Sem-4 March 2025. M.Sc. Maths.

Seat No. ______

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 4 MARCH 2025 (Regular) MARCH - 2025

NUMBER THEORY 2

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-04-00003

Time : 2.30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Define Pythagorean triplet with an example.

2 Find an integer n>1, such that $n^2 + (n+1)^2$ is a perfect square.

3 Find the value of <2,2,…,2,…> .

4 Define Pell's equations with an example.

5 Find continued fraction expansion for 253/51.

6 Write any two Primitive Pythagorean triplets (x,y,z), where z>100.

7 If n is a smallest positive integer such that 2/27 appears in the n^{th}- row of Farey fraction then n = ____.

8 Define Periodic Continued Fraction.

9 Find the general solution of the Diophantine equation 3x+15y = 8. (if exist).

10 Define Quadratic irrational.

Q.2 Answer the following (Any Two) 14

1 Prove that, there is a polynomial $f_n(x)$ of degree n with leading coefficient 1 such that $f_n(2\cos\theta) = 2\cos n\theta$, $\forall n \in \mathbb{N}$.

2 Let $< a_0, a_1, \ldots, a_n, \ldots >$ be the infinite simple continued fraction. Let h_n, k_n to be define in standard notation and $\theta > 1$ be any real number. Prove that, $< a_0, a_1, \ldots, a_{n-1}, \theta > = \frac{\theta h_{n-1} + h_{n-2}}{\theta k_{n-1} + k_{n-2}}$

3 Prove that, π is irrational number.

Q.3 Answer the following 14

1 Let x be a real number and $x > 1$. If $x + x^{-1} < \sqrt{5}$, then prove that, $x < \frac{\sqrt{5}+1}{2}$ and $x^{-1} > \frac{\sqrt{5}-1}{2}$.

2 Let ρ and σ are two positive real numbers. Suppose $\sigma < \sqrt{\rho}$ and (s,t) is a positive solution of $x^2 - \rho y^2 = \sigma$ with $(s,t) = 1$. Prove that, $s = h_n$ and $t = k_n$, for some n.

OR

Answer the following 14

1 Prove that, the equation $15x^2 - 7y^2 = 9$, does not have any integer solution.

2 State and prove, Hurwitz Inequality for Farey Fraction.

Q.4 Answer the following questions (Any Two)

1 Find the solution of the equation $x^2 - 18y^2 = 1$ as well as $x^2 - 18y^2 = -1$, if exist.

2 Let r and s be two positive integers such that (i) $r > s \geq 1$. (ii) $\gcd(r,s) = 1$. (iii) r is odd, s is even OR r is even, s is odd then for $x = r^2 - s^2$, $y = 2rs$ and $z = r^2 + s^2$. Prove that, (x, y, z) is primitive Pythagorean triplet.

Q.5 Answer the following (Any Two)

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with integer coefficients of degree n. Suppose $\frac{a}{b}$ is a rational number with $b > 0$ and $(a,b) = 1$ such that $f(\frac{a}{b}) = 0$. Prove that, b divides a_n as well as a divides a_0.

Let θ be an irrational and $\frac{a}{b}$ is a rational number with b > 0, (a, b) = 1 and $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$. Prove that $\frac{a}{b} = \frac{h_n}{k_n}$, for some $n \in \mathbb{N}$.

Find the values of <7,4,8,4,8,……> and <3,3,3,……>.

Define, Diophantine equation. Find the general solution of Diophantine equation 147x + 258y = 369.