SHREE H. N. SHUKLA GROUP OF COLLEGES
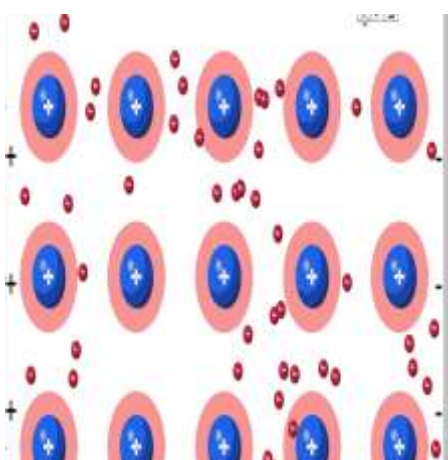
T.Y.B.Sc. SEM – VI

Subject: Physics

Paper- 602

Unit -3

FREE ELECTRON THEORY OF METAL

- Introduction
- Free electron model
- Density of states
- Thermal conductivity
- Hall effect
- Kroning peny model
- Weimmann frenz law
- Application

✚ INTRODUCTION:

1) What are the failures of Classical Free electron theory?

Heat Capacities: - The internal energy of a molar substance, $U = \frac{3}{2}KTN$

Molar specific heat $C_v = \frac{\partial V}{\partial T} = \frac{3}{2}KN = \frac{3}{2}R$

'N' is the Avogadro number, K is Boltzmann constant and 'R' is the universal gas constant. The molar specific heat is 1.5 R theoretically, where as the experimental value obtained is too low. This is due to the fact that all free electrons do not contribute significantly to thermal or electrical conductivity. Therefore classical free e⁻ theory can't hold good.

Mean free path: - It is calculated using the formula: $\lambda = C^- \times T_r$

$$\lambda = \sqrt{\frac{3kT}{m}} \times \frac{m}{\rho n e^2}$$

$$\lambda = \frac{\sqrt{3kTm}}{\rho n e^2}.$$

For cu at 20° c, $\rho = 1.69 \times 10^{-8}$ ohm-m^{-1}, e⁻ concentration n = $8.5 \times 10^{28}/m^3$.

$$\lambda = 2.5 \times 10^{-9} m$$

The experimental value of 'λ'was obtained nearly 10 times its theoretical value. So classical theory could not explain the large variation in 'λ' value.

Resistivity: - According to the classical free electron theory, the resistivity is given by the equation, $\rho = \frac{\sqrt{3kTm}}{\lambda n e^2}$

Which means the resistivity is proportional to the square root of absolute temperature. But according to theory at room temperature it does not change up to 10K and in intermediate range of temperature ρ is proportional to T^5.

- The conductivity of semiconductors and insulators cannot be explained by the free electron theory.

2) What are the applications of Hall Effect?

Determination of the type of Semi-conductors:
The Hall coefficient R_H is -ve for an n-type semiconductor and +ve for p-type semiconductor. Thus the sign of Hall coefficient can be used to determine whether a given Semi-conductor is n or p-type.

Calculation of carrier concentration:

$$R_H = \frac{1}{\rho} = \frac{1}{ne} \text{ (for e⁻ s)}$$
$$R_H = \frac{1}{\rho e} \text{ (for holes)}$$
$$\Rightarrow n = \frac{1}{eR_H} \qquad \Rightarrow \rho = \frac{1}{eR_H}$$

Determination of Mobility: $\sigma = ne\mu$

$$\mu = \frac{\sigma}{ne} = \sigma R_H$$
$$\mu = \sigma R_H \qquad | \quad |$$

Measurement of Magnetic Flux Density:
Hall Voltage is proportional to the magnetic flux density B for a given current I. So, Hall Effect can be used as the basis for the design of a magnetic flux density metal.

3) Define Fermi energy level.

The highest energy level that can be occupied by an electron at 0 K is called Fermi energy level. It is denoted by E_F .

4) Distinguish between conductors, Insulator and Semiconductors.

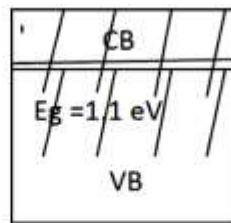Solids are classified into three types based on energy gap.
- Conductors(metals)
- Insulators
- Semiconductors
- In case of conductors, valence band and conduction band almost overlap each other and no significance for energy gap. The two allowed bands are separated by Fermi energy level. Here there is no role in Eg, as a result conduction is high.
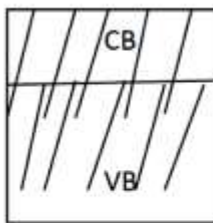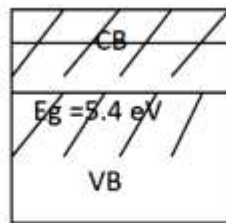
| Conductors | insulators | Semiconductors |

- In case of insulator, valence band and conduction band are separated by large energy gap, hence conductivity is zero.
- In case of semiconductors, the valence band and conduction band are separated by relatively narrow energy gap; hence the conductivity lies in between conductors and insulators.

5) Define the following terms.
i. Collision time ii. Relaxation time iii. Mean free path iv .Drift velocity v. Mobility

i. Collision time: The time taken by the electron to complete one collision with the +ve ion center.
ii.Relaxation time: The time taken by the electron to reduce its velocity to 1/e of its initial velocity.
iii.Mean free path: The average distance covered by the electron between two successive collisions.
iv.Drift velocity: The steady state velocity of the electrons in the presence of Electric field.
v.Mobility: The steady state velocity of the electrons per unit electric field.

Part- B (Descriptive- 10marks)

1) What are the salient features of classical free electron theory of metals? What are its drawbacks?

Drude and Lorentz proposed free electron theory of on the basis of some assumptions.

- In conductors (metals), there are large number of free electrons moving freely within the metal i.e. the free electrons or valence electrons are free to move in the metal like gaseous molecules, because nuclei occupy only 15% metal space and the remaining 85% space is available for the electrons to move.
- Since free electrons behave like gaseous molecules, the laws of kinetic theory of gases can be applied. The mean K.E of a free electron is equal to that of a gas molecule at same temperature.
- In the absence of any electric field, the electrons move randomly while undergoing scattering at +ve ion centers. The collisions are regarded as elastic (no loss of energy).
- The electron speeds are distributed according to the Maxwell- Boltzmann distribution law.
- When an electric filed is applied, the free electrons are accelerated in a direction opposite to that of the field.
- The free electrons are confined to the metal due to surface potential.
- The electrostatic force of attraction between the + ve ion cores and the free electrons is assumed to be negligible.

Drawbacks:

1. Heat capacities: - The internal energy of a molar substance, $U = \frac{3}{2}KTN$

Molar specific heat $C_v = \frac{\partial V}{\partial T} = \frac{3}{2}KN = \frac{3}{2}R$

'N' is the Avogadro number, K is Boltzmann constant and 'R' is the universal gas constant. The molar specific heat is 1.5 R theoretically, where as the experimental value obtained is too low. This is due to the fact that all free electrons do not contribute significantly to thermal or electrical conductivity. Therefore classical free e⁻ theory can't hold good.

2. Mean free path: - It is calculated using the formula, $\lambda = C^- \times Tr$

$\lambda = \sqrt{\frac{3kT}{m}} \times \frac{m}{\rho n e^2}$

$\lambda = \frac{\sqrt{3kTm}}{\rho n e^2}$.

For cu at 20° c , $\rho = 1.69 \times 10^-8$ ohm-m^{-1},e⁻ concentration n $= 8.5 \times 10^{28}/m^3$.

$$\lambda = 2.5 \times 10^{-9}m$$

The experimental value of 'λ' was obtained nearly 10 times its theoretical value. So classical theory could not explain the large variation in 'λ' value.

3. Resistivity: - According to the classical free electron theory, the resistivity is given by the equation. $\rho = \frac{\sqrt{3kTm}}{\lambda ne^2}$

Which means the resistivity is proportional to the square root of absolute temperature. But according to theory at room temperature it does not change up to 10K and in intermediate range of temperature ρ is proportional to T^5.

4. The conductivity of semiconductors and insulators cannot be explained by the free electron theory.

2) What are the assumptions of quantum free electron theory? State its drawbacks.

In 1929, Somerfield stated to apply quantum mechanics to explain conductivity phenomenon in metal. He has improved the Drude - Lorentz theory by quantizing the free electron energy and retained the classical concept of free motion of electron at a random.

ASSUMPTIONS:-

- The electrons are free to move within the metal like gaseous molecules. They are confined to the metal due to surface potential.
- The velocity distribution of the free electrons is described by Fermi-Dirac Statistics because electrons are spin half particles.
- The free electrons would go into the different energy levels by following Pauli's exclusion Principle which states that no two electrons have same set of Quantum numbers.
- The motion of electrons is associated with a complex wave called matter wave, according to De-Broglie hypothesis.
- The electrons cannot have all energies but will have discrete energies according to the equation, $E = n^2 h^2 / 8ma^2$.

Drawbacks:

Conductivity: According to Quantum free electron theory, the conductivity of a metal is $\sigma = \mu ne$, here 'µ' is the mobility of electrons, 'n' is the free electron concentration and 'e' is the electron charge.

According to the above equation, polyvalent metals like Aluminum (Al) should be more conductive than mono valent metals like copper (Cu). But experimentally it is not so.

Hall coefficient: According to the free electron theory, the hall coefficients for all metals is negative where as there are certain metals like Be, Cd, Zn for which the Hall coefficient is +

ve. Free electron theory could not explain why certain substances behave as insulators and some other substances as semiconductors; in spite of they have free electrons in them.

3. Define Fermi energy level. Explain Fermi Dirac distribution function.

Energy levels – Fermi Dirac Distribution:
According to the Quantum theory quantization leads to discrete energy levels. The electrons are distributed among these energy levels according to Pauli's exclusions principle i.e., it allows a maximum number of two electrons with spins in opposite directions in any energy level. The pair of electrons, one with sign up and the other with spin down occupy the lowest energy level. The next pair occupies the next level. This process goes on until all the electrons in the metal occupy their position.

- The highest energy level that can be occupied by an electron at 0 K is called Fermi energy level. It is denoted by E_F .

When the metal is not under the influence of an external field, all the levels above the Fermi energy level are empty; those lying below are completely filled.

Fermi – Dirac Distribution:
When the material is at a temperature higher than OK, it receives thermal energy from surroundings i.e. electrons are thermally excited. As a result, they move into the higher energy levels which are unoccupied at OK. The occupation obeys a statistical distribution called Fermi – Dirac distribution law.

According to this distribution law, the probability F(E) that a given energy state E is occupied at a temperature T is given by $\dfrac{1}{\exp(E-Ef)/KT + 1}$

Here F(E) is called Fermi – Dirac probability function. It indicates that the fraction of all energy state (E) occupied under thermal equilibrium 'K' is Boltzmann constant.

4) Explain the motion of an electron in periodic potential using Bloch theorem? (or) Explain Band theory of solids in detail. (or) Discuss the Kronig- penny model for the motion of an electron in a periodic potential.

Electrons in a periodic potential –Bloch Theorem:
An electron moves through + ve ions, it experiences varying potential. The potential of the electron at the +ve ions site is zero and is maximum in between two +ve ions sites.
The potential experienced by an e⁻, when it passes though +ve ions shown in fig.
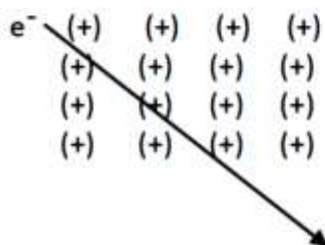
i.e. The potential experienced by an e⁻, in shown in Fig known as real periodic potential variation.
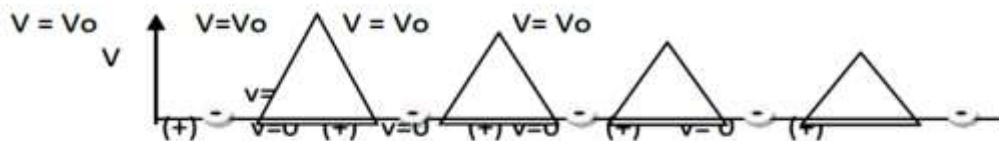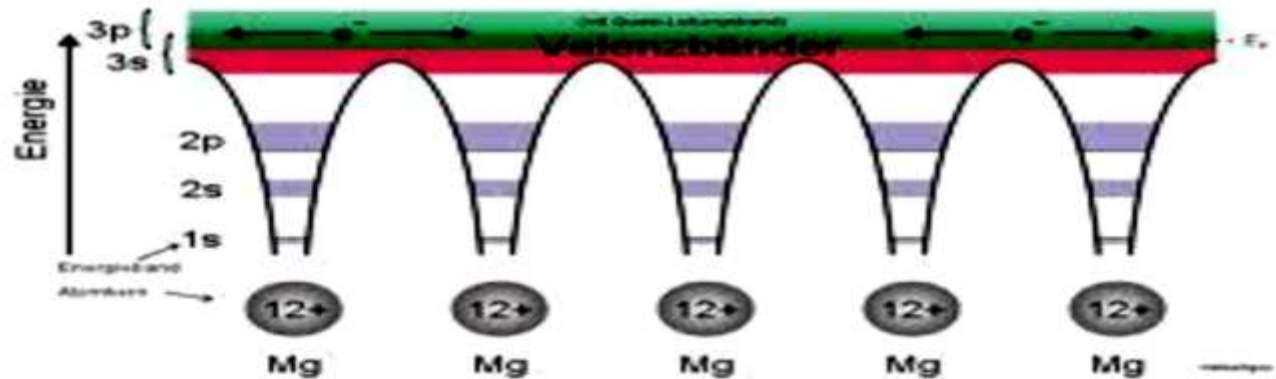
To study the motion of e^- in lattice and the energy states it can occupy, Schrodinger equation is necessary. Kronig-penny introduced a simple model for the shape of potential variation. The potential inside the crystal is approximated to the shape of rectangular steps.

KRONIG- PENNY MODEL:-

Kronig – penny consider a periodic arrangement of potential walls and barriers to represent the potential variation exhibited by the e^-, known as ideal-periodic square well potential as shown in figure. New forms of boundary conditions are developed to obtain a simple solution known as cyclic or periodic boundary conditions.
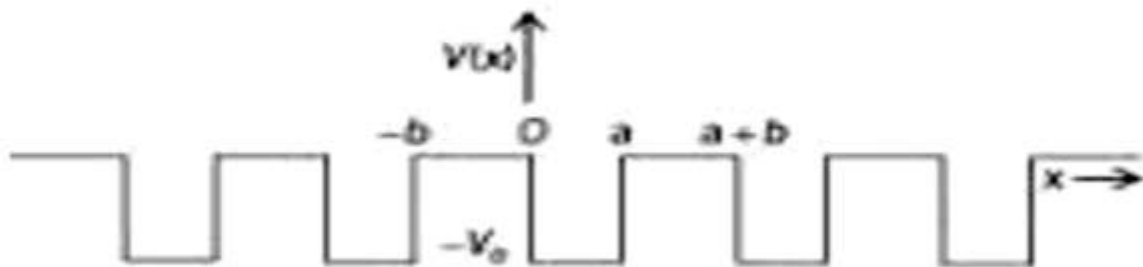
The wave functions associated with this model can be calculated by solving Schrödinger's eq for two regions 1 and 2.

ie $\frac{d^2\varphi}{dx^2} + \frac{8\pi^2 m}{h^2} E \varphi = 0$, $0 < x < a$

$\frac{d^2\varphi}{dx^2} + \alpha^2 \varphi = 0$, $\alpha^2 = \frac{8\pi^2 m}{h^2} E$

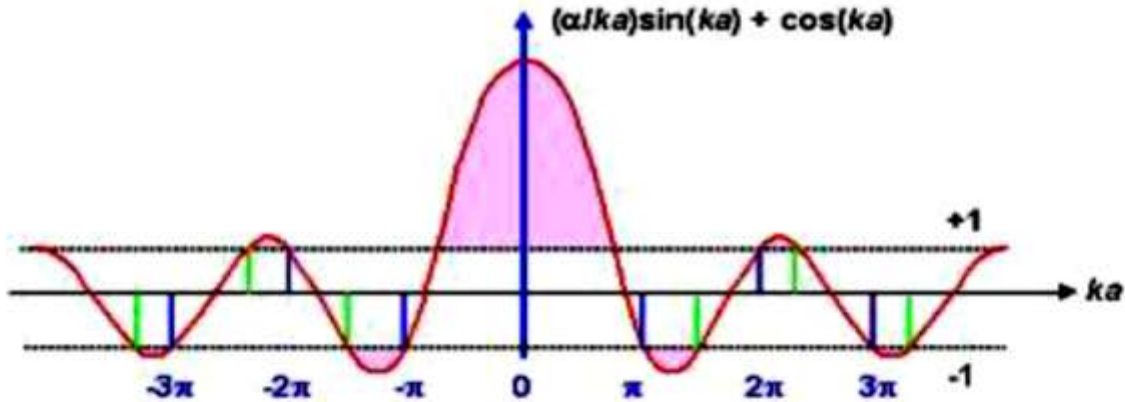$\frac{d^2\varphi}{dx^2} = \frac{8\pi^2 m}{h^2}(v_0 - E)\varphi = 0$, $-b < x < o$

$\frac{d^2}{dx^2} - \beta^2 \varphi = 0$, $\beta^2 = \frac{8\pi^2 m}{h^2}(v_0 - E)$

These two eqs are solved by using Bloch and Kronig-penny models, and applying boundary conditions the solution is

$p \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$ 　　　　　　　1

Here $p = \frac{mv_0 ba}{h^2}$ is scattering power

And 'v_0 b' is known as barrier strength.

$(\alpha/ka)\sin(ka) + \cos(ka)$

Conclusion from Kronig –Penny Model:

1) The energy spectrum of e⁻ consists of an infinite number of allowed energy bands separated by intervals in which, there are no allowed energy levels. These are known as forbidden regions.

2) When αa increase, the first term of eq(1) on LHS decrease , so that the width of the allowed energy bands is increased and forbidden energy regions become narrow.

3) The width of the allowed band decrease with the increase of p value. When p → α, the allowed energy regions become infinity narrow and the energy spectrum becomes line spectrum.

5. Explain the origin of energy band formation in solids based on band theory.

Energy band Formation in solids:

In isolated atom, the e⁻s are tightly bound and have discrete, sharp energy levels.

$$\underline{\quad\quad\bullet\quad\quad}\ E_1$$

When two identical atoms are brought closer the outermost orbits of these atoms overlap and interacts with the wave functions of the e⁻s of the different atoms, then the energy levels corresponding to those wave functions split in to two.

$$\underline{\quad\quad\quad\quad\quad}\ E_1$$
$$\underline{\quad\bullet\bullet\quad}\ E_2$$
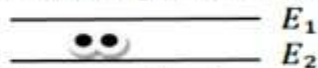
If more atoms are brought together more levels are formed and for a solid of N atoms, each of these energy levels of an atom splits into N levels of energy.

N atoms

N Energy levels

- The levels are so close together that they form almost continuous band.
- The e⁻ first occupies lower energy bands and are of no importance in determining many of the physical properties of solid.
- These e⁻ present in higher energy bands are important in determining many of the physical of solids.
- These two allowed energy bands are called as valence and conduction bands.
- The band corresponding to the outermost orbit is called conduction band and the gap between those two allowed bands is called forbidden energy gap or band gap.

6. What is effective mass of an electron? Derive an expression for the effective mass of an electron.

Effective mass of the electron moving in a crystal lattice:
- Consider a crystal (metal) subjected to an electric field 'E', so the force experienced by an electron of charge 'e' is Ee.
- Acceleration of the electron in the crystal is given by $a = F/m = Ee/m$
- But acceleration of the electron is not constant because of the velocity changes i.e., the electron move faster near the +ve ions in the crystal. Since the electric field and charge of the electron are invariant, the effective mass m* of the electron change accordingly.
  i.e $F = m^*a$ ————————(1)
- Consider a particle velocity 'v' is equal to group velocity 'v_g' of a wave packet, then

$V = v_g = \dfrac{dw}{dk}$, w = angular frequency, k = wave propagation vector

$W = 2\pi v$ $dw = 2\pi dv$

Frequency of the complex wave $v = E/h$ $dv = 1/h\, dE$

$dw = 2\pi\, dE/h = dE/\hbar$

$\therefore V = dw/dk = 1/\hbar \; dE/dk$

SO, $a = \dfrac{dk}{dt} = \left(\dfrac{1}{\hbar}\right) \dfrac{d^2E}{dk\, dt}$

$a = \dfrac{1}{\hbar} \dfrac{d^2E}{dk^2} \left(\dfrac{dk}{dt}\right)$

Wave propagation vector $k = \dfrac{2\pi}{\lambda}$

$$k = \dfrac{2\pi}{h} P = \dfrac{p}{\hbar}$$

'P' is momentum, 'λ' is de-Broglie wavelength.

$\therefore \dfrac{dk}{dt} = \dfrac{1}{\hbar} \left(\dfrac{dp}{dt}\right) = \dfrac{F}{\hbar}$.

Since $\dfrac{dp}{dt}$ is the rate of change of momentum, which is nothing but force 'F'.

$a = \dfrac{1}{\hbar} \dfrac{d^2E}{dk^2} \dfrac{1}{\hbar} F$
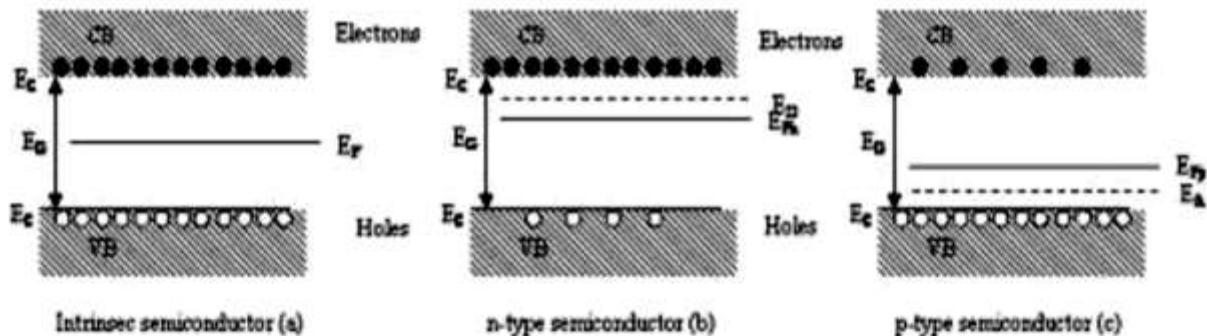
$a = \dfrac{F}{\hbar^2} \left(\dfrac{d^2E}{dk^2}\right)$

i.e. $F = \dfrac{\hbar^2}{\frac{d^2E}{dk^2}} a$ (2)

Compare 1 & 2

Effective mass $m^* = \dfrac{\hbar^2}{\frac{d^2E}{dk^2}}$

7. Derive an expression for carrier concentration of intrinsic semiconductors?

Intrinsic Semi conductors:

Intrinsec semiconductor (a)   n-type semiconductor (b)   p-type semiconductor (c)

- A semi conductor in which holes in the valance band and electrons in the conduction band are solely created by thermal excitations is called intrinsic semiconductors i.e., A pure semi-conductor is considered as intrinsic semiconductor.
- The no. of electrons moving into the conduction band is equal to the no. of holes created in the valence band.
- The Fermi level lies exactly in the middle of forbidden energy gap.
- Intrinsic semi-conductors are not of practical use in view of their poor conductivity.

Carrier concentration in intrinsic semi-conductors:

- In the conduction band, the level density D (E) at an energy E is given by the expression.

$$D(E)dE = \frac{4\pi}{h^3}(2m)^3 /^2 E^1/^2 dE$$

- The probability of an energy level filled with electrons is given by Fermi-Dirac function.

$$F(E) = \frac{1}{1+\exp(\frac{E-EF}{KT})}$$

- The no of electrons 'n' filling into energy level between the energies E and E+dE is

$$n = D(E) F(E) dE$$

- $n = \frac{4\pi}{h^3}(2me)^3/^2 E^1/^2 \frac{1}{1+\exp(\frac{E-EF}{KT})} dE$

- In the above expression, mass of the electron 'm' is replaced with effective mass m_e^* and factor '2' for the two possible spins of the electrons.
- The number of electrons in the conduction band is obtained by making integration between the limits Ec to ∞. Since minimum energy in the conduction band is Ec and at the bottom of the conduction band we write E- Ec for E

$$n = \frac{4\pi}{h^3}(2m_e^*)^3/^2 \int_0^\infty \frac{(E-E_c)^{\frac{1}{2}}}{1+\exp(\frac{E-E_F}{KT})} dE$$

For all possible temperatures E-E_F >>>> KT

Hence F (E) = exp - $(\frac{(E-E_F)}{KT})$ = exp $(\frac{E_F-E}{KT})$

Equation 1 becomes

$n = \frac{4\pi}{h^3}(2m_e^*)^3/^2 \int_{E_c}^\infty (E-Ec)1/2 \exp(\frac{EF-E}{KT})dE$

$n = \frac{4\pi}{h^3}(2m_e^*)^3/^2 \exp(E_F/KT)\int_{E_c}^\infty (E-Ec)1/2 \exp(\frac{-E}{KT})dE$

To solve this Integral Part

E-Ec = x

E = Ec+x

dE = dX

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp(E_F/KT) \int_0^\infty x^{\frac{1}{2}} \exp -\left(\frac{Ec+x}{KT}\right) dx$$

$$n = \frac{4\pi}{h^3} (2m_e^*)3^{/2} \exp\left(\frac{EF-Ec}{KT}\right) \int_0^\infty x^{\frac{1}{2}} \exp -\left(\frac{x}{KT}\right)$$

Using gamma function, it can be shown that

$$\int_0^\infty x^{\frac{1}{2}} \exp -\left(\frac{Ec+x}{KT}\right) = (KT)^{3/2} \frac{\pi}{2}^{\frac{1}{2}}$$

Hence, $n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{EF-Ec}{KT}\right) (KT)^{3/2} \frac{\pi}{2}^{\frac{1}{2}}$

No of electrons per unit volume is given by

$$n = 2\left(\frac{2\pi m_e^* KT}{h^2}\right) \exp\left(\frac{E_F-Ec}{KT}\right)$$

The expression for no of holes in the valance band is given by the expression

$$P = 2\left(\frac{2\pi m_p^* KT}{h^2}\right) \exp\left(\frac{Ev-E_F}{KT}\right)$$

In Intrinsic semi conductor n=p then the Intrinsic carrier concentration is n=p=ni;

$$ni^2 = 4 \left(\frac{2\pi KT}{h^2}\right)^3 (m_e^* m_p^*)^{3/2} \exp\left(\frac{Ev-Ec}{KT}\right)$$

$$ni^2 = 4 \left(\frac{2\pi KT}{h^2}\right)^3 (m_e^* m_p^*)^{3/2} \exp\left(\frac{-Eg}{KT}\right)$$

Here Ec - Ev = Eg (forbidden energy gap)

Hence $ni = 2 \left(\frac{2\pi KT}{h^2}\right)^{3/2} (m_e^* m_p^*)^{\frac{3}{4}} \exp\left(\frac{-Eg}{2KT}\right)$

Fermi Level: In Intrinsic semi conductor n=p and assuming the effective mass of e and hole to be same, i.e. $m_e^* = m_p^*$

$$\text{Exp}\left(\frac{Ef-Ec}{KT}\right) = \exp\left(\frac{Ev-Ef}{KT}\right)$$

$E_F - Ec = Ev - E_F$

$2 E_F = Ev + Ec$

$$E_F = \frac{Ev+Ec}{2}$$

Thus the Fermi level is located half way between the valance band and conduction band and its position is independent of the temperature.

8. Derive an expression for carrier concentration in n-type extrinsic semiconductors?

- When pentavalent impurities like P, As, Sb is added to the intrinsic semi-conductors, resultant semi conductor is called N-Type semi-conductor.
- The concentration of free electrons is more when compared to concentration of holes.

Expression for carriers' concentration in N-type semi conductors:

- In this type of semi conductor, there will be donor levels formed at an energy Ed.
- N_d represents no. of impurities per unit volume of semi conductor.
- At low temperature all donor levels are filled with electrons, with increase of temperature, more and more donor atoms get ionized and the density of electrons in the conduction band increases.
- Density of electrons in the conduction band is given by

$$n = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{EF-Ec}{KT}\right) \to 1$$

- The Fermi level (EF) lies in between Ed & Ec
- The density of empty donor levels is given by

$$Nd [1-F(E_d)] \approx N_d \left[1-1+\exp\left(\frac{Ed-Ec}{KT}\right)\right] = Nd [1-F(E_d)] \approx Nd \exp\left(\frac{Ed-Ec}{KT}\right)$$

- At low temperature, there are no excitations of the electrons from donor level to the conduction band.
- Hence, density of empty donors and the electron density in conduction band should be same

    i.e. $2\left(\dfrac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\dfrac{Ef-Ec}{KT}\right) = N_d \exp\left(\dfrac{Ed-Ef}{KT}\right)$

- Taking log on both the sides & rearranging

    $\left(\dfrac{Ef-Ec}{KT}\right) - \left(\dfrac{Ed-Ef}{KT}\right) - \log N_d - \log 2\left(\dfrac{2\pi m_e^* KT}{h^2}\right)^{3/2}$

    $E_F - Ec - Ed/KT = \log\left(\dfrac{N_d}{2\left(\frac{2\pi m_e^* KT}{h^2}\right)3/2}\right)$

    $2Ef - (E_c + E_d)\, KT \log\left(\dfrac{N_d}{2\left(\frac{2\pi m_e^* KT}{h^2}\right)3/2}\right) \rightarrow 2$

- At absolute zero $E_F = \dfrac{Ec+E_d}{2}$

    i.e. Fermi level lies exactly at the middle of donor level E_d and the bottom of the Conduction band E_c.

    Substituting equation 2 in eqn. 1 & re-arranging,

    $N = (2^{Nd})^{1/2} \left(\dfrac{2\pi m_e^* KT}{h^2}\right)^{3/4} \exp\left(\dfrac{E_d - Ec}{2KT}\right)$

- Hence the density of the electrons in the conduction band is proportional to the square root of the donor concentration.

9. Derive an expression for carrier concentration in p-type extrinsic semiconductors?

- P-type semi-conductors are fabricated by addition of trivalent atoms like Al as impurity to the intrinsic semi-conductor.
- Hence, holes are majority charge carriers and free electrons are minority charge carriers.

Expression for Carrier concentration in P type semi-conductors:

- In this type of semi-conductor, there will be there will be acceptor levels formed at an energy Ea.
- Na represents no. of impurities per unit volume of semi-conductor.
- At low temperatures, all the acceptor levels are empty.
- With increase of temperature, acceptor atoms get ionized i.e. the electrons moves from valance band and occupy the vacant sites in the acceptor energy levels, there by leaving holes in the valence band
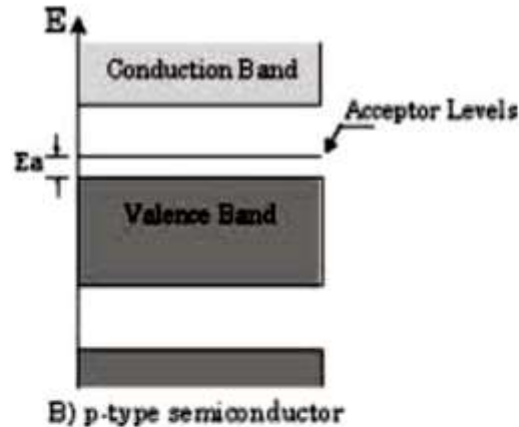- Density of holes in the valance band is given by

    $P = 2\left(\dfrac{2\pi m_p^* KT}{h^2}\right)^{3/2} \exp\left(\dfrac{Ev - Ef}{KT}\right)$

- Since Ef lies below the acceptor levels, the density of ionized acceptors is given by

    $N_a\, F(E_a) = N_a \exp\left(\dfrac{Ef - Ea}{KT}\right)$

- Hence, density of holes in the valance band is equal to the density of ionized acceptors.

    $2\left(\dfrac{2\pi m_p^* KT}{h^2}\right)^{3/2} \exp\left(\dfrac{Ev - Ef}{KT}\right) = N_a \exp\left(\dfrac{Ef - Ea}{KT}\right)$

B) p-type semiconductor

i.e. $2^{\frac{Ev-Ef-Ef+Ea}{KT}} = \dfrac{Na}{2\left(\frac{2\pi m_p^* KT}{h^2}\right)3/2}$

- Taking log, $\dfrac{Ev+Ea-2Ef}{KT} = \log \dfrac{Na}{2\left(\frac{2\pi m_p^* KT}{h^2}\right)3/2} \rightarrow 2$

- At 0° K, $E_f = \dfrac{Ev+Ea}{2}$

  i.e. At 0 K, Fermi level lies exactly at the middle of the acceptor level and in the top of the valance band.

- Sub. eqn. 2 in eqn. 1 & re-arranging, $P = (2Na)\; \frac{1}{2} \left(\frac{2\pi m_p^* KT}{h^2}\right)^{3/4} \exp\left(\frac{Ev-Ea}{KT}\right)$

- Thus the density of the holes in the valance band is proportional to the square root of the acceptor concentration.

10. Explain Hall Effect in detail? What are its applications?
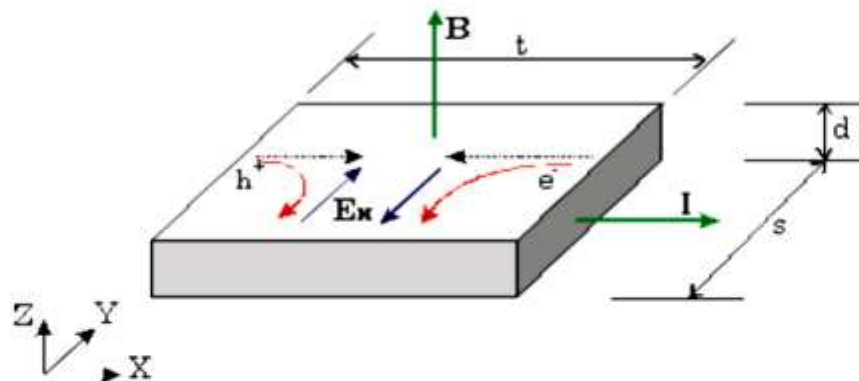
Hall-Effect:
- When a material carrying current is subjected to a magnetic field in a direction perpendicular to direction of current, an electric field is developed across the material in a direction perpendicular to both the direction of magnetic field and current direction. This phenomenon is called "Hall-effect".

Explanation:
- Consider a semi-conductor, and current passes along the X-axis and a magnetic field Bz is applied along the Z-direction, a field Ey is called the Hall field which is developed in the Y-direction.

- In P-tpe semi-conductor, holes move with the velocity "V" in the "+"ve X-direction. As they move across the semi conductor the holes experience a transverse force 'Bev' due to the magnetic field.
- This force drives the holes down to the lower face. As a result, the lower face becomes +vely charged and –ve charge on the upper surface creating the hall field in the Y-direction. The Hall field exerts an upward force on holes equal to Ee.
- In the steady sate, two forces just balance and as a result, no further increase of + ve charge occurs on Face1.
- In N type semiconductor, the majority charge carriers are electrons experiences a force in the downward direction and lower face gets – vely charged. As a result, Hall field will be in the Y – direction.

Demonstration:

- Consider a rectangular slab of n-type semi conductor carrying current in the + ve X-direction.
- If magnetic field "B' is acting in the Z-direction as shown then under the influence of magnetic field, electrons experience a force given by F_L = -Bev.
- As a result of force F_H acting on the electrons in the Y – direction as a consequence the lower face of the specimen gets – vely charged and upper surface is + vely charged.
- Hence a potential V_H called the Hall Voltage present between the top and bottom faces of the specimen.
- The Hall field F_H, exerts an upward force on the electrons given by F= -eE_H
- The two opposing forces F_L and F_H establish an equilibrium under which

$$|F_L| = |F_H| \quad \text{i.e} \quad -Bev = -eE_H$$

$$E_H = BV$$

- If 'd' is the thickness of the specimen, then $E_H = \frac{V_H}{d}$

$$V_H = E_H d$$
$$V_H = Bvd$$

- If 'W' is the width of the specimen, then $J = \frac{1}{wd}$

$$J = nev = \rho V$$
$$=> V_H = \frac{Bid}{\rho wd} = \frac{Bi}{pw}$$

Hall Coefficient:

- Hall field E_H, for a given material depends on the current density J and the applied magnetic field B.

$$\text{i.e.} \quad E_H \alpha JB$$
$$E_H = R_H \, \alpha \, JB$$

Since, $V_H = \frac{Bi}{\rho w}$, $E_H = \frac{V_H}{d}$

$$E_H = \frac{Bi}{\rho wd}$$

$$J = \frac{i}{wd}, \frac{Bi}{\rho wd} = R_H = (\frac{i}{wd}) B$$

i.e $R_H = \frac{1}{\rho}$

Applications:
Determination of the type of Semi-conductors:

The Hall coefficient R_H is -ve for an n-type semiconductor and +ve for p-type semiconductor. Thus the sign of Hall coefficient can be used to determine whether a given Semi-conductor is n or p-type.

Calculation of carrier concentration:

$$R_H = \frac{1}{\rho} = \frac{1}{ne} \text{ (for e}^- \text{ s)}$$

$$R_H = \frac{1}{\rho e} \text{ (for holes)}$$

$$\Rightarrow n = \frac{1}{eR_H}$$

$$\Rightarrow \rho = \frac{1}{eR_H}$$

Determination of Mobility:

If the conduction is due to one type carriers, ex: electrons

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} = \sigma R_H$$

$$\mu = \sigma R_H$$

Measurement of Magnetic Flux Density:

Hall Voltage is proportional to the magnetic flux density B for a given current I. so, Hall Effect can be used as the basis for the design of a magnetic flux density metal.