SHREE H.N.SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics)
Semester-II

IMP Questions of PDE

Q-1 Short Answer Questions.

1. Find the integral curve of the equation $\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{z}$.
2. Find the direction cosines of the normal to the surface $2x + 3y + 5z = 7$ at the point $(1, 0, 1)$.
3. Verify that the equation $ydx - xdy + 2y^2dz = 0$ is integrable.
4. Verify that the differential form $\frac{dx}{x^2} = \frac{dy}{y^4} = \frac{dz}{z^3}$ is an exact differential .
5. Eliminate the constants a and b from the equation $(x - a)^2 + (y - b)^2 + z^2 = 1$
6. If the equations $\frac{dx}{(x-y)y^2} = \frac{dy}{(y-x)x^2} = \frac{dz}{(x^2+x^2)z}$ have integrals $x^3 + y^3 = c_1, \frac{x-y}{z} = c_2$, then determine the particular solution to $(x - y)y^2p + (y - x)x^2q = (x^2 + x^2)z$ through the curve $xz = 27. y = 0$.
7. Find the complete integral of the equation $pq = 9$.
8. If $z = f(x + 5iy) + g(x - 5iy)$ where the function f and $g$are arbitrary, then prove that $25\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
9. Find the particular solution to $(D - 1)^3z = e^{3x+2y}$.
10. Find the solution to the Pfaffian differential equation $zy^4dx + zx^4 - x^4y^4dz = 0$.
11. Write down the set of parametric equation of sphere.
12. Define: a) equation of Normal Line and b) Complete Integral
13. Solve : $f\left(x + y, x - \sqrt{z}\right)$.
14. Define integrating factor of a Pfaffian differential equation with an example.
15. Determine the direction ratios at point P to the curve of intersection of $ax^2 + by^2 + cz^2 = 6$ and $x + y + z = 3$.
16. Determine the envelope of two parameter system of the surfaces $z^2 + (y - b)^2 + (x - a)^2 = 3$.
17. State the genera form of Wave equation and Laplacian equation.
18. Solve $\left(D^3 - 7DD'^2 + 6D'^3\right)z = 0$.
19. Justify whether the equation $2uvdu + uv^2dv = 0$ is integrable or not?
20. Find the complete integral of $pqz = p^2(xq + p^2) + q^2(yp + q^2)$.

21. Define: Complementary function and singular solution.
22. Verify the equation $z = \sqrt{zy+b} + \sqrt{2x+a}$ is the solution of $z = \frac{1}{p} + \frac{1}{q}$.
23. State Liptchiz condition for the function of three variable(x, y, z) from the point (a, b, c).
24. Find the complete integral of $p^3 + q^3 = 3$.
25. Find the direction cosines of the normal to the surface $4x - 6y - 10z = 7$ at the point (2,1,1).
26. Verify the equation is exact or not $y^2dx + x^2dy + 3x^2dz = 0$.
27. Determine the envelope of the two parameters system of surfaces$(x-a)^2 + (y-b)^2 + z^2 = 1$.
28. Define : Tangent plane and Pffafian Differential form.

Q-2 Long answer Questions.

1. State and prove the necessary and sufficient condition under which the pfaffian differential equation (for three variables) is integrable.
2. Solve the partial differential equation $zq = (p^2 + q^2)y$ using Charpits method.
3. Find the general form of the complete integral of $f(u_x, u_y, u_z) = 0$ and illustrate for the method for the equation $u_x + u_y + u_z = u_x u_y u_z$.
4. Find the integral surface of $2x(y+z^2)p + y(2y+z^2)q = z^3$ and deduce the solution to the form $yz(yz + z^2 - 2y) = x^2$ provided $\frac{c_2+1}{{c_1}^2} = 1$.
5. Solve $\cos(x+y)\, p + \sin(x+y)\, q = z$.
6. Find the particular integral of $(3D^2 - 2D' + 6DD')z = \sin(x+2y)$.
7. Prove that $F(D, D')|e^{ax+by}| = e^{ax+by}F(a, b)$.
8. Solve the partial differential equation $z^2 = pqxy$ using Jacobi's method.
9. Prove necessary and sufficient condition that there exists between two function $u(x, y)$ and $v(x, y)$ a relation $f(u, v) = 0$ not involving x and y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$
10. Find the integral surface of the partial differential equation$(x-y)p + (y-x-z)q = z$ which passes through circle $x^2 + y^2 = 1$ and line $z = 1$.
11. Using Nattani's method solve the partial differential equation $z(z+y^2)dx + z(z+x^2)dy = xy(x+y)dz$.
12. Using Nattani's method solve the partial differential equation $2yzdx - 2xzdy -$

$(x^2 - y^2)(z-1)dz = 0.$

13. Find the particular integral of $\left(D^2 + 2D'^2 - 2DD'\right)z = \cos(x+y)$.

14. Classify the equation and convert it into canonical form $2r - 5s + 3t = x$.

15. Find the general solution of equation $\left(x^2D^2 - 2y^2D'^2 - xD - yD'\right)z = x - y$.

16. Find the particular integral of $\left(D^2 - D'^2\right)z = x^2 - y$.

17. If $(\alpha D + \beta D' + \gamma)^n$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')z = 0$ is $z = e^{\frac{-\gamma}{\alpha}x}\left(\phi_1(\beta x - \alpha y) + x\phi_2(\beta x - \alpha y) + \cdots + x^n\phi_n(\beta x - \alpha y)\right)$. Where $\phi_i = \phi_i(\xi)$ is an arbitrary function of single variable $(i = 1, 2, 3, \dots, n)$.

18. If $(\alpha D + \beta D' + \gamma)$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')z = 0$ is $z = e^{\frac{-\gamma}{\alpha}x}\left(\phi(\beta x - \alpha y)\right)$. Where $\phi = \phi(\xi)$ is an arbitrary function of single variable.

19. Find the general solution of equation $(D - D')(D + D')z = e^{2x-y}(x + 2y)$.

20. Find the primitive solution of $2y(a - x)dx + [(z - y)^2 + (a - x)^2]dy - ydz = 0$.

21. Solve the equation $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

22. Find the integral curves of the equation $\frac{dx}{cy-bz} = \frac{dy}{az-cx} = \frac{dz}{bx-ay}$ and show that they are circles.

23. Prove that for any non-zero functions $\mu = \mu(x, y, z)$ and $X = (P, Q, R)$ where P, Q, R are the function x, y, z then $X \cdot curlX = 0$ if and only if $(\mu X) \cdot curl(\mu X) = 0$.

24. Find the equation of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersection with the hyperbola of one-parameter system $xy = z + c$.

25. Solve the partial differential equation $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$.

26. Determine the partial differential equation from the relation $F(u, v) = 0$, where u and v function of x, y and z, with z is dependent of x and y.

27. Describe the Jacobi's method.

28. Solve $2(z + xp + yp) = yp^2$ using Charpits's method.

29. Find the orthogonal trajectories on the curve $x^2 + y^2 + 2fyz + d = 0$ of its curve intersection with the plane parallel to $x - y$.

30. Find the orthogonal trajectories on the curve $x^2 + y^2 = z^2 tan^2\alpha$ of its intersection with the family of plane parallel to $z = c$.

31. Prove that the equation $yz(y+z)dx + zx(z+x)dy + xy(x+y)dz = 0$ is integrable and find its solution.

32. Find the surface which intersects the system of surface $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$ and $z = 1$.

33. Find the general solution of $(x^2 + y^2)p + 2xyq = (x+y)z$.

34. Find the integral surface of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through $x = 1$ and $y = 0$.