Shree H.N.Shukla College of Science Rajkot

MATHEMATICS

T.Y.B.Sc. (Sem.VI) (CBCS)

PRELIMS EXAM

PAPER- 601

Graph Theory & Complex Analysis-II

Time: 2.5 hour] **[Total Marks: 70**

Instruction: (i) All questions are compulsory.

(ii) Figures to the right indicate full marks of the question.

1. (A) Answer the following: **[04]**

1) Write a degree of a pendent vertex.
2) Define: Simple Graph
3) Find the Nullity of connected graph with 4 vertices and 8 edges.
4) Write the number of internal vertices in a binary tree with 13 vertices.

(B) Attempt any one: **[02]**

1) Prove that the number of vertices n in a binary tree is always odd.
2) Define: (i) Circuit (ii) Minimally connected graph

(C) Attempt any one: **[03]**

1) Prove that a graph is a tree iff it is minimally connected.
2) State and prove first theorem of Graph theory.

(D) Attempt any one: **[05]**

1) Prove that a Simple graph with n-vertices and k-components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.
2) Explain Konigsberg Bridge Problem.

2. (A) Answer the following: [04]

1) Define: Acyclic Digraph
2) Define: Self dual graph
3) What is the chromatic number of a complete graph with 5 vertices?
4) Regions of a connected planar graph with 4 vertices and 6 edges are _____.

(B) Attempt any one: [02]

1) Define: Path Matrix
2) Define Cut-set vector with example.

(C) Attempt any one: [03]

1) A connected simple planner graph G with n-vertices, e-edges and f-region then prove that (i) $e \geq \frac{3}{2} f$

   (ii) $e \leq 3n - 6$

2) Prove that Complete graph K_4 is Self dual graph.

(D) Attempt any one: [05]

1) Prove that the complete graph of 5-vertices is non-planner graph.
2) Define: Incidence Matrix and its properties

3. (A) Answer the following: [04]

1) Define: Power series
2) Define: Complex Series
3) State Maclaurin series of an analytic function f(z).
4) Find radius of convergence of series $\sum \frac{z^n}{2^{n-1}}$.

(B) Attempt any one: [02]

1) Expand $\frac{1}{1+z}$ in Maclaurin's series.
2) Discuss about the convergence of series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{4^n (n+1)^3}$.

(C) Attempt any one: **[03]**

1) Expand $\frac{1}{z(z^2-3z+2)}$ in Laurent's series for (i) 1<|z|<2 (ii) 0<|z|<1
2) Expand e^z in term of (z-1).

(D) Attempt any one: **[05]**

1) State and prove Taylor's infinite series of an analytic function f(z).
2) Prove that

$$\cosh\left(z + z^{-1}\right) = a_0 + \sum_{n=1}^{\infty} a_n\left(z^n + z^{-n}\right)$$

Where,

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} \cosh(2\cos\theta)\cos n\theta \, d\theta$$

4. (A) Answer the following: **[04]**

1) Define: Residue of f(z) at pole Z_0.
2) Find $Res\left(\frac{\cos z}{z}, 0\right)$
3) Write an isolated singular point for $f(z) = \frac{1}{z-2}$.
4) Define: Singular point

(B) Attempt any one: **[02]**

1) Find the residue of $f(z) = \frac{z+2}{(z-1)(z-2)}$ at Simple pole.
2) Find Res(f(z), 1), where

$$f(z) = \frac{e^{2z}}{(z-1)^2}$$

(C) Attempt any one: **[03]**

1) Obtain the formula for finding the residue of f(z) at m^{th} order pole.
2) Find the value of integral $\int_C \frac{dz}{Z^3(Z+4)}$ $where\ C: |Z| = 2$

(D) Attempt any one: **[05]**

1) Prove that

$$\int_0^\infty \frac{dx}{(x^2+a^2)^{n+1}} = \frac{\pi(2n)!}{(n!)^2(2a)^{2n+1}}\ , where\ a > 0.$$

2) State and prove Cauchy residue theorem.

5. (A) Answer the following: **[04]**

1) Define: Linear mapping
2) Find fixed point of the bilinear transformation $w = \frac{3z-4}{z-1}$.
3) Define: Mobius mapping
4) Find the critical point of $w = \frac{1}{z-1}$.

(B) Attempt any one: **[02]**

1) Show that x+y=2 transform into the parabola $u^2=-8(v-2)$ under the transformation $W=Z^2$.
2) Find critical point of $w = \frac{z-1}{z+1}$.

(C) Attempt any one: **[03]**

1) Prove that the transformation $w=2z+z^2$ maps the unit circle $|z|=1$ of z-plane into cardiod to w-plane.
2) Show that the composition of bilinear maps is again a bilinear.

(D) Attempt any one: **[05]**

1) Discuss the bilinear mapping $W=Z^2$.
2) Discuss the mapping of $W=e^z$ in Cartesian system.

******BEST OF LUCK******