Shree H.N. Shukla College of Science

M.Sc. (Mathematics) Sem-3

Prelims Test

MATH.CMT-3002: Functional Analysis

[Time: 2.5 Hours] [Total Marks: 70]

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1 Answer any seven 7x2=14

(a) Define $\|\cdot\|_p$ on $L^p[a,b]$ space and $\|\cdot\|_\infty$ on $L^\infty[a,b]$ space.

(b) Define C_0 and C_{00}

(c) State Riez-Reprasention theorem for Hilbert Space.

(d) Give an example of n.l. space which is complete.

(e) Define norm on B(X,Y).Where X and Y are norm linear spaces. .

(f) Prove that Every orthonormal set in inner product space is L.I.

(g) Given example of Hilbert Space.

(h) $(S+T)^*=S^*+T^*$, Where S and T are bounded linear operators.

(i) $(ST)^*= T^* S^*$, Where S and T are bounded linear operators.

(j) $(\alpha T)^*= \bar{\alpha}T^*$

2 Answer any two 2x7=14

(a) Every finite dimensional n.l.s over $\mathbb{K}$ is Banach space.

(b) Show that (C[a,b], $\|\cdot\|_\infty$) is Banach space

(c) Show that $(l^p, \|\cdot\|_p)$ is Banach space.

3 Answer any four

(a) State and prove Riesz lemma. 7

(b) State and prove Riesz-Reprasentation theorem for bounden sesquilinear mapping on Hilbert Space. 7

(c) Let X and Y are norm linear spaces and Y is Banach Sapce then Show that B(X,Y) is Banach Space. 7

(d) State and prove Schwarz inequality.

(e) Prove that $(\mathbb{R}^n, \|\cdot\|)$ is Banach space. Where $\|\cdot\|$ is Euclidean norm 7

4 All are compulsory

(a) State and Prove Bessel's inequality. 7

(b) State and Prove Hahn-Banach theorem for norm linear space. 7

or

(b) An orthonormal set M in Hilbert space H is total in H if and only if for all $x \epsilon H$ the Perseval relation holds. 7

BEST OF LUCK