003-1163004

Seat No. ______

MASTER OF SCIENCE MATHEMATICS(W.E.F.-2016) MSC MATHS(2016) Semester - 3 Examination

October - 2024

DISCRETE MATHEMATICS

Faculty Code : 003

Subject Code : 003-1163004

Time : 2.30Hours] **[Total Marks : 70**

Q.1 Answer the following : (Any seven out of ten, each of 02 marks) **14**

1 Define: Subsemigroup and Submonoid.

2 Does there exist surjective homomorphism from (Z,+) to (N,+)? Justify.

3 Give two examples of modular lattice which are not distributive.

4 Define: Boolean expression.

5 Define: Predicate with example.

6 Write any two valid argument forms.

7 Construct finite automata of language of all stings containing even number of zeros.

8 Define extended transition function in NFA.

9 Define the role of encoder and decoder in communication system.

10 Find distance between codewords 11101 and 01011.

Q.2 Answer the following : (Any two out of three, each of 07 marks) **14**

1 Show that $(N \cup \{0\}, +)$ is isomorphic to some quotient semigroup of $\{0,1\}^*$.

2 Let $n \geq 1$ then $(D_n, \leq_{div})$ is complemented iff n is a product of unique primes.

3 Show that a binary code C can correct up to k errors in any codeword if and only if $d(C) \geq 2k + 1$.

Q.3 Answer the following : (1 & 2 Both are compulsory, each of 07 marks) **14**

1 State pumping lemma for regular languages. Using it show that $L = \{0^i 1^i | i \geq 0\}$ is not regular.

2 Show that a lattice $(L, \leq)$ is distributive iff for all $a, b, c \in L$,
$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

OR

Answer the following : (1 & 2 Both are compulsory, each of 07 marks) **14**

1 Show that for any NFA $M = (Q, \Sigma, q_0, A, \delta)$ accepting a language $L \subseteq \Sigma^*$, there is an FA $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$ that also accepts L.

2 A lattice $(L, \leq)$ is modular if and only if the following statement holds:
"If M is any sublattice of $(L, \leq)$, then M can not be isomorphic to the pentagon lattice"

Q.4 Answer the following :

1. Define argument and check whether the following argument is logically valid or not."If I save money, I will buy house. I did not buy house. Therefore I did not save money".
2. State and prove the generalized DeMorgan's laws in first order logic.

Q.5 Answer the following : (Any two out of four, each of 07 marks)

1. State and prove the fundamental theorem of homomorphism of semigroups.
2. Let G be a group and H is a normal subgroup of G then the relation R defined on G by $g_1 R g_2$ iff $g_1 g_2^{-1} \in H$ is a congruence relation.
3. Suppose L_1 and L_2 are subset of $\{0,1\}^*$ such that, $L_1 = \{x \mid 00 \textit{ is not a substring of } x\}$ $L_2 = \{x \mid x \textit{ ends in } 01\}$. Find FA that recognize $L_1 - L_2$ and $L_1 U L_2$.
4. Devise a binary Hamming code of length seven with three parity bits which can correct a single error if any due to noise.