M. Sc (Mathematics) (Sem. 1)

Test

MATH.CMT-1002: Real Analysis

[Time: 2.30 Hours] [Total Marks: 70]

1 Answer any seven 7x2=14

(a) Let $A \subseteq \mathbb{R}$ and $m^*(A) = 0$ then show that A is measurable.

(b) Let $A \subseteq \mathbb{R}$ and A be G_δ set then show that

A^c is F_σ set.

(c) If a measurable function f is integrable over a measurable set E , then

show that $|f|$ is integrable over E.

(d) State Monotone convergence theorem.

(e) f: [0,1]→R be a increasing function verify that f is function of

bounded variation over [0,1]. .

(f) Let $(X, \|\cdot\|)$ be a norm linear space. Let d: X x X → R be given by

$d(x, y) = \|x-y\|$ then show that d is metric.

(g) Define Lebesgue outer measure. $m^*((0,1)\cup(2,3))= ?$

(h) Show that every continues function is measurable.

(i) Define Lebesgue integral of bounded measurable function define on

measurable set E.

(j) When we say that f :[a , b] → R. is function of bounded variation

over [a , b].

2 Answer any two 2x7=14

(a) Let $A \subseteq \mathbb{R}$. Let $E_1, E_2, E_3, \ldots, E_n$ be finite sequence of disjoint measurable sets.

Show that $m^*(A \cap (\bigcup_{i=1}^{n} Ei)) = \sum_{i=1}^{n} m^*(A \cap E_i)$.

(b) Let $a, b \in \mathbb{R}$ be a such that $a \leq b$ prove that $m^*[a,b] = b - a$.

(c) Let f ,g : R → R be a measurable function. Prove that the function f+g, fg and f+c are measurable. Where $c \in$ R.

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(a) f and g are bounded measurable function define on a set E of finite measure.

If $f \le g$ a.e on E, then prove that $\int_E f \le \int_E g$. 5

(b) If $f \in BV[a,b]$, then show that $P_a^b(f) - N_a^b = f(b) - f(a)$. 4

(c) If f be measurable function define on measurable set E and f = g a.e on E , then prove that g is measurable. 5

OR

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(a) Prove that every borel set is measurable. 5

(b) State Fatou's lemma. Show that we may have strict inequality in Fatou's lemma. 4

(c) $g : [0,1] \rightarrow R$ define by

$g(x) = x^2(\sin(1/x^2))$ if $x \neq 0$

$= 0$ if $x = 0$

then show that g is not function of bounded variation. 5

4 Answer any two 2x7=14

(a) State and Prove monotone convergence theorem.

(b) prove that a function f is function of bounded variation on [a,b] if and only if there are two monotonically increasing function g and h define on [a,b] such that f = g-h.

(c) State and prove Minkowski inequality.

5 Answer any two 2x7=14

(a) State and prove Holder's inequality.

(b) If f is integrable over on [a,b] and $\int_a^b f(t)dt = 0$ for all $x \in [a,b]$ then show that f(t) = 0 a.e on [a,b].

(c) Let $< f_n >$ be a sequence of measurable function defined on E and f be A measurable real valued function define on E such that $f_n \rightarrow f$ in measure On E. Show that there is a subsequence $< f_{n_k} >$ which is converges to f a.e on E.

BEST OF LUCK