SHREE H. N. SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics) Semester-2

IMP Questions of Complex Analysis

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1. State and prove fundamental theorem of algebra.
2. State and prove Liouvilles theorem.
3. State Orientation principle. And show that the concept of symmentric points with respect to a circle Γ is independent of choice of three points $z_2, z_3, z_4 \in \Gamma$.
4. Find the Laurent's series expansion in the powers of z for function $f(z) = \frac{z+2}{z^2-2z+3}$ in
   (i) $|z| < 1$;
   (ii) $1 < |z| < 3$;
   (iii) $|z| > 3$.
5. State and prove Cauchy integral formula for second version.
6. State and prove Cauchy's theorem.
7. Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z}\, ds = 2\pi, \forall z \in \mathbb{C}, |z| < 1$.
8. State and prove maximum modulus theorem.
9. Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\pi}{\sqrt{2}}$.
10. Find the bilinear transformation S taking $1 \to 2,\ 3 \to \infty,\ \infty \to 0$.
11. Define removable singularity of complex function "f" of a complex variable and give an example. Prove that $a \in \mathbb{C}$ is a removable singularity of f if and only if $a_n = 0, \forall\, n \leq -1$ in the Laurent's expansion $\sum_{n=-\infty}^{\infty} a_n (z-a)^n$ of f at a.
12. State Rouche's theorem. Using the Rouche's theorem prove that $3z^7 + 5z - 1$ has exactly one zero in $|z| < 1$ and is a real zero in $(0,1)$.

13. Show that the set $M = \{S : S \text{ is a bilinear transformation}\}$ is a group under composition.

14. Prove that every bilinear transformation can be written as composition of translation, dilation and inversion.

15. State and prove minimum modulus theorem. Also give an example of a non-constant analytic function in $\mathbb{C}$ which may attain its minimum value.

16. Prove that for analytic function $: G \to \mathbb{C}$; where G be an open connected subset of $\mathbb{C}$ and $G^* = \{\bar{z} : z \in \mathbb{C}\}$, then $f^*: G^* \to \mathbb{C}$ defined by $f^*(z) = \overline{f(\bar{z})}$; is an analytic function.

17. Prove that if $f : G - \{a\} \to \mathbb{C}$ is an analytic function a is a pole of f then there exists $m \in \mathbb{N}$ and $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}$.

18. Find $\int_\gamma \frac{1}{z}\, dz$, where $\gamma = [1-i, 1+i, -1+i, -1-i, 1-i]$.

19. Define the right side, left side of a circle Γ in $\mathbb{C}_\infty$ w.r.t. an orientation of Γ. Find The right side of the imaginary axis L w.r.t. the orientation $(-i, 0, i)$ and the left side of the unit circle Γ with center at 0 w.r.t. the orientation $(-i, -1, i)$.

20. If γ is a rectifiable curve in $\mathbb{C}$, $f : \{\gamma\} \to \mathbb{C}$ is continuous and $c \in \mathbb{C}$, then prove that $\int_\gamma f(z)\, dz = \int_{\gamma+c} f(z-c)\, dz$.

------------------------------------ALL THE BEST------------------------------------