(1,1), (1,0)} . Find [T;B_1, B_2].

(D) Attempt any one: [05]

1) Find the Eigen value and Eigen vector for the linear transformation T: $R^3 \to R^3$, T(x, y, z)=(-2y-2z, -2x-3y-2z, 3x+6y+5z), $\forall$(x,y,z)ϵ R^3 by considering the standard basis of R^3.
2) Let T: V$\to$V be a linear transformation and let B be any basis of V. Then T is singular if and only if det ([T; B]) =0.

****BEST OF LUCK****