SHREE SYAMJI KRISHNA VARMA B.ED. COLLEGE.

RAJKOT

(Affiliated To Saurashtra University & NCTE)

Mathematic Method Book B.ed. Sem-1

Unit .1. Mathematic education's value and Mathematician.

1.1. Value of Teaching Mathematics

1.cultural value.

Mathematics has got immense cultural value and this value is steadily increasing day by day. It is said – "mathematics is the father of civilization" It has made a significant contribution in bringing he man to such an advanced stage of development. The prosperity of man and his cultural advancement have depended considerably upon the advancement of mathematic.

Modern civilization owes it advancement to the progress of various occupation such as agriculture, engineering, aviation, navigation, medicine etc .Mathematic has contribution extensively to the advancement of these occupation. It shapes our culture as a playback pioneer. Modern materialistic attitude in everything is perhaps the outcome of the deep influence of mathematics on life and culture.

The history of mathematics reflects the civilization and culture of different countries at different time. The greatness of Indian culture and heritage can be viewed from the glory of Indian Mathematics of olden day.

Apart from acquainting us without civilization and culture mathematic helps in there preservation, transmission , and promotion. Mathematics is also a pivot for culture arts like music, poetry, and painting .It might not be altogether a matter of chance that the Greeks, the greatest geometers were also great artist.

2.disciplinarian value.

Acoording to Locke-"Mathematics is a way to settle in the mind a habit reasoning ". Due to its very nature, it has a real disciplinary value. If taught the real sense, it develop reasoning and thinking power more and demand less from memory. Its study result in the development of power rather than the acquisition of knowledge.

Acquisition of knowledge is not the major purpose for which a child is sent to school. Knowledge often out of data .In every sphere of knowledge , new concepts , ideas and theories are being developed rapidly . In this situation of ever advancing knowledge , the important is not only to learn facts, but also to know how to learn facts. It is not the acquisition of knowledge but the acquisition of the power of acquiring knowledge which is more important.

Knowledge becomes real and useful only when one is able to apply it to new situation. In mathematics, there is again a vast scope for application. Ability to apply knowledge to new situation is inculcation in the students.

The concentration of the mind which is necessitates and develops is another disciplinary value of this object.

Reasoning in mathematic possesses certain characteristics which are suitable for the training of the learner ‘s mind. If properly emphasized , there characteristics develop the corresponding habit and disciplines in the learner.

3.utilitarian value.

It is also known as practical value .Mathematic touches our life al every point. Every one uses some form of mathematic directly or indirectly in his daily life. Even the commonest man cannot pull on without counting and calculation. On account of his ignorance in mathematic he will be at the mercy of other and will be easily cheated and exploited. The knowledge of its fundamental processes and the skill to use them are the preliminary requirements of the present day citizen. Its need is felt al every moment in a day. One gets up at the appointed time, has to reach the place of work at the right time, has to take one ‘s meals at specific moments in the day, ha to keep many appointments at specified times, has to distribute his time between various activities and duties, has to draw the maximum benefit out of time available to him and has to go to bed at the appointed hour.

The housewife also need mathematics for the smooth running of domestic life, preparing family budget, keeping various accounts, making purchases, controlling expenses etc.

A person may belong to the lowest, the highest or any other class of society , he is bound to utilize mathematics in one form or another. He may be a finance minister a financier, an industrialist a businessman , a banker , a contractor or even a labourer - he has to calculate his wages , makes purchases in a bazaar, adjust his expenditure to the income and save something for the rainy day.

Various fundamental processes of mathematic like counting notation four fundamental operations measurements, areas, percentage, ratio, frication ,etc. have got an immense practical value in life. The knowledge and skill in these processes can be provided in an effective and systematic manner only by teaching mathematics in the schools.

In many occupation such as banking, accountancy, auditing, booking, tailoring, carpentry, taxation, postal jobs, insurance by which important function of life are carried out are intimately dependent on mathematic.

Budgeting is indispensable for the individual, the family, the panchayat , the school, the industry, the shop, the state and so on.

Mathematical knowledge in individual in understanding and controlling the force of nature.

Mathematic is at the base of all essential knowledge and is the basis of the progress of science , technology, economy, production and society in general.

Young has said "wherever we turning these days of iron, steam and electricity we find that mathematics has been the pioneer''.

Let us for a moment imagine the situation when all sort of mathematical knowledge is made to disappear. Then we would not be having any sort of business system, accounts ,banks, booking offices, tickets , salaries, timings etc. all our wheels will be jammed. We would have no houses to live in, no bridges to cross the rivers, no dams for producing electricity and no agency to look after economy. The structure of our civilization will collapse and he so called achievements and the amenities of this civilization will be nowhere to be seen.

1.2. Indian Mathematician's contribution in Mathematic.

1.Bhaskaracharya-2 .

Bhaskara is also known as **Bhaskara II** or as **Bhaskaracharya**, this latter name meaning "Bhaskara the Teacher". Since he is known in India as Bhaskaracharya we will refer to him throughout this article by that name. Bhaskaracharya's father was a Brahman named Mahesvara. Mahesvara himself was famed as an astrologer. This happened frequently in Indian society with generations of a family being excellent mathematicians and often acting as teachers to other family members.

Bhaskaracharya became head of the astronomical observatory at Ujjain, the leading mathematical centre in India at that time. Outstanding mathematicians such as Varahamihira and Brahmagupta had worked there and built up a strong school of mathematical astronomy.

In many ways Bhaskaracharya represents the peak of mathematical knowledge in the 12^{th} century. He reached an understanding of the number systems and solving equations which was not to be achieved in Europe for several centuries.

Six works by Bhaskaracharya are known but a seventh work, which is claimed to be by him, is thought by many historians to be a late forgery. The six works are: *Lilavati* (The Beautiful) which is on mathematics; *Bijaganita* (Seed Counting or Root Extraction) which is on algebra; the *Siddhantasiromani* which is in two parts, the first on mathematical astronomy with the second part on the sphere; the *Vasanabhasya* of *Mitaksara* which is Bhaskaracharya's own commentary on the *Siddhantasiromani* ; the *Karanakutuhala* (Calculation of Astronomical Wonders) or *Brahmatulya* which is a simplified version of the *Siddhantasiromani* ; and the *Vivarana* which is a commentary on the *Shishyadhividdhidatantra* of Lalla. It is the first three of these works which are the most interesting, certainly from the point of view of mathematics, and we will concentrate on the contents of these.

Given that he was building on the knowledge and understanding of Brahmagupta it is not surprising that Bhaskaracharya understood about zero and negative numbers. However his understanding went further even than that of Brahmagupta. To give some examples before we examine his work in a little more detail we note that he knew that $x^2 = 9$ had two solutions. He also gave the formula

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

Bhaskaracharya studied Pell's equation $px^2 + 1 = y^2$ for p = 8, 11, 32, 61 and 67. When p = 61 he found the solutions x = 226153980, y = 1776319049. When p = 67 he found the solutions x = 5967, y = 48842. He studied many Diophantine problems.

Let us first examine the *Lilavati*. First it is worth repeating the story told by Fyzi who translated this work into Persian in 1587. We give the story as given by Joseph in [5]:-

Lilavati was the name of Bhaskaracharya's daughter. From casting her horoscope, he discovered that the auspicious time for her wedding would be a particular hour on a certain day. He placed a cup with a small hole at the bottom of the vessel filled with water, arranged so that the cup would sink at the beginning of the propitious hour. When everything was ready and the cup was placed in the vessel, Lilavati suddenly out of curiosity bent over the vessel and a pearl from her dress fell into the cup and blocked the hole in it. The lucky hour passed without the cup sinking. Bhaskaracharya believed that the way to console his dejected daughter, who now would never get married, was to write her a manual of mathematics!

This is a charming story but it is hard to see that there is any evidence for it being true. It is not even certain that Lilavati was Bhaskaracharya's daughter. There is also a theory that Lilavati was Bhaskaracharya's wife. The topics covered in the thirteen chapters of the book are: definitions; arithmetical terms; interest; arithmetical and geometrical progressions; plane geometry; solid geometry; the shadow of the gnomon; the kuttaka; combinations.

In dealing with numbers Bhaskaracharya, like Brahmagupta before him, handled efficiently arithmetic involving negative numbers. He is sound in addition, subtraction and multiplication involving zero but realised that there were problems with Brahmagupta's ideas of dividing by zero. Madhukar Mallayya in [14] argues that the zero used by Bhaskaracharya in his rule $(a.0)/0 = a$, given in *Lilavati*, is equivalent to the modern concept of a non-zero "infinitesimal". Although this claim is not without foundation, perhaps it is seeing ideas beyond what Bhaskaracharya intended.

Bhaskaracharya gave two methods of multiplication in his *Lilavati*. We follow Ifrah who explains these two methods due to Bhaskaracharya in [4]. To multiply 325 by 243 Bhaskaracharya writes the numbers thus:

```
243  243  243
```

```
 3 2 5
-------------------
```

Now working with the rightmost of the three sums he computed 5 times 3 then 5 times 2 missing out the 5 times 4 which he did last and wrote beneath the others one place to the left. Note that this avoids making the "carry" in ones head.

```
 243 243 243
 3 2 5
-------------------
 1015
 20
-------------------
```

Now add the 1015 and 20 so positioned and write the answer under the second line below the sum next to the left.

```
 243 243 243
 3 2 5
-------------------
 1015
 20
-------------------
 1215
```

Work out the middle sum as the right-hand one, again avoiding the "carry", and add them writing the answer below the 1215 but displaced one place to the left.

```
 243 243 243
 3  2  5
-------------------
 4  6 1015
 8  20
-------------------
 1215
 486
```

Finally work out the left most sum in the same way and again place the resulting addition one place to the left under the 486.

```
 243 243 243
 3  2  5
-------------------
 6  9  4  6  1015
 12  8  20
-------------------
 1215
 486
 729
-------------------
```

Finally add the three numbers below the second line to obtain the answer 78975.

```
 243 243 243
 3 2 5
-------------------
 6 9 4 6 1015
 12 8 20
-------------------
 1215
 486
 729
-------------------
 78975
```

Despite avoiding the "carry" in the first stages, of course one is still faced with the "carry" in this final addition.

The second of Bhaskaracharya's methods proceeds as follows:

```
 325
 243
--------
```

Multiply the bottom number by the top number starting with the left-most digit and proceeding towards the right. Displace each row one place to start one place further right than the previous line. First step

```
 325
 243
```

```
--------
  729
```

Second step

```
  325
  243
--------
  729
  486
```

Third step, then add

```
  325
  243
--------
  729
  486
  1215
--------
  78975
```

Bhaskaracharya, like many of the Indian mathematicians, considered squaring of numbers as special cases of multiplication which deserved special methods. He gave four such methods of squaring in *Lilavati*.

Here is an example of explanation of inverse proportion taken from Chapter 3 of the *Lilavati*. Bhaskaracharya writes:-

In the inverse method, the operation is reversed. That is the fruit to be multiplied by the augment and divided by the demand. When fruit increases or decreases, as the demand is augmented or diminished, the direct rule is used. Else the inverse.

Rule of three inverse: If the fruit diminish as the requisition increases, or augment as that decreases, they, who are skilled in accounts, consider the rule of three to be inverted. When there is a diminution of fruit, if there be increase of requisition, and increase of fruit if there be diminution of requisition, then the inverse rule of three is employed.

As well as the rule of three, Bhaskaracharya discusses examples to illustrate rules of compound proportions, such as the rule of five (Pancarasika), the rule of seven (Saptarasika), the rule of nine (Navarasika), etc. Bhaskaracharya's examples of using these rules are discussed in [15].

An example from Chapter 5 on arithmetical and geometrical progressions is the following:-

Example: On an expedition to seize his enemy's elephants, a king marched two yojanas the first day. Say, intelligent calculator, with what increasing rate of daily march did he proceed, since he reached his foe's city, a distance of eighty yojanas, in a week?

Bhaskaracharya shows that each day he must travel $^{22}/_{7}$ yojanas further than the previous day to reach his foe's city in 7 days.

An example from Chapter 12 on the kuttaka method of solving indeterminate equations is the following:-

Example: Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted.

Bhaskaracharya is finding integer solution to $195x = 221y + 65$. He obtains the solutions $(x, y) = (6, 5)$ or $(23, 20)$ or $(40, 35)$ and so on.

In the final chapter on combinations Bhaskaracharya considers the following problem. Let an n-digit number be represented in the usual decimal form as

$d_1 d_2 \ldots d_n$ (*)

where each digit satisfies $1 \le d_j \le 9$, $j = 1, 2, \ldots, n$. Then Bhaskaracharya's problem is to find the total number of numbers of the form (*) that satisfy

$d_1 + d_2 + \ldots + d_n = S$.

In his conclusion to *Lilavati* Bhaskaracharya writes:-

Joy and happiness is indeed ever increasing in this world for those who have Lilavati clasped to their throats, decorated as the members are with neat reduction of fractions, multiplication and involution, pure and perfect as are the solutions, and tasteful as is the speech which is exemplified.

The *Bijaganita* is a work in twelve chapters. The topics are: positive and negative numbers; zero; the unknown; surds; the kuttaka; indeterminate quadratic equations; simple equations; quadratic equations; equations with more than one unknown; quadratic equations with more than one unknown; operations with products of several unknowns; and the author and his work.

Having explained how to do arithmetic with negative numbers, Bhaskaracharya gives problems to test the abilities of the reader on calculating with negative and affirmative quantities:-

Example: Tell quickly the result of the numbers three and four, negative or affirmative, taken together; that is, affirmative and negative, or both negative or both affirmative, as separate instances; if thou know the addition of affirmative and negative quantities.

Negative numbers are denoted by placing a dot above them:-

The characters, denoting the quantities known and unknown, should be first written to indicate them generally; and those, which become negative should be then marked with a dot over them.

Example: Subtracting two from three, affirmative from affirmative, and negative from negative, or the contrary, tell me quickly the result ...

In *Bijaganita* Bhaskaracharya attempted to improve on Brahmagupta's attempt to divide by zero (and his own description in *Lilavati*) when he wrote:-

A quantity divided by zero becomes a fraction the denominator of which is zero. This fraction is termed an infinite quantity. In this quantity consisting of that which has zero for its divisor, there is no alteration, though many may be inserted or extracted; as no change takes place in the infinite and immutable God when worlds are created or destroyed, though numerous orders of beings are absorbed or put forth.

So Bhaskaracharya tried to solve the problem by writing $n/0 = \infty$. At first sight we might be tempted to believe that Bhaskaracharya has it correct, but of course he does not. If this were true then 0 times ∞ must be equal to every number n, so all numbers are equal. The Indian mathematicians could not bring themselves to the point of admitting that one could not divide by zero.

Equations leading to more than one solution are given by Bhaskaracharya:-

Example: Inside a forest, a number of apes equal to the square of one-eighth of the total apes in the pack are playing noisy games. The remaining twelve apes, who are of a more serious disposition, are on a nearby hill and irritated by the shrieks coming from the forest. What is the total number of apes in the pack?

The problem leads to a quadratic equation and Bhaskaracharya says that the two solutions, namely 16 and 48, are equally admissible.

The kuttaka method to solve indeterminate equations is applied to equations with three unknowns. The problem is to find integer solutions to an equation of the form $ax + by + cz = d$. An example he gives is:-

Example: The horses belonging to four men are 5, 3, 6 *and* 8. *The camels belonging to the same men are* 2, 7, 4 *and* 1. *The mules belonging to them are* 8, 2, 1 *and* 3 *and the oxen are* 7, 1, 2 *and* 1. *all four men have equal fortunes. Tell me quickly the price of each horse, camel, mule and ox*.

Of course such problems do not have a unique solution as Bhaskaracharya is fully aware. He finds one solution, which is the minimum, namely horses 85, camels 76, mules 31 and oxen 4.

Bhaskaracharya's conclusion to the *Bijaganita* is fascinating for the insight it gives us into the mind of this great mathematician:-

A morsel of tuition conveys knowledge to a comprehensive mind; and having reached it, expands of its own impulse, as oil poured upon water, as a secret entrusted to the vile, as alms bestowed upon the worthy, however little, so does knowledge infused into a wise mind spread by intrinsic force.

It is apparent to men of clear understanding, that the rule of three terms constitutes arithmetic and sagacity constitutes algebra. Accordingly I have said ... The rule of three terms is arithmetic; spotless understanding is algebra. What is there unknown to the intelligent? Therefore for the dull alone it is set forth.

The *Siddhantasiromani* is a mathematical astronomy text similar in layout to many other Indian astronomy texts of this and earlier periods. The twelve chapters of the first part cover topics such as: mean longitudes of the planets; true longitudes of the planets; the three problems of diurnal rotation; syzygies; lunar eclipses; solar eclipses; latitudes of the planets; risings and settings; the moon's crescent; conjunctions of the planets with each other; conjunctions of the planets with the fixed stars; and the patas of the sun and moon.

The second part contains thirteen chapters on the sphere. It covers topics such as: praise of study of the sphere; nature of the sphere; cosmography and geography; planetary mean motion; eccentric epicyclic model of the planets; the armillary sphere; spherical trigonometry; ellipse calculations; first visibilities of the planets; calculating the lunar crescent; astronomical instruments; the seasons; and problems of astronomical calculations.

There are interesting results on trigonometry in this work. In particular Bhaskaracharya seems more interested in trigonometry for its own sake than his predecessors who saw it only as a tool for calculation. Among the many interesting results given by Bhaskaracharya are:

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

and

$\sin(a - b) = \sin a \cos b - \cos a \sin b$.

Bhaskaracharya rightly achieved an outstanding reputation for his remarkable contribution. In 1207 an educational institution was set up to study Bhaskaracharya's works. A medieval inscription in an Indian temple reads:-

Triumphant is the illustrious Bhaskaracharya whose feats are revered by both the wise and the learned. A poet endowed with fame and religious merit, he is like the crest on a peacock.

It is from this quotation that the title of Joseph's book [5] comes.

2.Aryabhatt.

Aryabhatta

Aryabhatta, also known as **Aryabhatta I** or **Aryabhata** (476-550?), was a famous Indian mathematician and astronomer, born in a place called Taregana, in Bihar (though some people do not agree with the evidence). Taregana (also spelled as Taragna) which literally means songs of stars in Bihari, is a small place situated nearly 30 km from Patna, which was then known as Kusumpura later **Pataliputra**, the capital of the Gupta Empire. This is the very empire that has been dubbed as the "golden period in Indian history". The best introduction to the genius of past

is seen in the words of Bhaskara I who said, "A*ryabhatta is the master who, after reaching the furthest shores and plumbing the inmost depths of the sea of ultimate knowledge of mathematics, kinematics and spherics, handed over the three sciences to the learned world*".
Aryabhatta, the Indian mathematician head of Nalanda University at Kusumpura (modern Patna)

What was his name?

Varahamihira, the younger contemporary of Aryabhatta also mentions him as "Aryabhata". In addition to this, Bhaskara I too mentions him as Aryabhata. It seems as if the correct name was **Aryabhata** and not **Aryabhatta**. This could mean that "Bhatta" was not his surname but as part of his first name. In fact, there is a lot of confusion about his name too. Perhaps he was called Arya and his surname was Bhat or Bhatta!

Where did Aryabhatta come from?

There is some disagreement about this birth place. Some are of the view that he was born in Patliputra while some are of the view that he was born in Kerala and moved to Patliputra and lived there. Those who say that he was in Bihar is because of this name. His name "Arya" and "Bhatta" indicates that he was from North India. His suffix "Bhatta" could have been either part of his name or his surname, till date it's not known if this is correct or not. It is interesting to note that Aryabhatta himself have mentioned himself at only 3 places and as "Aryabhata" in his work Aryabhatiya.

The reason for not considering Kerala as his birthplace is that nowhere in his works he has mentioned Kerala. In addition, all works of Aryabhatta is in Sanskrit and Sanskrit was not used in Kerala. So to claim that Aryabhatiya was written in Kerala has no credibility. Furthermore, he has been identified by numerous mathematicians and in Arabic translations as someone who hailed from Kusumpura (modern Patna), the capital of Magadha. It therefore appears that Aryabhatta was born, lived, flourished and worked in Magadha. He has also been described as the head of the Nalanda University.

praṇipatyaikam anekaṃ kaṃ satyāṃ devatāṃ paraṃ brahma /
Āryabhaṭas trīṇi gadati gaṇitaṃ kālakriyāṃ golam //
(*Āryabhaṭīya, Gītikāpāda,* verse 1)

brahma-ku-śaśi-budha-bhṛgu-
kuja-guru-koṇa-bhagaṇān namaskṛtya /
Āryabhaṭas tviha nigadati
Kusumapure 'bhyarcitaṃ jñānam //
(*Gaṇitapāda,* verse 1)

Āryabhaṭīyaṃ nāmnā
pūrvaṃ svāyambhuvaṃ sadā satyam /
sukṛtāyuṣoḥ praṇāśaṃ
kurute pratikañcukaṃ yo 'sya //
(*Golapāda,* verse 50)

Aryabhatta mentions himself as Aryabhata

Influence of Aryabhatta on science and mathematics

Aryabhatta is considered to be one of the mathematicians who changed the course of mathematics and astronomy to a great extent. He is known to have considerable influence on Arabic science world too, where he is referred to as Arjehir. His notable contributions to the world of science and mathematics includes the theory that the earth rotates on its axis, explanations of the solar and lunar eclipses, solving of quadratic equations, place value system with zero, and **approximation of pie (π)**.

π Aryabhatta approximatted pi

Aryabhatta exerted influence on the Indian astronomical tradition to such an extent that his presence was felt in neighboring countries and cultures also. There have been various translations of his work among which the Arabic translation during the 820CE is very significant.

When mathematical students are confused with trigonometry even today, Aryabhatta had defined sine, cosine, versine and inverse sine back in his era, influencing the birth of trigonometry. The signs were originally known as jya, kojya, utkrama-jya and otkram jya. In Arabic they were translated as jiba and kojiba, which later when being translated into Latin was misunderstood to be ‘fold in a garment’ by Gerard of Cremona, who stated it as sinus, which meant fold in Latin. Aryabhatta was the first mathematician to detail both sine and versine (1 − cos x) tables, in 3.75° intervals from 0° to 90°, to 4 decimal places.

Aryabhatta’s astronomical calculations influenced the Arabians, who used the trigonometric tables to compute many astronomical tables. His calendared calculation has been in continuous use in India, on which the present day Panchangam is based. His studies are also base for the national calendars of Iran and Afghanistan today.

The Story of Numbers (0 and 1) Indian Numerals or Arabic?

Aryabhatiya

It is known that Aryabhatta has authored at least three astronomical books, in addition he also wrote some free stanzas. Among them “**Aryabhatiya**” is the only text that has survived to this day, whereas unfortunately his other works have been extinct. It is a small treatise written is 118 verses, which summarizes the Hindu mathematics of that time. This great mathematical masterpiece of the past starts with 10 verse introduction, which is then followed by mathematical section which is written in 33 verses that gives out 66 mathematical rules, but there is no proof to go with it. The mathematical part of the **Aryabhatiya** is about algebra, arithmetic, plane trigonometry and spherical trigonometry in addition to advanced mathematics on continued fractions, quadratic equations, sums of power series and a table of sines.

equation $Tx^2 + Ax = AB$

$$x = \frac{\sqrt{BAT + \left(\frac{A}{2}\right)^2} - \frac{A}{2}}{T}$$

Quadratic equation by Aryabhatta

The next section consists of 25 verses which gives us glimpse into the planetary models. The final section of the book is dedicated to sphere and eclipses which runs into 50 verses. He states that the moon and planets shine by reflected sunlight. Instead of the prevailing cosmogony where eclipses were believed to be caused by pseudo-planetary nodes Rahu and Ketu, he explains eclipses in terms of shadows cast by earth or those shadows that fall on earth. It is amazing how Aryabhatta could explain both lunar and solar eclipse so accurately.

Statue of Aryabhatta at Inter-University Centre for Astronomy and Astrophysics at Pune (India)

There is some argument over the claim of Aryabhatta being the inventor of place value system that made use of zero. Georges Ifrah, in his work 'Universal history of numbers: From prehistory to the invention of the computer (London, 1998)' writes in work, "..it is extremely likely that Aryabhatta knew the sign for zero and the numerals of the place value system". Georges Ifrah has studied the works of Aryabhatta and found that the counting and mathematical work carried out by him would have been not possible without zero or place value system.

Honouring Aryabhatta

The Indian ISRO (Indian Space Research Organization) named its first satellite after the genius mathematician and astronomer. A research establishment has been set up in Nainital, called the Aryabhatta Research Institute of Observational Sciences (ARIOS) to honor his contribution to the field of science. There is also a lunar crater and a species of bacteria discovered by ISRO named after Aryabhatta.

Some of the works of Aryabhatta include

- Aryabhatta worked out the value of pi.
- He worked out the area of a triangle. His exact words were, "*ribhujasya phalashariram samadalakoti bhujardhasamvargah*" which translates "for a triangle, the result of a perpendicular with the half side is the area".
- He discussed the idea of sin.
- He worked on the summation of series of squares and cubes (square-root and cube-root).
- He talks about the "rule of three" which is to find the value of x when three numbers a, b and c is given.
- Aryabhatta calculates the volume of a sphere.
- Aryabhatta described the model of the solar system, where the sun and moon are each carried by epicycles that in turn revolve around the Earth. He also talks about the number of rotations of the earth, describes that the earth rotating on its axis, the order of the planets in terms of distance from earth.
- Aryabhatta describes the solar and lunar eclipses scientifically.
- Aryabhatta describes that the moon and planets shine by light reflected from the sun.
- Aryabhatta calculated the sidereal rotation which is the rotation of the earth with respect to the stars as 23 hours, 56 minutes and 4.1 seconds.
- He calculated the length of the sidereal year as 365 days, 6 hours, 12 minutes and 30 seconds. The actual value shows that his calculations was an error of 3 minutes and 20 seconds over a year.

अनुलोमगतिर्नौस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥

Mention of rotation of the earth on its axis by Aryabhatta

Although we know nothing about the personal history of Aryabhatta, he was the genius who continues to baffle mathematicians even to this day.

A new ebook (paperback coming soon) has been published called, "Life and Works of Aryabhata" which is available on Amazon.

Aryabhata

September 22, 2013 | bob 2 minutes

Born: 476, probably in Ashmaka

Died: 550 (at age 74), location unknown

Nationality: Indian

Famous For: Early mathematician who calculated the value of pi

Aryabhata (476-550) was an Indian mathematician and astronomer. He is generally considered to have begun the line of great Indian astronomer-mathematicians that flourished during the country's classical period. Several of his calculations showed remarkable accuracy for the era, with some remaining the best available for many centuries. He is sometimes referred to as Aryabhata I, since several later scientists of the same name also produced notable works.

Aryabhata's Early Life

Aryabhata came from southern India, but his precise place of birth is not known. Some authorities suggest that Kerala is the most likely location, while others believe that Dhaka or Maharashtra are more probable. It is, however, generally accepted that he studied at an advanced level in Kusumapura in modern-day Patna, where he remained for some years.

A contemporary poem places Aryabhata as the manager of a scientific institution; the precise nature of the body is not given, but there are grounds for suspecting that it may have been linked to the astronomical observatory that was maintained there by the University of Nalanda.

The Aryabhatiya

While studying at the university, Aryabhata produced the *Aryabhatiya*, his major work. Written at the age of just 23, it ranges widely across mathematics and astronomy, but is particularly notable for its calculations regarding planetary periods. The value given for the length of the Earth's astronomical day differs from the true value by only a matter of minutes.

Aryabhata also worked out a value for pi that equates to 3.1416, very close to the approximations still used today. Using this value, he was able to calculate that the Earth had a circumference of 24,835 miles. This is correct to within 0.2%, and remained the best figure available well into medieval times.

While working on the calculation of pi, it is possible that Aryabhata may also have discovered that number's irrationality. The relevant text is inconclusive on this point, but if he did establish the irrational nature of pi, he beat the first European mathematicians to do this by many hundreds of years.

The *Aryabhatiya* also contains solid work regarding the solar system. It states correctly that the light cast by planets and the moon is caused by sunlight reflecting off their surfaces, and that all planets follow elliptical orbits. Aryabhata was also able to describe accurately the processes that lead to both solar and lunar eclipses.

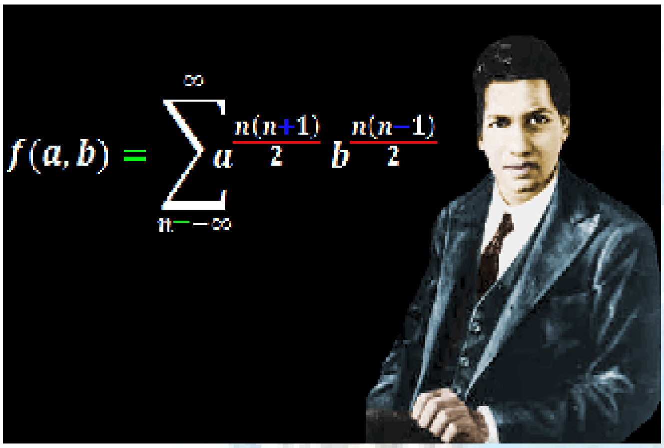
Aryabhata's Legacy

For several hundred years after its author's death, the *Aryabhatiya* was unknown in the West, although its Arabic translation in the 9th century was of great use to the scientists of the Islamic Golden Age. The book was eventually translated into Latin shortly after 1200. The mathematical ideas contained within it were quickly adopted by Europeans, especially those dealing with areas and volumes, and with finding cube and square roots.

However, Aryabhata's astronomical findings had less impact, and it was left to later men such as Copernicus and Galileo to bring about the Western astronomical revolution. The first Indian artificial was named Aryabhata in his honor, as was a new university in the state of Bihar

3.Ramanujan.

Srinivasa Ramanujan



Lived 1887 – 1920.

Srinivasa Ramanujan was a largely self-taught pure mathematician. Hindered by poverty and ill-health, his highly original work has considerably enriched number theory and, more recently, physics.

Beginnings

Srinivasa Ramanujan was born on December 22, 1887 in the town of Erode, in Tamil Nadu, in the south east of India. His father was K. Srinivasa Iyengar, an accounting clerk for a clothing merchant. His mother was Komalatammal, who earned a small amount of money each month as a singer at the local temple.

His family were Brahmins, the Hindu caste of priests and scholars. His mother ensured the boy was in tune with Brahmin traditions and culture. Although his family were high caste, they were very poor.

Ramanujan's parents moved around a lot, and he attended a variety of different elementary schools. By the age of 10, he was the top student, not just in his school, but in his district.

Early Mathematics

Ramanujan moved on to high school at the age of 10: the Kumbakonam Town High School. With access to mathematical books in the school's library, he quickly found his vocation. By the time he was 12, he had begun serious self-study of mathematics, working through arithmetic and geometric series and cubic equations. He discovered his own method of solving quartic equations.

As Ramanujan's mathematical knowledge developed, his main source of inspiration and expertise became *Synopsis of elementary results in pure mathematics* by George S. Carr. This book presented a very large number of mathematical results – over 4000 theorems – but generally showed little working, cramming into its pages as many results as possible.

2478 $$\int_0^{\pi} \cos^p x \cos nx\, dx = C\left(p, \frac{p-n}{2}\right)\frac{\pi}{2^p},$$

when p and n are either *both odd* or *both even*, and n is not greater than p.

Entry 2478 from Carr's*Synopsis of elementary results in pure mathematics*

With little other guidance, Ramanujan came to believe this was how mathematics was done, so he himself learned to show little working. Also, he could afford only a small amount of paper, doing most of his work on slate with chalk, transferring a minimal amount of his working and his results to paper.

His memory for mathematical formulas and constants seems to have been boundless: he amazed classmates with his ability to recite the values of irrational numbers like π, e, and √2 to as many decimal places as they asked for.

An Apparently Bright Future Fizzles Out

In 1904 Ramanujan left high school; his future looked promising: he had won the school's mathematics prize and, more importantly, a scholarship allowing him to study at the Government Arts College in the town of Kumbakonam.

Obsessed with mathematics, Ramanujan failed his non-mathematical exams and lost his scholarship. In 1905 he traveled to Madras and enrolled at Pachaiyappa's College, but again failed his non-mathematical exams.

The Discovery of Ramanujan as a Mathematician of Genius

The Hungry Years

At the beginning of 1907, at the age of 19, with minimal funds and a stomach all too often groaning with hunger, Ramanujan continued on the path he had chosen: total devotion to mathematics. The mathematics he was doing was highly original and very advanced.

Even though (or some might say because) he had very little formal mathematical education he was able to discover new theorems. He also independently discovered results originally discovered by some of the greatest mathematicians in history, such as Carl Friedrich Gauss and Leonhard Euler.

Ill-health was Ramanujan's constant companion – as it would be for much of his short life.

By 1910 he realized he must find work to stay alive. In the city of Madras he found some students who needed mathematics tutoring and he also walked around the city offering to do accounting work for businesses.

And then a piece of luck came his way. Ramanujan tried to find work at the government revenue department, and there he met an official whose name was Ramaswamy Aiyer. Ramanujan did

not have a resume to show Ramaswamy Aiyer; all he had was his notebooks – the results of his mathematical work.
Ramanujan's good fortune was that Ramaswamy Aiyer was a mathematician. He had only recently founded the Indian Mathematical Society, and his jaw dropped when he saw Ramanujan's work.

"I was struck by the extraordinary mathematical results contained in it. I had no mind to smother his genius by an appointment in the lowest rungs of the revenue department."

V. Ramaswamy Aiyer, 1871 – 1936

Mathematician

Things Begin to Look Up
Ramaswamy Aiyer contacted the secretary of the Indian Mathematical Society, R. Ramachandra Rao, suggesting he provide financial support for Ramanujan. At first Rao resisted the idea, believing Ramanujan was simply copying the work of earlier great mathematicians. A meeting with Ramanujan, however, convinced Rao that he was dealing with a genuine mathematical genius. He agreed to provide support for Ramanujan, and Ramaswamy Aiyer began publishing Ramanujan's work in the *Journal of the Indian Mathematical Society*.
Ramanujan's work, however, was hard to understand. The style he had adopted as a schoolboy, after digesting George S. Carr's book, contributed to the problem. His mathematics often left too few clues to allow anyone who wasn't also a mathematical genius to see how he obtained his results.
In March 1912 his financial position improved when he got a job as an accounting clerk with the Madras Port Trust.
There he was encouraged to do mathematics at work after finishing his daily tasks by the port's Chief Accountant, S. Narayana Iyer, who was treasurer of the Indian Mathematical Society, and by Sir Francis Spring, an engineer, who was Chairman of the Madras Port Trust.
Francis Spring began pressing for Ramanujan's mathematical work to be supported by the government and for him to be appointed to a research position at one of the great British universities.
A Crank or a Genius?
Ramanujan and his supporters contacted a number of British professors, but only one was receptive – an eminent pure mathematician at the University of Cambridge – Godfrey Harold Hardy, known to everyone as G. H. Hardy, who received a letter from Ramanujan in January 1913. By this time, Ramanujan had reached the age of 25.

Professor Hardy puzzled over the nine pages of mathematical notes Ramanujan had sent. They seemed rather incredible. Could it be that one of his colleagues was playing a trick on him? Hardy reviewed the papers with J. E. Littlewood, another eminent Cambridge mathematician, telling Littlewood they had been written by either a crank or a genius, but he wasn't quite sure which. After spending two and a half hours poring over the outlandishly original work, the mathematicians came to a conclusion. They were looking at the papers of a mathematical genius:
mathematicians came to a conclusion. They were looking at the papers of a mathematical genius:

"I had never seen anything in the least like them before. A single look at them is enough to show that they could only be written by a mathematician of the highest class. They must be true because, if they were not true, no one would have the imagination to invent them."
G. H. Hardy, 1877 – 1947
Mathematician

Hardy was eager for Ramanujan to move to Cambridge, but in accordance with his Brahmin beliefs, Ramanujan refused to travel overseas. Instead, an arrangement was made to fund two years of work at the University of Madras. During this time, Ramanujan's mother had a dream in which the goddess Namagiri told her she should give her son permission to go to Cambridge, and this she did. Her decision led to several very heated quarrels with other devout family members.

Ramanujan at Cambridge

Ramanujan arrived in Cambridge in April 1914, three months before the outbreak of World War 1. Within days he had begun work with Hardy and Littlewood. Two years later, he was awarded the equivalent of a Ph.D. for his work – a mere formality.

Srinivasa Ramanujan at Cambridge

Ramanujan's prodigious mathematical output amazed Hardy and Littlewood.
The notebooks he had brought from India were filled with thousands of identities, equations and theorems which he had discovered for himself in the years 1903 – 1914.
Some had been discovered by earlier mathematicians; some, through inexperience, were mistaken; many were entirely new.

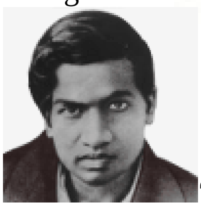
"It was his insight into algebraical formulae, transformations of infinite series, and so forth that was most amazing. On this side most certainly I have never met his equal, and I can compare him only with Euler or Jacobi."

G. H. Hardy, 1877 – 1947

Mathematician

Explaining Ramanujan's Extraordinary Mathematical Output

Ramanujan had very little formal training in mathematics, and indeed large areas of mathematics were unknown to him. Yet in the areas familiar to him and in which he enjoyed working, his output of new results was phenomenal.
Ramanujan said that the Hindu goddess Namagiri – who had appeared in his mother's dream telling her to allow him to go to Cambridge – had appeared in one of his own dreams.



"While asleep, I had an unusual experience. There was a red screen formed by flowing blood, as it were. I was observing it. Suddenly a hand began to write on the screen. I became all attention. That hand wrote a number of elliptic integrals. They stuck to my mind. As soon as I woke up, I committed them to writing."
Srinivasa Ramanujan, 1887 – 1920
Mathematician

Synopsis

Srinivasa Ramanujan was born in southern India in 1887. After demonstrating an intuitive grasp of mathematics at a young age, he began to develop his own theories and in 1911 published his first paper in India. Two years later Ramanujan began a correspondence with British mathematician G. H. Hardy that resulted in a five-year-long mentorship for Ramanujan at Cambridge, where he published numerous papers on his work and received a B.S. for research. His early work focused on infinite series and integrals, which extended into the remainder of his career. After contracting tuberculosis, Ramanujan returned to India, where he died in 1920 at 32 years of age.

Intuition

Srinivasa Ramanujan was born on December 22, 1887, in Erode, India, a small village in the southern part of the country. Shortly after this birth, his family moved to Kumbakonam, where his father worked as a clerk in a cloth shop. Ramanujan attended the local grammar school and high school, and early on demonstrated an affinity for mathematics.

When at age 15 he obtained an out-of-date book called *A Synopsis of Elementary Results in Pure and Applied Mathematics*, Ramanujan set about feverishly and obsessively studying its thousands of theorems before moving on to formulate many of his own. At the end of high school, the strength of his schoolwork was such that he obtained a scholarship to the Government College in Kumbakonam.

A Blessing and a Curse

But Ramanujan’s greatest asset proved also to be his Achilles heel. He lost his scholarship to both the Government College and later at the University of Madras because his devotion to math caused him to let his other courses fall by the wayside. With little in the way of prospects, in 1909 he sought government unemployment benefits.

Yet despite these setbacks, Ramanujan continued to make strides in his mathematical work, and in 1911 published a 17-page paper on Bernoulli numbers in the *Journal of the Indian Mathematical Society*. Seeking the help of members of the society, in 1912 Ramanujan was able to secure a low-level post as a shipping clerk with the Madras Port Trust, where he was able to make a living while building a reputation for himself as a gifted mathematician.

Cambridge

Around this time, Ramanujan had become aware of the work of British mathematician G. H. Hardy — who himself had been something of a young genius — with whom he began a correspondence in 1913 and shared some of his work. After initially thinking his letters a hoax, Hardy became convinced of Ramanujan’s brilliance and was able to secure him both a research scholarship at the University of Madras as well as a grant from Cambridge.

The following year, Hardy convinced Ramanujan to come study with him at Cambridge. During their subsequent five-year mentorship, Hardy provided the formal framework in which Ramanujan's innate grasp of numbers could thrive, with Ramanujan publishing upwards of 20 papers on his own and more in collaboration with Hardy. Ramanujan was awarded a bachelor of sciences for research from Cambridge in 1916 and in 1918 became a member of the Royal Society of London.

Doing the Math

"[Ramanujan] made many momentous contributions to mathematics especially number theory," states George E. Andrews, an Evan Pugh Professor of Mathematics at Pennsylvania State University. "Much of his work was done jointly with his benefactor and mentor, G. H. Hardy. Together they began the powerful "circle method" to provide an exact formula for p(n), the number of integer partitions of n. (e.g. p(5)=7 where the seven partitions are 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1). The circle method has played a major role in subsequent developments in analytic number theory. Ramanujan also discovered and proved that 5 always divides p(5n+4), 7 always divides p(7n+5) and 11 always divides p(11n+6). This discovery led to extensive advances in the theory of modular forms."

Bruce C. Berndt, Professor of Mathematics at the University of Illinois at Urbana-Champaign, adds that: "the theory of modular forms is where Ramanujan's ideas have been most influential. In the last year of his life, Ramanujan devoted much of his failing energy to a new kind of function called mock theta functions. Although after many years we can prove the claims that Ramanujan made, we are far from understanding how Ramanujan thought about them, and much work needs to be done. They also have many applications. For example, they have applications to the theory of black holes in physics."

But years of hard work, a growing sense of isolation and exposure to the cold, wet English climate soon took their toll on Ramanujan and in 1917 he contracted tuberculosis. After a brief period of recovery, his health worsened and in 1919 he returned to India.

The Man Who Knew Infinity

Srinivasa Ramanujan died of his illness on April 26, 1920, at the age of 32. And even on his deathbed had been consumed by math, writing down a group of theorems that he said had come to him in a dream. These and many of his earlier theorems are so complex that the full scope of Ramanujan's legacy has yet to be completely revealed and his work remains the focus of much mathematical research. His collected papers were published by Cambridge University Press in 1927.

Of Ramanujan's published papers — 37 in total — professor Bruce C. Berndt reveals that "a huge portion of his work was left behind in three notebooks and a 'lost' notebook. These notebooks contain approximately 4000 claims, all without proofs. Most of these claims have now been proved, and like his published work, continue to inspire modern-day mathematics."

A biography of Ramanujan titled *The Man Who Knew Infinity* was published in 1991 and a movie of the same name starring Dev Patel as Ramanujan and Jeremy Irons as Hardy, premiered in September 2015 at the Toronto Film Festival.

Fact Check

We strive for accuracy and fairness. If you see something that doesn't look right, contact us!

Birth

Srinivasa Ramanujan, an Indian mathematician was born in 22nd December, 1887 in Madras, India. Like Sophie Germain, he received no formal education in mathematics but made important contributions to advancement of mathematics. His acquaintance G.H. Hardy summed up his achievement in following words:

“The limitations of his knowledge were as startling as its profundity. Here was a man who could work out modular equations and theorems…to orders unheard of, whose mastery of continued fraction was… beyond that of any mathematician in the world, who had found for himself the functional equation of zeta function and the dominant terms of many of the most famous problems in analytical theory of numbers; and yet he had never heard of a doubly periodic function or of Cauchy’s theorem, and had indeed but the vaguest idea of what a function of complex variable was…”

Contribution to Mathematics

His chief contribution in mathematics lies mainly in analysis, game theory and infinite series. He made in depth analysis in order to solve various mathematical problems by bringing to light new and novel ideas that gave impetus to progress of game theory. Such was his mathematical genius that he discovered his own theorems. It was because of his keen insight and natural intelligence that he came up with infinite series for π

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

This series made up the basis of certain algorithms that are used today. One such remarkable instance is when he solved the bivariate problem of his roommate at spur of moment with a novel answer that solved the whole class of problems through continued fraction. Besides that he also led to draw some formerly unknown identities such as by linking coefficients of and providing identities for hyperbolic secant.

He also described in detail the mock theta function, a concept of mock modular form in mathematics. Initially, this concept remained an enigma but now it has been identified as holomorphic parts of maass forms. His numerous assertions in mathematics or concepts opened up new vistas of mathematical research for instance his conjecture of size of tau function that has distinct modular form in theory of modular forms. His papers became an inspiration with later mathematicians such as G. N. Watson, B. M. Wilson and Bruce Berndt to explore what Ramanujan discovered and to refine his work. His contribution towards development of mathematics particularly game theory remains unrivaled as it was based upon pure natural talent and enthusiasm. In recognition of his achievements, his birth date 22 December is celebrated in India as Mathematics Day. It would not be wrong to assume that he was first Indian mathematician who gained acknowledgment only because of his innate genius and talent.

His Publications

It was after his first publication in the "Journal of the Indian Mathematical Society" that he gained recognition as genius mathematician. With collaboration of English mathematician G. H. Hardy, with whom he came in contact with during his visit to England, he brought forward his divergent series that later stimulated research in that given area thus refining the contribution of Ramanujan. Both also worked on new asymptotic formula that gave rise to method of analytical number theory also called as "Circle Method" in mathematics.

It was during his visit to England that he got worldwide recognition after publication of his mathematical work in European journals. He also achieved the distinction of becoming second Indian, who was elected as Fellow of Royal Society of London in 1918.

Death

He died on 26 April 1920 at hands of dreadful disease of tuberculosis. Although he couldn't get recognition of world at large but in field of mathematics, his contribution is duly recognized

1.3.Other mathematician's contribution in Mathematic .

1.Eculid

Euclid

Born: **about 325 BC**
Died: **about 265 BC in Alexandria, Egypt**

Click the picture above
to see six larger pictures

Euclid of Alexandria is the most prominent mathematician of antiquity best known for his treatise on mathematics *The Elements*. The long lasting nature of *The Elements* must make Euclid the leading mathematics teacher of all time. However little is known of Euclid's life except that he taught at Alexandria in Egypt. Proclus, the last major Greek philosopher, who lived around 450 AD wrote (see [1] or [9] or many other sources):-

Not much younger than these [pupils of Plato] is Euclid, who put together the "Elements", arranging in order many of Eudoxus's theorems, perfecting many of Theaetetus's, and also bringing to irrefutable demonstration the things which had been only loosely proved by his predecessors. This man lived in the time of the first Ptolemy; for Archimedes, who followed closely upon the first Ptolemy makes mention of Euclid, and further they say that Ptolemy once asked him if there were a shorted way to study geometry than the Elements, to which he replied that there was no royal road to geometry. He is therefore younger than Plato's circle, but older than Eratosthenes and Archimedes; for these were contemporaries, as Eratosthenes somewhere says. In his aim he was a Platonist, being in sympathy with this philosophy, whence he made the end of the whole "Elements" the construction of the so-called Platonic figures.

There is other information about Euclid given by certain authors but it is not thought to be reliable. Two different types of this extra information exists. The first type of extra information is that given by Arabian authors who state that Euclid was the son of Naucrates and that he was born in Tyre. It is believed by historians of mathematics that this is entirely fictitious and was merely invented by the authors.

The second type of information is that Euclid was born at Megara. This is due to an error on the part of the authors who first gave this information. In fact there was a Euclid of Megara, who was a philosopher who lived about 100 years before the mathematician Euclid of Alexandria. It is not quite the coincidence that it might seem that there were two learned men called Euclid. In fact Euclid was a very common name around this period and this is one further complication that makes it difficult to discover information concerning Euclid of Alexandria since there are references to numerous men called Euclid in the literature of this period.

Returning to the quotation from Proclus given above, the first point to make is that there is nothing inconsistent in the dating given. However, although we do not know for certain exactly what reference to Euclid in Archimedes' work Proclus is referring to, in what has come down to us there is only one reference to Euclid and this occurs in *On the sphere and the cylinder*. The obvious conclusion, therefore, is that all is well with the argument of Proclus and this was assumed until challenged by Hjelmslev in [48]. He argued that the reference to Euclid was added to Archimedes' book at a later stage, and indeed it is a rather surprising reference. It was not the tradition of the time to give such references, moreover there are many other places in Archimedes where it would be appropriate to refer to Euclid and there is no such reference. Despite Hjelmslev's claims that the passage has been added later, Bulmer-Thomas writes in [1]:-

Although it is no longer possible to rely on this reference, a general consideration of Euclid's works ... still shows that he must have written after such pupils of Plato as Eudoxus and before Archimedes.

For further discussion on dating Euclid, see for example [8]. This is far from an end to the arguments about Euclid the mathematician. The situation is best summed up by Itard [11] who gives three possible hypotheses.

(i) Euclid was an historical character who wrote the *Elements* and the other works attributed to him.

(ii) Euclid was the leader of a team of mathematicians working at Alexandria. They all contributed to writing the 'complete works of Euclid', even continuing to write books under Euclid's name after his death.

(iii) Euclid was not an historical character. The 'complete works of Euclid' were written by a team of mathematicians at Alexandria who took the name Euclid from the historical character Euclid of Megara who had lived about 100 years earlier.

It is worth remarking that Itard, who accepts Hjelmslev's claims that the passage about Euclid was added to Archimedes, favours the second of the three possibilities that we listed above. We should, however, make some comments on the three possibilities which, it is fair to say, sum up pretty well all possible current theories.

There is some strong evidence to accept (i). It was accepted without question by everyone for over 2000 years and there is little evidence which is inconsistent with this hypothesis. It is true that there are differences in style between some of the books of the *Elements* yet many authors vary their style. Again the fact that Euclid undoubtedly based the *Elements* on previous works means that it would be rather remarkable if no trace of the style of the original author remained.

Even if we accept (i) then there is little doubt that Euclid built up a vigorous school of mathematics at Alexandria. He therefore would have had some able pupils who may have helped out in writing the books. However hypothesis (ii) goes much further than this and would suggest that different books were written by different mathematicians. Other than the differences in style referred to above, there is little direct evidence of this.

Although on the face of it (iii) might seem the most fanciful of the three suggestions, nevertheless the 20th century example of Bourbaki shows that it is far from impossible. Henri Cartan, André Weil, Jean Dieudonné, Claude Chevalley and Alexander Grothendieck wrote collectively under the name of Bourbaki and Bourbaki's *Eléments de mathématiques* contains more than 30 volumes. Of course if (iii) were the correct hypothesis then Apollonius, who studied with the pupils of Euclid in Alexandria, must have known there was no person 'Euclid' but the fact that he wrote:-

.... Euclid did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it ...

certainly does not prove that Euclid was an historical character since there are many similar references to Bourbaki by mathematicians who knew perfectly well that Bourbaki was fictitious. Nevertheless the mathematicians who made up the Bourbaki team are all well known in their own right and this may be the greatest argument against hypothesis (iii) in that the 'Euclid team' would have to have consisted of outstanding mathematicians. So who were they?

We shall assume in this article that hypothesis (i) is true but, having no knowledge of Euclid, we must concentrate on his works after making a few comments on possible historical events. Euclid must have studied in Plato's Academy in Athens to have learnt of the geometry of Eudoxus and Theaetetus of which he was so familiar.

None of Euclid's works have a preface, at least none has come down to us so it is highly unlikely that any ever existed, so we cannot see any of his character, as we can of some other Greek mathematicians, from the nature of their prefaces. Pappus writes (see for example [1]) that Euclid was:-

... most fair and well disposed towards all who were able in any measure to advance mathematics, careful in no way to give offence, and although an exact scholar not vaunting himself.

Some claim these words have been added to Pappus, and certainly the point of the passage (in a continuation which we have not quoted) is to speak harshly (and almost certainly unfairly) of Apollonius. The picture of Euclid drawn by Pappus is, however, certainly in line with the evidence from his mathematical texts. Another story told by Stobaeus [9] is the following:-

... someone who had begun to learn geometry with Euclid, when he had learnt the first theorem, asked Euclid "What shall I get by learning these things?" Euclid called his slave and said "Give him threepence since he must make gain out of what he learns".

Euclid's most famous work is his treatise on mathematics *The Elements*. The book was a compilation of knowledge that became the centre of mathematical teaching for 2000 years. Probably no results in *The Elements* were first proved by Euclid but the organisation of the material and its exposition are certainly due to him. In fact there is ample evidence that Euclid is using earlier textbooks as he writes the *Elements* since he introduces quite a number of definitions which are never used such as that of an oblong, a rhombus, and a rhomboid.

The *Elements* begins with definitions and five postulates. The first three postulates are postulates of construction, for example the first postulate states that it is possible to draw a straight line between any two points. These postulates also implicitly assume the existence of points, lines and circles and then the existence of other geometric objects are deduced from the fact that these exist. There are other assumptions in the postulates which are not explicit. For example it is assumed that there is a unique line joining any two points. Similarly postulates two and three, on producing straight lines and drawing circles, respectively, assume the uniqueness of the objects the possibility of whose construction is being postulated.

The fourth and fifth postulates are of a different nature. Postulate four states that all right angles are equal. This may seem "obvious" but it actually assumes that space in homogeneous - by this we mean that a figure will be independent of the position in space in which it is placed. The famous fifth, or parallel, postulate states that one and only one line can be drawn through a point parallel to a given line. Euclid's decision to make this a postulate led to Euclidean geometry. It was not until the 19^{th} century that this postulate was dropped and non-euclidean geometries were studied.

There are also axioms which Euclid calls 'common notions'. These are not specific geometrical properties but rather general assumptions which allow mathematics to proceed as a deductive science. For example:-

Things which are equal to the same thing are equal to each other.

Zeno of Sidon, about 250 years after Euclid wrote the *Elements*, seems to have been the first to show that Euclid's propositions were not deduced from the postulates and axioms alone, and Euclid does make other subtle assumptions.

The *Elements* is divided into 13 books. Books one to six deal with plane geometry. In particular books one and two set out basic properties of triangles, parallels, parallelograms, rectangles and squares. Book three studies properties of the circle while book four deals with problems about circles and is thought largely to set out work of the followers of Pythagoras. Book five lays out the work of Eudoxus on proportion applied to commensurable and incommensurable magnitudes. Heath says [9]:-

Greek mathematics can boast no finer discovery than this theory, which put on a sound footing so much of geometry as depended on the use of proportion.

Book six looks at applications of the results of book five to plane geometry.

Books seven to nine deal with number theory. In particular book seven is a self-contained introduction to number theory and contains the Euclidean algorithm for finding the greatest common divisor of two numbers. Book eight looks at numbers in geometrical progression but van der Waerden writes in [2] that it contains:-

... cumbersome enunciations, needless repetitions, and even logical fallacies. Apparently Euclid's exposition excelled only in those parts in which he had excellent sources at his disposal.

Book ten deals with the theory of irrational numbers and is mainly the work of Theaetetus. Euclid changed the proofs of several theorems in this book so that they fitted the new definition of proportion given by Eudoxus.

Books eleven to thirteen deal with three-dimensional geometry. In book eleven the basic definitions needed for the three books together are given. The theorems then follow a fairly similar pattern to the two-dimensional analogues previously given in books one and four. The main results of book twelve are that circles are to one another as the squares of their diameters and that spheres are to each other as the cubes of their diameters. These results are certainly due to Eudoxus. Euclid proves these theorems using the "method of exhaustion" as invented by Eudoxus. The *Elements* ends with book thirteen which discusses the properties of the five regular polyhedra and gives a proof that there are precisely five. This book appears to be based largely on an earlier treatise by Theaetetus.

Euclid's *Elements* is remarkable for the clarity with which the theorems are stated and proved. The standard of rigour was to become a goal for the inventors of the calculus centuries later. As Heath writes in [9]:-

This wonderful book, with all its imperfections, which are indeed slight enough when account is taken of the date it appeared, is and will doubtless remain the greatest mathematical textbook of all time. ... Even in Greek times the most accomplished mathematicians occupied themselves with it: Heron, Pappus, Porphyry, Proclus and Simplicius wrote commentaries; Theon of Alexandria *re-edited it, altering the language here and there, mostly with a view to greater clearness and consistency...*

It is a fascinating story how the *Elements* has survived from Euclid's time and this is told well by Fowler in [7]. He describes the earliest material relating to the *Elements* which has survived:-

Our earliest glimpse of Euclidean material will be the most remarkable for a thousand years, six fragmentary ostraca containing text and a figure ... found on Elephantine Island in 1906/07 *and* 1907/08*... These texts are early, though still more than* 100 *years after the death of Plato* (*they are dated on palaeographic grounds to the third quarter of the third century BC*)*; advanced* (*they deal with the results found in the "Elements"* [*book thirteen*] *... on the pentagon, hexagon, decagon, and icosahedron*)*; and they do not follow the text of the Elements. ... So they give evidence of someone in the third century BC, located more than* 500 *miles south of Alexandria, working through this difficult material... this may be an attempt to understand the mathematics, and not a slavish copying ...*

The next fragment that we have dates from 75 - 125 AD and again appears to be notes by someone trying to understand the material of the *Elements*.

More than one thousand editions of *The Elements* have been published since it was first printed in 1482. Heath [9] discusses many of the editions and describes the likely changes to the text over the years.

B L van der Waerden assesses the importance of the *Elements* in [2]:-

Almost from the time of its writing and lasting almost to the present, the Elements has exerted a continuous and major influence on human affairs. It was the primary source of geometric reasoning, theorems, and methods at least until the advent of non-Euclidean geometry in the 19*th century. It is sometimes said that, next to the Bible, the "Elements" may be the most translated, published, and studied of all the books produced in the Western world.*

Euclid also wrote the following books which have survived: *Data* (with 94 propositions), which looks at what properties of figures can be deduced when other properties are given; *On Divisions* which looks at constructions to divide a figure into two parts with areas of given ratio; *Optics* which is the first Greek work on perspective; and *Phaenomena* which is an elementary introduction to mathematical astronomy and gives results on the times stars in certain positions will rise and set. Euclid's following books have all been lost: *Surface Loci* (two books), *Porisms* (a three book work with, according to Pappus, 171 theorems and 38 lemmas), *Conics* (four books), *Book of Fallacies* and *Elements of Music*. The *Book of Fallacies* is described by Proclus [1]:-

Since many things seem to conform with the truth and to follow from scientific principles, but lead astray from the principles and deceive the more superficial, [*Euclid*] *has handed down methods for the clear-sighted understanding of these matters also ... The treatise in which he gave this machinery to us is entitled Fallacies, enumerating in order the various kinds, exercising our intelligence in each case by theorems of all sorts, setting the true side by side with the false, and combining the refutation of the error with practical illustration*.

Elements of Music is a work which is attributed to Euclid by Proclus. We have two treatises on music which have survived, and have by some authors attributed to Euclid, but it is now thought that they are not the work on music referred to by Proclus.

Euclid may not have been a first class mathematician but the long lasting nature of *The Elements* must make him the leading mathematics teacher of antiquity or perhaps of all time. As a final personal note let me add that my [EFR] own introduction to mathematics at school in the 1950s was from an edition of part of Euclid's *Elements* and the work provided a logical basis for mathematics and the concept of proof which seem to be lacking in school mathematics today.

Euclid

Euclid was an ancient Greek mathematician who lived in the Greek city of Alexandria in Egypt during the 3rd century BCE. After Alexander the Great conquered Egypt, he set up Alexandria as the political and economic center, and many Greeks lived or worked there. **Euclid** is often referred to as the 'father of geometry' and his book *Elements* was used well into the 20th century as the standard textbook for teaching geometry.

Euclid's Background

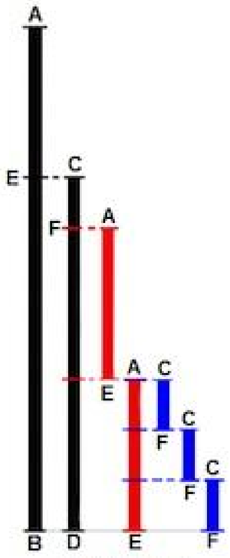
There is a lot about Euclid's life that is a mystery, including the exact dates of his birth and death, and in many historical accounts he is simply referred to as 'the author of *Elements*'. This is not a reflection of his importance, just a testament of how hard it is to maintain good records over 2,300 years. Euclid seems to have known, worked with, or influenced other major Greek figures, including Plato and Archimedes. There are at least six major works attributed to Euclid. Most of them deal with mathematical formulas, but also delve into things like the math of mirrors and reflections, astronomy, and optical illusions.

Elements: Euclid & Geometry

The most famous work by Euclid is the 13-volume set called ***Elements***. This collection is a combination of Euclid's own work and the first compilation of important mathematical formulas by other mathematicians into a single, organized format. Thus, it made mathematical learning much more accessible. *Elements* also contains a series of **mathematical proofs**, or explanations of equations that will always be true, which became the foundation for Western math.

Among these are Euclid's **theorems**, or statements proven by compounding different previously proven statements. Two of Euclid's theorems form foundational understandings about arithmetic and number theory. The first theorem is that every positive integer greater than 1 can be written as a product of prime numbers. For example, 21=3x7 or 31= 31x1. Euclid's second theorem states that there are an infinite number of prime numbers. These theorems may sound basic, but Euclid had to develop formulas to prove them. In fact, these are some of the fundamental concepts of arithmetic and had to be proven before more advanced theorems could be built upon them.

Euclid's *Elements* contains several **axioms**, or foundational premises so evident they must be true, about geometry. These include such basic principles as when two non-parallel lines will meet, that opposite angles of an isosceles triangle are equal, and how to find the area of a right triangle. *Elements* also contains geometric interpretations of algebra, such as ideas like a(b+c)=ab+ac. Most important among these is Euclid's algorithm, a formula for devising the greatest common factor of two integers.



Euclid's example

2.Paythagorsh.

Pythagoras

Introduction

Pythagoras of Samos is one of the most famous names in the history of mathematics and is recognized as the first true mathematician. Most of the information we have today on this legendary mathematician were compiled centuries after he lived and thus many are considered to be unreliable. His early biographies are written by authors who wanted to present him as some supernatural or god like figure. It is said that before the birth of Pythagoras, it was prophesized that his pregnant mother would give birth to a man supremely beautiful, wise, and beneficial to humankind.

Early Life

He was born on the Greek Island – Samos in the eastern Aegean. His birth date is estimated to be somewhere in 570 BC. His father Mnesarchus was a merchant and travelled a lot for business, Pythagoras also accompanied his father in various expeditions. When Pythagoras was 18, he visited Miletus- an ancient Greek city on the western coast of Anatolia; where he met Thales– the first know Greek philosopher and scientist. By that time Thales was very old and is not believed to have taught Pythagoras a great deal. However, it was this meeting which triggered his interest in the science of mathematics and astronomy. Thales advised him to travel to Egypt and explore these avenues. Pythagoras is believed to have had strong desire to learn and for this he had undertaken extensive travels. He was taught by wide range of teachers and philosophers. He spent years in Egypt in search of all available knowledge and received wisdom from an Egyptian priest Oenuphis of Heliopolis.

Works

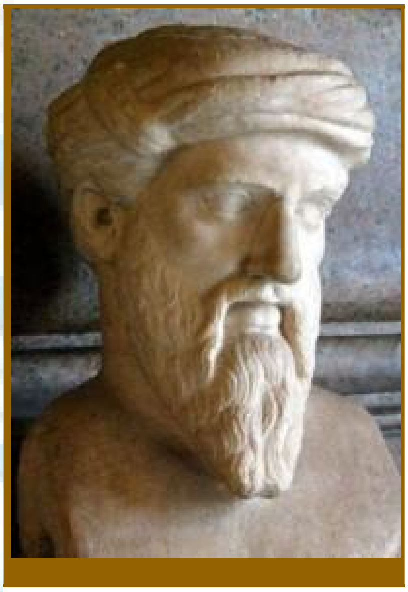
In around 530 BC Pythagoras settled in Croton- Italy, where he founded a philosophical and religious school that instantly attracted many followers. He established and headed a society called mathematikoi. The members of his society lived permanently together and followed strict rules. Pythagoras taught all the members of the society personally. It is due to the strict rules, secrecy and communal system of his school that there is not much known of Pythagoras's actual work or it is really hard to distinguish his work from that of his followers.

Pythagoras has commonly been credited for discovering the Pythagorean Theorem of geometry. Though this theorem was previously utilized by Babylonians and Indians; it is widely believed that Pythagoras or his students were the first to construct its proof. Pythagoras believed that numbers had personalities like perfect or incomplete, masculine or feminine, beautiful or ugly. He also studied properties of numbers which would be familiar to mathematicians today like even and odd numbers.

Pythagoras desired to stay out of politics, yet his society was always affected by politics. In 510 BC Croton attacked and defeated its neighbor Sybaris and there are certainly some suggestions that Pythagoras became involved in the dispute. Then in around 508 BC the Pythagorean Society at Croton was attacked by Cylon, a noble from Croton itself. Pythagoras escaped to Metapontium and the most authors say he died there, some claiming that he committed suicide because of the attack on his Society. The evidence is unclear as to when and where the death of Pythagoras occurred but his society expended rapidly after 500 BC and its contributions to mathematics are still recognized and respected.

GREEK MATHEMATICS - PYTHAGORAS

It is sometimes claimed that we owe pure mathematics to Pythagoras, and he is often called the first "true" mathematician. But, although his contribution was clearly important, he nevertheless remains a controversial figure. He left no mathematical writings himself, and much of what we know about Pythagorean thought comes to us from the writings of Philolaus and other later Pythagorean scholars. Indeed, it is by no means clear whether many (or indeed any) of the theorems ascribed to him were in fact solved by Pythagoras personally or by his followers.

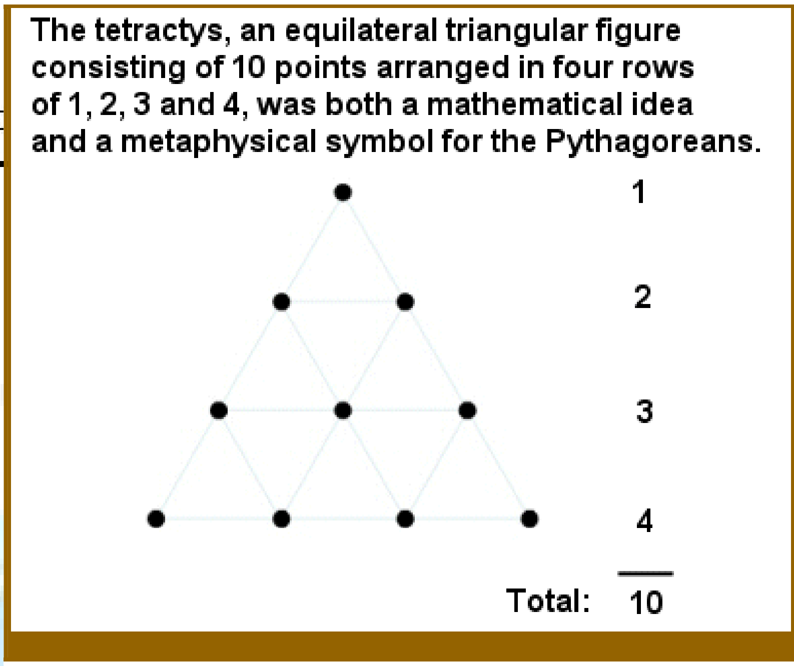


Pythagoras of Samos (c.570-495 BCE)

The school he established at Croton in southern Italy around 530 BCE was the nucleus of a rather bizarre Pythagorean sect. Although Pythagorean thought was largely dominated by mathematics, it was also profoundly mystical, and Pythagoras imposed his quasi-religious philosophies, strict vegetarianism, communal living, secret rites and odd rules on all the members of his school (including bizarre and apparently random edicts about never urinating towards the sun, never marrying a woman who wears gold jewellery, never passing an ass lying in the street, never eating or even touching black fava beans, etc) .

The members were divided into the "mathematikoi" (or "learners"), who extended and developed the more mathematical and scientific work that Pythagoras himself began, and the "akousmatikoi" (or "listeners"), who focused on the more religious and ritualistic aspects of his teachings. There was always a certain amount of friction between the two groups and eventually the sect became caught up in some fierce local fighting and ultimately dispersed. Resentment built up against the secrecy and exclusiveness of the Pythagoreans and, in 460 BCE, all their meeting places were burned and destroyed, with at least 50 members killed in Croton alone.

The over-riding dictum of Pythagoras's school was “All is number” or “God is number”, and the Pythagoreans effectively practised a kind of numerology or number-worship, and considered each number to have its own character and meaning. For example, the number one was the generator of all numbers; two represented opinion; three, harmony; four, justice; five, marriage; six, creation; seven, the seven planets or “wandering stars”; etc. Odd numbers were thought of as female and even numbers as male.

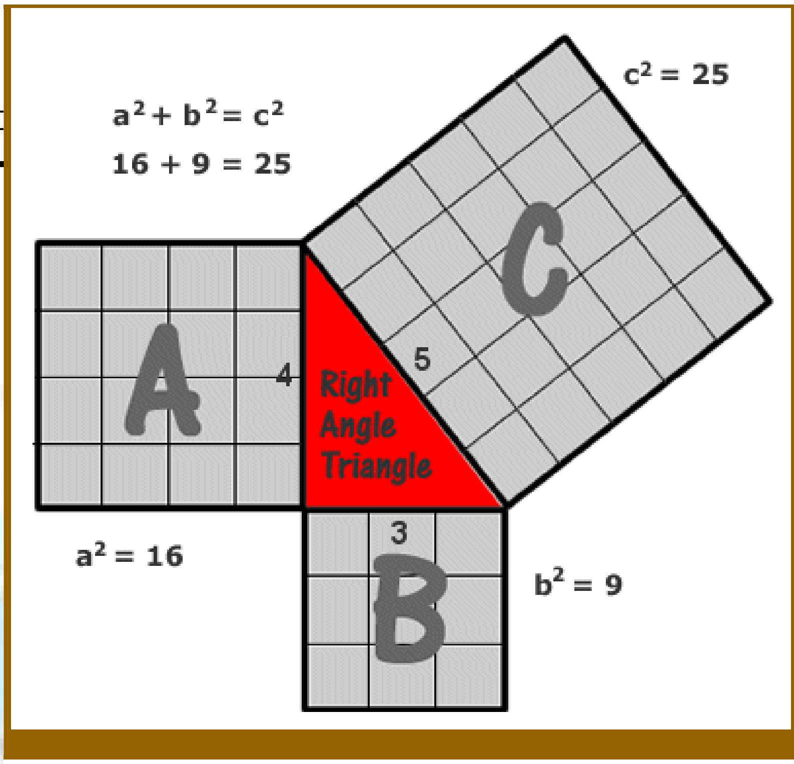


The Pythagorean Tetractys

The holiest number of all was "tetractys" or ten, a triangular number composed of the sum of one, two, three and four. It is a great tribute to the Pythagoreans' intellectual achievements that they deduced the special place of the number 10 from an abstract mathematical argument rather than from something as mundane as counting the fingers on two hands.

However, Pythagoras and his school - as well as a handful of other mathematicians of ancient Greece - was largely responsible for introducing a more rigorous mathematics than what had gone before, building from first principles using axioms and logic. Before Pythagoras, for example, geometry had been merely a collection of rules derived by empirical measurement. Pythagoras discovered that a complete system of mathematics could be constructed, where geometric elements corresponded with numbers, and where integers and their ratios were all that was necessary to establish an entire system of logic and truth.

He is mainly remembered for what has become known as Pythagoras' Theorem (or the Pythagorean Theorem): that, for any right-angled triangle, the square of the length of the hypotenuse (the longest side, opposite the right angle) is equal to the sum of the square of the other two sides (or "legs"). Written as an equation: $a^2 + b^2 = c^2$. What Pythagoras and his followers did not realize is that this also works for any shape: thus, the area of a pentagon on the hypotenuse is equal to the sum of the pentagons on the other two sides, as it does for a semi-circle or any other regular (or even irregular(shape.



Pythagoras' (Pythagorean) Theorem

The simplest and most commonly quoted example of a Pythagorean triangle is one with sides of 3, 4 and 5 units ($3^2 + 4^2 = 5^2$, as can be seen by drawing a grid of unit squares on each side as in the diagram at right), but there are a potentially infinite number of other integer "Pythagorean triples", starting with (5, 12 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), (9, 40, 41), etc. It should be noted, however that (6, 8, 10) is not what is known as a "primitive" Pythagorean triple, because it is just a multiple of (3, 4, 5).

Pythagoras' Theorem and the properties of right-angled triangles seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry, and it was touched on in some of the most ancient mathematical texts from Babylon and Egypt, dating from over a thousand years earlier. One of the simplest proofs comes from ancient China, and probably dates from well before Pythagoras' birth. It was Pythagoras, though, who gave the theorem its definitive form, although it is not clear whether Pythagoras himself definitively proved it or merely described it. Either way, it has become one of the best-known of all mathematical theorems, and as many as 400 different proofs now exist, some geometrical, some algebraic, some involving advanced differential equations, etc.

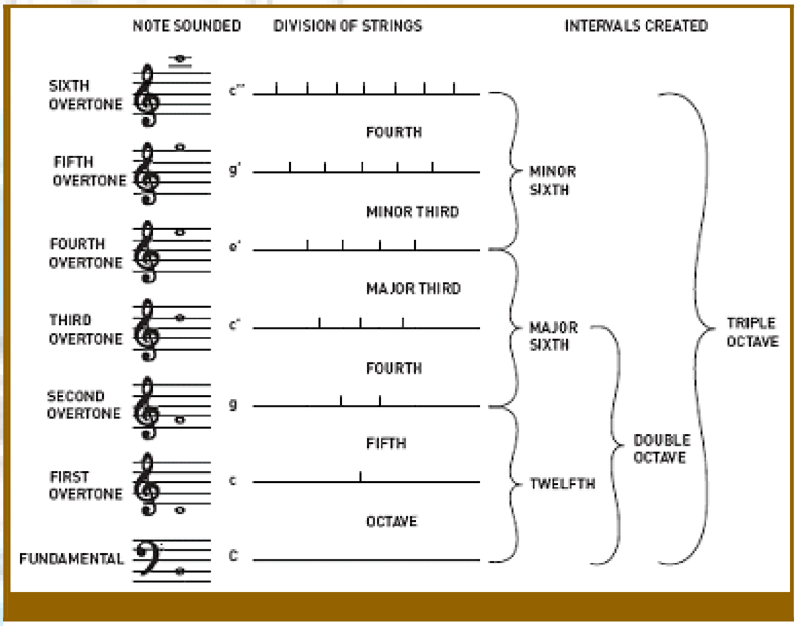
It soom became apparent, though, that non-integer solutions were also possible, so that an isosceles triangle with sides 1, 1 and $\sqrt{2}$, for example, also has a right angle, as the Babylonians had discovered centuries earlier. However, when Pythagoras's student Hippasus tried to calculate the value of $\sqrt{2}$, he found that it was not possible to express it as a fraction, thereby indicating the potential existence of a whole new world of numbers, the irrational numbers (numbers that can not be expressed as simple fractions of integers). This discovery rather shattered the elegant mathematical world built up by Pythagoras and his followers, and the existence of a number that could not be expressed as the ratio of two of God's creations (which is how they thought of the integers) jeopardized the cult's entire belief system.

Poor Hippasus was apparently drowned by the secretive Pythagoreans for broadcasting this important discovery to the outside world. But the replacement of the idea of the divinity of the integers by the richer concept of the continuum, was an essential development in mathematics. It marked the real birth of Greek geometry, which deals with lines and planes and angles, all of which are continuous and not discrete.

Among his other achievements in geometry, Pythagoras (or at least his followers, the Pythagoreans) also realized that the sum of the angles of a triangle is equal to two right angles (180°), and probably also the generalization which states that the sum of the interior angles of a polygon with n sides is equal to $(2n - 4)$

right angles, and that the sum of its exterior angles equals 4 right angles. They were able to construct figures of a given area, and to use simple geometrical algebra, for example to solve equations such as $a(a - x) = x^2$ by geometrical means.

The Pythagoreans also established the foundations of number theory, with their investigations of triangular, square and also perfect numbers (numbers that are the sum of their divisors). They discovered several new properties of square numbers, such as that the square of a number n is equal to the sum of the first n odd numbers (e.g. $4^2 = 16 = 1 + 3 + 5 + 7$). They also discovered at least the first pair of amicable numbers, 220 and 284 (amicable numbers are pairs of numbers for which the sum of the divisors of one number equals the other number, e.g. the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220).



Pythagoras is credited with the discovery of the ratios between harmonious musical tones

Pythagoras is also credited with the discovery that the intervals between harmonious musical notes always have whole number ratios. For instance, playing half a length of a guitar string gives the same note as the open string, but an octave higher; a third of a length gives a different but harmonious note; etc. Non-whole number ratios, on the other hand, tend to give dissonant sounds. In this way, Pythagoras described the first four overtones which create the common intervals which have become the primary building blocks of musical harmony: the octave (1:1), the perfect fifth (3:2), the perfect fourth (4:3) and the major third (5:4). The oldest way of tuning the 12-note chromatic scale is known as Pythagorean tuning, and it is based on a stack of perfect fifths, each tuned in the ratio 3:2.

The mystical Pythagoras was so excited by this discovery that he became convinced that the whole universe was based on numbers, and that the planets and stars moved according to mathematical equations, which corresponded to musical notes, and thus produced a kind of symphony, the “Musical Universalis” or “Music of the Spheres”.

Unit.2.

2.1.concept of microteaching and importance..

MICRO-TEACHING

DEFINITIONS OF MICRO- TEACHING

Micro-teaching has been defined in a number of ways. Some selected definitions are given below:

- Micro-teaching is a scaled down teaching encounter in class size and class time.

- Micro-teaching is defined as a system of controlled practice that makes it possible to concentrate on specified teaching behaviour and to practices teaching under controlled conditions.

- Micro-teaching is a teacher education technique which allows teachers to apply clearly defined teaching skills to carefully prepared lessons in a planned series of 5-10 minutes encounter with a small group of real students, often with an opportunity to observe the result on video-tape.

- Micro-teaching is a scaled down teaching encounter in which a teacher teaches a small unit to a group of five pupils for a small period of 5-20 minutes. Such a situation offers a helpful setting for an experienced or inexperienced teacher to acquire new teaching skills and to refine old ones.

THE BEGGININGS OF MICRO- TEACHING

Stanford University developed Microteaching in 1963 as a part of an experimental program. It was viewed as feasible in making student- teachers aware of the realities of teaching. It also served as a measurable tool in identifying teaching skills prior to actual teaching

PURPOSES OF MICRO- TEACHING

There are two purposes of Microteaching: (a) for student- teachers to develop teaching skills under controlled conditions without risking the learning of the pupils, and (b) for experienced teachers to examine and refine their techniques.

PHASES OF MICRO- TEACHING:

According to J.C. Clift and others, micro-teaching procedure has three phases:

1. Knowledge acquisition phase: In this phase, the student teacher attempt to acquire knowledge about the skill- it's rational, its role in class room and its component behaviours. For this he reads relevant literature. He also observes demonstration lesson-mode of presentation of the skill. The student teacher gets theoretical as well as practical knowledge of the skill.

2. Skill acquisition phase: On the basis of the model presented to the student-teacher, he prepares a micro-lesson and practices the skill and carries out the micro-teaching cycle. There are two components of this phase:

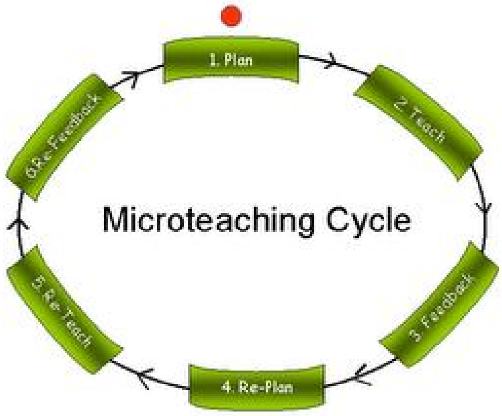
(a) feedback

(b) micro-teaching settings.

Micro-teaching settings include conditions like the size of the micro-class, duration of the micro-lesson, supervisor, types of students etc.

3. Transfer phase: Here the student-teacher integrates the different skills. In place of artificial situation, he teaches in the real classroom and tries to integrate all the skills.

MICROTEACHING CYCLE



The above diagram gives us an outlook about Micro teaching process. The cycle continues up to the extent when a trainee will able to master a specific skill.

COMPARISONS BETWEEN MICROTEACHING AND TRADITIONAL TEACHING

	MICRO- TEACHING	TRADITIONAL TEACHING
	1. Objectives are specified in behavioural terms.	1. Objectives are general and not specified in behavioural terms.
	2. Class consists of small group of 5-10 students.	2. Class consists of 40-60 students.
	3. The teacher takes up one skill at a time.	3. The teacher practices several skills at a time.
	4. Duration time for teaching is 5-10 minutes.	4. The duration is 40-50 minutes.
	5. There is immediate feed-back.	5.Immediate feed-back is not available
	6. Teaching is carried on under controlled situation.	6. There is no control over situation.
	7. Teaching is relatively simple.	7. Teaching become complex.

	8. The role of supervisor is specific and well defined to improve teaching.	8. The role of the supervisor is vague.
	9. Patterns of class room interaction can be studied objectively.	9. Patterns of classroom interactions cannot be studied objectively.

Microteaching

Tanja Gavrilović, Maja Ostojić, Dario Sambunjak, Michael Kirschfink, Thorsten Steiner, Veronika Strittmatter [Блог шаблоны joomla 3].

1. Introduction

Why microteaching?

Medical teachers most often do not receive a special training in pedagogic techniques, as it is usually not considered necessary for their recruitment or for an efficient continued performance. Their ability to teach therefore largely depends on self training, either by trial and error while teaching or by observation of colleagues, who may or may not be helpful examples.
Getting in front of students is a trying experience for a budding teacher. One may earnestly try to prepare him or herself: read books about teaching methods, attend lectures and take courses on didactics. Yet, in theory everything seems much simpler than in practice. The complexity of a teaching situation can be overwhelming. To deal effectively with it, teachers must not only have a good knowledge of the subject in hand, but also some communication skills such as ability to observe, supervise, lead a discussion and pose questions. Furthermore, a teacher should be aware of how students perceive him or her. This perception is sometimes quite different from the teacher's self-image. It is difficult to self assess one's own abilities and we benefit from colleagues' feed back to recognize our strength and identify areas for possible improvement. Evaluation of teaching by students is becoming a common practice, and a constructive feedback could be an effective way to improve one's rating as a teacher. Even the experienced educators may sometimes reflect about strengths and weaknesses of their teaching style.

What is microteaching?

Microteaching is an excellent way to build up skills and confidence, to experience a range of lecturing/tutoring styles and to learn and practice giving constructive feedback. Microteaching gives instructors an opportunity to safely put themselves "under the microscope" of a small group audience, but also to observe and comment on other people's performances. As a tool for

teacher preparation, microteaching trains teaching behaviors and skills in small group settings aided by video-recordings. In a protected environment of friends and colleagues, teachers can try out a short piece of what they usually do with their students, and receive a well-intended collegial feedback. A microteaching session is a chance to adopt new teaching and learning strategies and, through assuming the student role, to get an insight into students' needs and expectations. It is a good time to learn from others and enrich one's own repertoire of teaching methods.
A microteaching session is much more comfortable than real classroom situations, because it eliminates pressure resulting from the length of the lecture, the scope and content of the matter to be conveyed, and the need to face large numbers of students, some of whom may be inattentive or even hostile. Another advantage of microteaching is that it provides skilled supervisors who can give support, lead the session in a proper direction and share some insights from the pedagogic sciences.

Historic context

The history of microteaching goes back to the early and mid 1960's, when Dwight Allen and his colleagues from the Stanford University developed a training program aimed to improve verbal and nonverbal aspects of teacher's speech and general performance. The Stanford model consisted of a three-step (teach, review and reflect, re-teach) approach using actual students as an authentic audience. The model was first applied to teaching science, but later it was introduced to language teaching. A very similar model called Instructional Skills Workshop (ISW) was developed in Canada during the early 1970's as a training support program for college and institute faculty. Both models were designed to enhance teaching and promote open collegial discussion about teaching performance.

In the last few years, microteaching as a professional development tool is increasingly spreading in the field of medical education.

2. Planning a Microteaching Session

The duration of a Microteaching session depends on the number of participants. Microteaching should take place in two separate classrooms where the second room is required for videotape viewing. It is helpful to organize professional videotaping, although this can also be done (taken over) by the participants upon instruction.

Equipment for Microteaching session:

- TV/Computer set
- video recorder/camcorder
- camera

- tapes for camera
- black- or whiteboard, flipchart, pin board, markers with different colors

One-day plan for Microteaching (an example):

- 09:00-09:30 Introduction to microteaching given by a professional supervisor
- 09:30-10:00 Preparation of the micro lessons
- 10:00-... Microteaching session (each segment about 20-30 min)

3. Steps in Microteaching and Rotating Peer Supervision

I. Preparation

Each participant of the session prepares a teaching segment. The presenter gives a brief statement of the general objectives of his/her presentation to be addressed. The group may be asked to focus their attention to particular elements of the lesson or of the teaching style. This may include pace, clarity of explanation, use of media, voice and body language, level of group interaction.

II. Presentation and Observation

Each participant presents his/her 10-minute teaching segment. He/she is allowed to use the media available. During the presentation, other participants serve as members of a supervisory team and take notes for the group feedback. Special assessment forms (Tables 1 and 2) may be helpful in standardizing the observation and feedback process. Each lesson is videotaped. Although the lesson is short, objective and procedures should be clear to generate useful discussions.

III. Videotape Viewing

The presenter watches the tape of his/her presentation and decides whether or not the objectives were accomplished. He/she also makes a list of strengths and suggestions for personal improvement. Then he/she again joins the supervisory team. In the meantime the supervisory team discussed and made conclusions about the teacher's lecturing.

IV. Discussion and Analysis

While the presenter goes to another room to view the videotape, the supervisory team discusses and analyses the presentation. Patterns of teaching with evidence to support them are presented. The discussion should focus on the identification of recurrent behaviors of the presenter in the act of teaching. A few patterns are chosen for further discussions with the presenter. Only those patterns are selected which seem possible to alter and those which through emphasis or omission would greatly improve the teacher's presentation. Objectives of the lesson plan are also examined to determine if they were met. It is understood that flexible teaching sometimes includes the modification and omission of objectives. Suggestions for improvement and alternative methods for presenting the lesson are formulated. Finally, a member of the supervisory team volunteers to be the speaker in giving the collected group feedback.

4. Appendices

Characteristic	Aim	Observed
Duration of presentation	Approx. 10 minutes	Start time....... Finish time........ Total duration...........minutes
Comprehensibility	The presentation should be given in comprehensible language.	The presentation is sufficiently comprehensible. Comprehensibility should be improved.
Visualization	The presentation should be accompanied by selected elements of visualization.	The following forms of visualization have been used: • slides • handouts for the participants • pin board • flipchart • white/black board The visual elements assist the understanding. The visual elements should be improved.
Density of information	Density of information should be high. However, it must not	The density of information seems to demand too much of the learner.

	overtax the learner.	Density of information is rather high. Density of information is rather low. The density of information seems to demand too little of the learner.
Further observations	-	-

Table 2. Characteristics of a good quality presentation. (Tick Yes or No when assessing)			
Is the presentation comprehensible?	- speaks freely	yes	no
	- short sentences	yes	no
	- terminology is comprehensible	yes	no
	- presentation is well-structured	yes	no
	- conciseness	yes	no
	- use of examples	yes	no
Is the presentation stimulating?	- eye contact	yes	no
	- speaker varies his position	yes	no

	- participants are encouraged to contribute	yes	no
	- use of humor to create a relaxed atmosphere	yes	no
	- presented with commitment	yes	no
	- friendly/respectful behavior	yes	no
Is the visualization helpful?	- visualization is clear and well-structured	yes	no
	- includes graphic elements and optical stimuli	yes	no
	- easily legible writing	yes	no
	- colors help to focus on the important aspects	yes	no
	- comprehensible visualization	yes	no
	- affectionate layout	yes	no

2.2.introduction of skill, introducation, questionnaire ,blackboard work, example, reinforcement skill

Skill of Explanation

Meaning : To present the subject-matter in the simplified form before the pupils and making it acquirable is termed as skill of explanation. It is considered necessary in all the subjects. In its absence the presentation of the subject-matter is not possible. In it such words are used in the statements by which the statements exhibit the clarity of their meanings. Sometimes when a word is removed form the statements, it becomes vague.

List of Teaching Skills

Different educationists have presented various lists of teaching skills based on their research outcomes.

1 Dr. B.K. Passi (1975)

A) Writing Instructional objectives

B) Introduction of the lesson or set Introduction

C) Fluency of questioning

D) Probing questions

E) Explaining

F) Illustrating

G) Stimulus variation

H) Silence and non-verbal curves

I) Reinforcement

J) Increasing students, participation

K) Use of Blackboard

L) Achieving closure

M) Attending behaviour of the pupils

2. Stanford University:

A) Stimulus variation

B) Closure

C) Silence and non-verbal cues

D) Reinforcing Pupil's Participation

E) Set Induction

F) Fluency in question

G) Probing questions

H) Higher order questions

I) Divergent questions

J) Illustrating and use of examples

K) Lecturing

L) Planned repetition

M) Completeness of communication

N) Recognising Attending behaviour

Meaning of Various Teaching Skills:

1) Set Induction : It means the introduction of the lesson. It links previous knowledge with the present knowledge. It is know as the skill of introduction or set-induction skill.

2) Stimulus Variations: It means changing of gestures and positions by the teacher. If a teacher does not change his gestures and positions during the teaching process it becomes bore and lacks in interest. Hence, it is necessary to provide the training to the teachers in the skill of changing the gestures.

3) Probing Questions: It is concerning with the questions to be asked about the content in more depth. It stimulates the cognitive development of the Punjab.

4) Illustration: There are two teaching methods – continuous lecturing method and demonstration method. The pupil teachers should explain the concepts through examples and by displaying pictures and charts. It is called the illustrating skills.

5) Lecturing: It is concerned with the effective presentation of the content. The teacher leaves has impressions by using many techniques and tactics through the skill. It is also known as 'Communication Skill'.

6) Skill of Explaining: It means – use of explaining or connecting links to link the statements or systematic information. When a teacher shows his behaviour while explaining the pupils about 'What' 'Why' and 'How' regarding some facts, principles and concepts, that behaviour constitutes the skill of explaining.

7) Use of Blackboard: It is very essential in the class. Its use also needs special training. The necessary components of blackboard work are clarity of handwriting, legibility and rationale of blackboard work etc.

8) Closure: It means to finish some task, i.e. in class the pupil-teacher exhibit various behaviours. If we divide thee behaviours in smaller units, these are termed as 'skills, when a pupil-teacher delivers lecture and sums up properly and in an attractive way, the skill is termed as 'Closure Skill'. The lesson remains ineffective in the absence of proper closure.

9) Use of A.V. Aids: It is essential to make the teaching task more attractive and effective. As its use also needs a skill, the training of using A.V. aids is also desirable for the teachers.

10) Skills for Class Management: Both- social as well as educational activities performed in order to create proper environment for learning in the classroom. The performance of these activities needs special skill. As these activities manage the class it is called 'Skill for Class Management'.

11) Increasing Pupil Participation: It is concerned with increasing pupil participation which means – pupils, direct behaviour which is observable. This includes both responses and reactions of the pupils along with their own new activities.

12) Recognising Attending Behaviour: On the basis of pupils' behaviour. the teacher selects his own activities and also distinguishes the interesting and boring activities.

PRACTISING SOME TEACHING SKILLS

1) Introduction Skill

Meaning:

Introduction skill is also known as self-induction skill, it is concerned with the lesson's initiation. If the beginning of the lesson is effective, its success is almost definite. The introduction of the lesson, keeps active the imaginative and creative powers of the teacher.

ELEMENTS:

(i) Previous Knowledge : Awareness of previous knowledge of the pupils is must before starting the teaching of new content. It should concentrate on the same topic which is to be started for teaching. It will create interest in the pupil-teachers for teaching new contents.

(ii) Proper Sequence : Coordination among ideas, questions and statements to be used is a must while starting the lesson.

(iii) Proper Use of Devices and Aids : Various aids are used keeping in mind the objectives of the lesson. Monotonous type of teaching bores the pupils, which can be controlled by selecting properly and attractive use of audio-visual aids.

(iv) Relationship between Contents, Objectives and Statements/ Relevance of the content : While teaching the lesson, the statements to be used must have some relationship with the new contents to be taught and these contents must be selected to the pre-determined objectives.

(v) Technique for creating Interest and Motivation : The teacher should have the capacity of creating interest and motivation in the pupils.

(vi) Duration of Introduction : Introduction should be neither lengthy nor too short. Its duration should be restricted to the reaction of interest and motivation in the pupils.

Micro-Lesson Plan

PT's R.No…… **Class: 10+2**

Skill: Induction/ Introduction **Teach/ Reteach**

Subject: Economics **Date :**

Topic: Factor of Green Revolution

Sl. No.	**Teacher's Activity**	**Pupils' Activity**	**BBW/ A.V.Aids**
1	Where the most population of our country lives?	In the village	
2	What is the main occupation of villagers?	Agriculture	
3	Which crops the farmers grow in their	Wheat, Rice, cotton, sugar-cane etc.	

	fields?		
4	What we call to the sudden growth in Production of rice and wheat in 1966-67 in Punjab ?	Green Revolution	
5	What are the major factors behind green revolution?	*NO Response*	
5	What is difference in act of Medium of Exchange and deferred payments?		**(Refocusing**)
6	1. What are secondary function of money? 2.How money is helpful in Capital formation?		**Increasing Critical Awareness**

Micro-Lesson Plan

RATING SCALE

PT's R.No…… **Class: 10+2**

Skill: Skill of Probing Question **Teach/ Reteach**

Subject: Economics **Date :**

Topic: Functions of Money

Components of Skill	Grading					*Supervisor Remarks*
Prompt	***A***	***B***	***C***	***D***	***E***	
Seeking Further Information	***A***	***B***	***C***	***D***	***E***	
Redirecting	***A***	***B***	***C***	***D***	***E***	
Refocusing	***A***	***B***	***C***	***D***	***E***	
Increasing Critical Awareness:	***A***	***B***	***C***	***D***	***E***	
	A	***B***	***C***	***D***	***E***	

NOTE:

“A” means 95 to 100 per cent correct use of component.

“B” means 85 to 94 per cent correct use of component.

“C” means 75 to 84 per cent correct use of component.

“D” means 65 to 74 per cent correct use of component.

“E” means below 65 per cent correct use of component.

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3) Lecturing Skill

Meaning : The verbal communication of self-ideas, concepts and principles is called lecturing. The teachers use lecturing skill for most of the time to import their ideas and knowledge to the pupils. Most of the time in the class is absorbed by lecturing. Inspite of its demerits, it is still used frequently.

Components of Lecturing Skill

A) Ability to start lecturing

B) The simplicity of the language used in lecture

C) Speed of the lesson

D) Use of A.V. Aids

E) Repetition of teaching points

F) Change in Inter-action

G) Use of interesting tactics

H) Clarity of the voice

I) Clarity and relationship of the statements

J) Ability to sum up the lecture

The observation and evaluation sheets for lecturing skill can be prepared following the pattern of introduction skill.

4) Skill of Discussion

Meaning : The teacher has to seek the help of discussion method in the teaching process in order to clarify the subject-matter. The teacher invites the views of the pupils about some teaching point presented before them. The pupils express their view one by one. In teaching process, it is called discussion. Creation of an environment charged with discussion is a part of teacher's ability. A teacher who has discussion skill is a successful and impressive teacher. It is one which also involves other skill such as skill of introduction, skill of stimulus variation skill of silence and non-verbal cities, skill of reinforcement. The practice of discussion skill accompanies the practice of all these skills.

Components of Skill of Discussion

A) Creation of proper environment.

B) Asking questions.

C) Stimulus variation

D) Increasing pupil participation

E) Silence and non-verbal cues

F) Variation in inter-action style

G) Increasing critical awareness

H) Developing lecturing skills

I) Reinforcement

5) Skill of Demonstration

Meaning : The teaching process can not be completed verbally. The teacher has to do something in the classroom in order to clarify the subject-matter. Sometimes he has to seek the help of demonstration method, specifically in science subjects. For example if a method of preparing some gas is to be explained in the class, the teacher will demonstrate the method of preparing gas in the class. Then he will ask other pupils to do the same themselves.

Components :

(i) Relevancy with the contents,

(ii) Pupil Participation,

(iii) Handling of equipments and materials,

(iv) Appropriateness of the demonstration,

(v) Visibility of the demonstration work,

(vi) Emphasizing Cause-Effect Relationship,

(vii) Capability of Drawing results.

6) Skill of Illustrating with Examples

Meaning : In class, complex concepts, thoughts etc. are to be explained to the pupils after simplification. For this, the teacher uses the skill of illustration. When a teacher seeks the help of pictures, clarification, examples etc. to simplify and clarify the subject matter, it is known as a skill of illustrating with examples. Also known as skill of interpretation.

Objective of Illustration :

(i) To make the lesson interesting,

(ii) To link unknown knowledge to known,

(iii) To simplify and familiarize with the complex and unfamiliar knowledge,

(iv) To use various senses of the pupils.

Precautions while Using Illustration :

(i) It should be related to the specific objectives,

(ii) It should be related to the main concept,

(iii) It should be interesting,

(iv) It should be according to mental level of the pupils,

(v) It should be of various types.

Approaches to Illustrate :

(i) Non-Verbal : Non-Verbal approach includes real objectives, model, map, picture, figure etc.

(ii) Verbal : Verbal approach includes example, word-picture, comparisons, story etc.

Components of Skill of Illustration :

(A) Selection of simple examples,

(B) Interesting examples,

(C) Selection of appropriate medium for examples,

(D) Selection of proper methods for examples,

(E) Appropriate number of illustrations,

(F) Relationship of concepts and thoughts with the examples,

(G) Illustrations or examples given by the pupils,

(H) Understanding of concepts.

Evaluation Sheet

The evaluation sheet of the skill is prepared on the basis of various components of the skill.

(7) Skill of Explanation

Meaning : To present the subject-matter in the simplified form before the pupils and making it acquirable is termed as skill of explanation. It is considered necessary in all the subjects. In its absence the presentation of the subject-matter is not possible. In it such words are used in the statements by which the statements exhibit the clarity of their meanings. Sometimes when a word is removed form the statements, it becomes vague.

Components of Skill of Explanation

(A) Coordination in Statement : Used during the explanation it is very essential as otherwise there will be all hotch-potch.

(B) Lack of Irrelevant Statement : While presenting the subject matter, only the concerned statements should be used.

(C) Fluency in Language : The teacher should use such fluent language that the pupils may listen and understand the thoughts of the teachers.

(D) Connecting Links : The use of words, idioms or connecting links is essential to link the different thoughts or statements, such as 'therefore' as a result of etc.

(E) Clear Beginning Statement : Before starting any explanation., the teacher should make the pupils aware of what he is to teach on that day through a clear beginning statement.

(F) Use of Proper Words : The teacher should use proper words for explaining an object or an event otherwise he would be in a state of confusion.

Precautions for Skill of Explaining :

1. It should be in simple language.
2. It should not be given the shape of an advice.
3. The thoughts included in it should be in a sequence.
4. Irrelevant things should not be included in it.
5. It should be according to the age, experience and mental level of the pupils.
6. It should be complicated, lengthy and small according to the objectives of the lesson.

(8) Skill of Stimulus Variation

Meaning : The skilful changes in the stimuli is known as the skill of Stimulus Variation. The teacher's teaching in the classroom seeks to make the lesson impressive. For this, he uses various types of methods and techniques. The teacher may present various types of stimuli in order to attract the pupils. Thus he can motivate them. He presents various stimuli such as movement of the body, gesture, changes in speech, focusing of the feeling, change in the interaction style in the pupils, pause and change in the order of audio-visual aids. The teacher can attract the pupils by changing all these aspects.

Components of Skill of Stimulus Variation

(A) Body Movements : The physical movements of the teacher in the class carry much importance. While excess of physical activities is undesirable, the teacher without these activities like a stone-idol.

(B) Gestures : Gestures also prove helpful is making the lesson effective in the classroom. These include facial gestures (laughing, raising eyebrows, emotions etc.) signals, of eyes, nodding, hand signals (signal to stop, signal to continue the task and signal to keep quiet) etc.

(C) Changes in Speech Pattern : Teacher should bring fluctuations in his voice. The pupils feel boredom with the speech at the same pitch, and they get deviated from the lesson.

(D) Focusing : It is used to concentrate the attentions of the pupils on some specific point or event. In it verbal focusing, gesture focusing and verbal or oral-gesture focusing are included. In the verbal focusing, the attention. In the gesture-focussing, the attention of the pupils is concentrated with the help of gestures towards some desirable direction or an object.

(E) Change in Interaction Style : Interaction between the teacher and the pupils is very essential in the classroom. The style of interaction in the class-room should go on changing.

(F) Change in Audio-Visual Sequence : A continuous change in the sequence of using audio-visual aids concentrates the attention of the pupils upon the teacher. The teacher should use sometimes visual and at the other times audio-aids.

(G) Pause : As and where the needs arises the teacher should use pauses in his teaching process.

SKILL OF REINFORCEMENT

Introduction

Every responding pupil of the class needs social approval of his behaviour. To satisfy his this need, he is always eager to answer each question known to him. If the teacher is encouraging the pupils by statements like, “good”; that is very good and certain non-verbal expressions, as smiling, nodding the head; and paying attention to the responding pupil, the pupil participation in the class is maximised. The main theme of the skill is that encouraging remarks of the teacher increases and discouraging remarks decreases the pupil-participation in the development of the learning process.

Components of Skill

1. Positive Verbal Reinforcement.

2. Positive Non-Verbal Reinforcement.

3. Negative Verbal Reinforcement.

4. Negative Non-Verbal Reinforcement.

5. Wrong use of Reinforcement.

6. Inappropriate use of Reinforcement.

Let us discuss these expressions.

Positive -Verbal Reinforcement : These are the positive comments given by the teacher on the correct response of the pupil. They are :

(i) Using words and phrases like, “good”, “very good” and excellent.

(ii) Repeating and rephrasing pupil’s response.

(iii) Using pupils idea in the development of the lesson.

(iv)Using extra-verbal cues, like “um”, “um”, “aha” to encourage pupils.

(v) Using prompts like carry on, think again etc. to help the pupil give correct response.

Positive Non -Verbal Reinforcement : The teacher gives comments to pupils on their correct response without using words :

This he does by:

nodding the head,

smiling,

patting,

looking attentively at the responding pupil,

writing pupil's answer on the black boards.

The teacher encourages the pupils to participate maximally in the development of the lesson.

Negative Verbal Reinforcement : The teacher gives comments on the incorrect or partially incorrect response by telling that the pupil's response is incorrect or making sarcastic remarks like "**idiots"**, **"stupid"** etc. Such behaviour of the teacher discourages pupil-participation and should not be used.

Negative Non -Verbal Reinforcement : The teacher shows his disapproval without using words. This involves, frowning, staring, looking angrily at the responding pupil, when he gives wrong response.

This type of behaviour of the teacher creates fear in the minds of the pupil and decreases pupilparticipation.

Wrong use of Reinforcement : This is the situation, where the teacher does not give reinforcement when the situation is demanding encouragement.

Inappropriate use of Reinforcement : This is the situation when the teacher does not encourage the pupil with respect to quality of his response. He uses same type of comment for every response. After going through above components and the skill, you have understood it well. Let us use them in the following Micro-lesson plan.

Announcement of the Topic:- After getting the satisfactory or unsatisfactory response of the students. P.T. will announce the topic by saying , "Well students, Today we shall study about '**Factors of Green revolution**".

RATING SCALE

PT's R.No…… **Class: 10+2**

Skill: Introduction/ Induction **Teach/ Reteach**

Subject: Economics **Date**
:

Topic: Factor of Green Revolution

Components of Skill	Grading					*Supervisor Remarks*
Utilization of previous knowledge	***A***	***B***	***C***	***D***	***E***	
Using appropriate devices and techniques	***A***	***B***	***C***	***D***	***E***	
Continuity/ Proper sequence	***A***	***B***	***C***	***D***	***E***	
Relevance of verbal and non-verbal	***A***	***B***	***C***	***D***	***E***	

behaviour						
Duration of the announcement the of the Topic	***A***	***B***	***C***	***D***	***E***	

NOTE:

“A” means 95 to 100 per cent correct use of component.

“B” means 85 to 94 per cent correct use of component.

“C” means 75 to 84 per cent correct use of component.

“D” means 65 to 74 per cent correct use of component.

“E” means below 65 per cent correct use of component.

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(2) SKILL OF PROBING QUESTION

Meaning : Probing questions are those which help the pupils to think in depth about the various aspects of the problem. By asking such questions again, the teacher makes the pupils more thoughtful. He enables the pupils to understand the subject deeply.

Situations for Use of Probing Question :

(i) When a pupil express his inability to answer some question in the class or his answer is incomplete, the teacher can ask such questions which **“Prompt”** the pupils in solving the already asked questions.

(ii) This technique is known as **‘Seeking further information’** technique, when the pupils answer correctly in the class, but the teacher wants to seek more information.

(iii) Sometimes, the teacher can ask probing questions to **concentrate the attention** of the pupils. Similarly, for the very same purpose, the teacher may ask the same question from other pupil. This is known as **'Refocusing'**.

(iv) In classroom, if the teacher wants to introduce the pupils with various aspects of the problem, then he can ask the **same question after slight changes in the language**. This is known as **'Redirection'.**

(v) In class, the teacher can ask questions bearing '**why'** in order **to develop the reasoning power** of the pupils. By getting motivated from such questions, pupils involve themselves in the process of reasoning. This is known as **'Critical Awareness Technique**'.

Components of Probing Question Skill

(i) Prompting : The teacher can ask such questions when the pupil expresses his inability to answer or accepts that 'he does not know'. In such questions pupils get some prompting regarding the answer.

(ii) Seeking Further Information : In class, when the pupils are unable to answer any question or answer partially, then in order to receive complete and correct answer, the teacher can ask such questions by accepting that the answer given is correct, but the pupil should reveal more. There can be alternate answer to the asked question. In this way, the teacher can seek maximum information from the pupils.

(iii) Refocusing : Sometimes, the teachers are not satisfied with the pupils, answers, they draw the attention of the pupils towards different situations where similar problems can arise. This makes the transfer of learning possible.

(iv) Redirection : In class, the teacher tries to develop the reasoning power in the pupils by asking various questions. This enables the teacher to encourage the pupils for maximum participation.

(v) Critical Awareness : In this component, the questions bearing 'Why' and 'How' are asked. By asking such questions, the teacher can develop critical awareness in the pupils.

Prompting Technique: This technique means to go deep into the pupil's response when it is incorrect or no response. Then a series of hints or prompts are given to pupil through step by step questioning in order to lead the pupil to the desired correct response. Let us take the following example:

Example :

T : What are the functions of Money ?

P : No response.

T : What you pay purchase a chocolate?

P : Money or Rupees or Paisa.

Seeking Further Information :

This technique is used when the response of pupil is incomplete or partially correct. The teacher helps the pupil to clarify or elaborate or explain his initial response by asking more small questions or creating situation in which the pupil is made to think and respond.

Example :
T : What are the functions of Money ?
P : No response.
T : What you pay purchase a chocolate?
P : Money or Rupees or Paisa.
T : What are the other functions of Money ?

Redirection :

This technique involves asking the same question from another pupil. The main purpose of this technique is to increase more and more pupil participation. When the situation is of no response or incorrect response prompting should be preferred to redirection.

Example :
T: What are the characteristic of Money ?

Ram: No response
Sohan: It is transferable (Redirection)
Mohan: It act as Medium of Exchange (Redirection)
Krihsan: It is act as store of Value(Redirection)
Gopal: It is helpful in deferred payments(Redirection)

Refocusing:

It is used when the pupil's response is correct. This involves comparing the phenomena in his response with other phenomena either for similarity/difference or relationship between the two situations. How one thing in point is different from the other thing? How one response of the pupil is related to any other point? How one thing is similar to another thing ? Such type of questions are put to the pupil.

Example:

T: What is difference in act of Medium of Exchange and deferred payments (**Refocusing**)
T: How act of store of Value is difference form act of Medium of Exchange **(Refocusing**)

Increasing Critical Awareness:

This technique is used when the pupil's response is correct. The teacher puts higher order questions to stimulate the pupil to think beyond what the pupil knows. This involves the 'how' and 'why' and sometimes 'what' type of questions on the point under discussion.

Example:

1. What are secondary function of money?
2. How money is helpful in Capital formation? etc.

Now you have understood the skill and its components. How to practice these components has been illustrated by the following micro-lesson plan.

Micro-Lesson Plan

PT's R.No…… **Class: 10+2**
Skill: Skill of Probing Question **Teach/ Reteach**
Subject: Economics **Date**
:

Topic: Functions of Money

Sl. No.	**Teacher's Activity**	**Pupils' Activity**	**Components of Skill**
1	What are the functions of Money ?	*No Response*	
2	What you pay purchase a chocolate?	*Money or Rupee*	***Prompt***
3	What are the other functions of Money ?	Pay Loans, Help in saving Capital formation	**Seeking Further Information**
4	What are the characteristic of Money ?	Ram: No response Sohan: It is transferable (Redirection) Mohan: It act as Medium of Exchange (Redirection) Krihsan: It is act as store of Value (Redirection) Gopal: It is helpful in deferred payments (Redirection)	***Redirecting***

2.3.aims structure

1.State the concept of general objective.

An example of a general objective is, "To make the student of information science capable of identifying the needs of users of a particular documentation system." A specific objective derived from this general objective is, "The student must be able to identify different types of documentary information networks." From these examples, it is evident that specific objectives are usually derived from general objectives.

2.state the concept of specific objectives.

Specific objective are usually expressed in terms of the student, and they are unequivocal, which means that they are expressed clearly and have only one interpretation. They also only describe behaviors that can be observed in the subject. UNESCO also indicates that specific objectives detail the unique conditions for the manifestation of certain behaviors and the criteria that must be met to determine whether the objective has been attained.

Unit.3.the development of lesson planning and approach

3.1.clarify the concept of bridge lesson and importance

The bridge lesson is connect of micro lesson and stray lesson. there for to bridge between both them. There are use two or three skills in of this lesson. There is not revision of education in bridge lesson. Bridge lesson is mini teaching so called bridge lesson. There is trainer use all skills in bridge lesson and teaching the student in the classroom. There is 12 to15 minutes in bridge lesson. The bridge lesson is increase the trainers confidence.

Importance.

1.bridge lesson is connected of micro lesson and stray lesson.
2. There are use many skills in the bridge lesson.
3.The trainer's really teaching method put to wait in classroom training .
4.The student gets real experience in teaching work.

5.The trainer is increase his confidence .
6.There are increase the students in this lesson.

3.2.The bridge's planning.

3.2.bridge lesson planning.

1.It is 15 to 20 minutes lesson plan
2. It works like bridge between stray lesson and micro lesson.
3.Each skill has been fully developed in this lesson.
4.The teacher can use the different teaching aids.
5.By this lesson, teacher gets confidence in teaching.

3.3.Explain the facts, definition, principle, mathematic education.

Facts.

A "basic fact" in math is defined as any mathematical number, fact or idea instantly recalled without resorting to strategies, according to NZCER.org. The main basic facts encountered in math are "whole-number" basic facts, in particular multiplication, addition, division and subtraction.

A fact is a true statement under the set conditions. A thing that is indisputably the case. Facts can generally be reproduced to be verified. A single counter example will immediately demote a fact to a false claim.

Math Diagram

A math diagram is any diagram that conveys mathematical concepts. This includes basic charts and graphs as well as sophisticated logic and geometrical diagrams.

Math and science concepts are often easier to understand with a visual aid. Mathematical diagrams are often created to illustrate concepts in textbooks or for presentation posters used at conferences. Some may also find it useful to create a math diagram for personal reference to work through a difficult problem visually.

diagram

Also found in: Thesaurus, Medical, Legal, Acronyms, Encyclopedia, Wikipedia.

di·a·gram

(dī′ə-grăm′)

*n.*1. A plan, sketch, drawing, or outline designed to demonstrate or explain how something works or to clarify the relationship between the parts of a whole.

2. *Mathematics* A graphic representation of an algebraic or geometric relationship.

3. A chart or graph.

PRINCIPAL.

Fully investigation and after thought really proof s right decision is called to principal.

Exa. Pythagorash principal

A rule is an informal axiom that expresses a philosophical point. A rule governs how a subject behaves under certain circumstances. A rule indicates increased chance of a certain outcome when a subject is in a certain state. Newton's "rules" or Occam's Razor are examples.

Laws are statements which are inferred by observation. Laws are not proved. Laws are demonstrated based on repeated observations. It expresses a causal relationship between entities under certain conditions, and is often expressed mathematically. The Laws of Thermodynamics (so often quoted without the mathematical context and misstated for that very reason) are good examples.

3.4.The arrangement of proper number for contents .

The content make analysis so content divided many small part. there for content is not arrange proper and logically number so contents small parts does not understanding clearly. The student did not understanding the content. Because the student like difficult content.so necessary the content analysis after arrange the content in proper. we attended logical number because the contents proceed does not logically so the student does not under standing this topic.The avoided the student problem and processed the content simply So need arrangement proper and logical number.

The content arrangement after teaching-learning activities, educational steps, reference books, methods and technique use in content.

3.5.approaches..inductive-Deductive ,problem solving, Analytic-synthetic,

Inductive approach.

One of the important part of searching new knowledge in science is to formulate and test hypotheses. Hypotheses are simply tentative explanations put forth to account for observed phenomena. Formulating testable hypotheses draws heavily on the scientist's creativity and imagination. One of the general pattern to formulate hypotheses is inductive logic or induction.

If a person tastes a green apple he finds it sour. if he repeats such experience he related the characteristics of green apple to the other characteristics of the apple. namely sour. thus he draws a generalization all green apples are sour. this is known as inductive method. it has four characteristics.

It begins with observations
It leads to hypothesis development.
It proceeds from specific to general.
It is a method of discovery.

Procedure of inductive Method.

=observation of the particular instances.

Various instances are provided for study to the learners in this stage .The more the instances the surer in results.

=Analysis of the instances

In this stage all the instances are stuied analytically. Their natures with references to the aim are studied.

=Finding out the points of difference and similarity.

In the third stage the points of difference and similarity are discovered and noted.

=Classification.

Now it the proper stage to classify the differences and similarities to build up the final solution of problem.

=Generalization.

When all the points of differences and similarities have been classified the generalization is started.

Merits of inductive Method.

1.learning effective and enduring
2.Learning meaningful.
3.learning motivated.
4.Learning learn self dependence.
5.Useful for practical life
6.It encourages creative thinking
7.The method is psychological.
8.A natural method.
9.no problems of discipline, inattention and so on.
10.Promotes teacher –pupil relation
11.promotes co-operation

Limitations of inductive Method.

1.Not complete in itself
2.Very slow and lengthy.
3.Not possible to learn all.
4.not suitable for small children.
5.learners may go astray.

6.not applicable in all subjects.

Deductive Method.

How do experiments test hypothesis? Experiments test hypothesis by testing the correctness of prediction that can be derived from them. This process involves the deductive logic deduction.

Deduction reasoning is the heart and soul of mathematics. Mathematics generally deal with symbols where deduction approach is helpful.

Some of the charateritics of deduction logic as follow:

*Deduction approach begins with hypothesis which lead to prediction.

*It proceeds from general to specific.

*It is a method of verification.

Ausbel believes that structured concepts can be taught to student. He regards the human mind as a system for receiving, processing, and storing information. Ausubel advocate learning new ideas relating them to available concepts or anchoring ideas. Ausubel s Advance Organizers helps learner's link new ideas with existing concepts. Advance organisers fit well in a direct instruction method called deduction teaching, the opposite of Bruner s inductive teaching.

The figure shows Advance Organisers and Deductive Teaching: Teach from general to specific. (Martin Jr., Allen, Bacon, p.43) An organizer helps to provide a foundation before the teacher presents the abstraction or generality of a lesson (1), then proceed to clarify terms (2), provide example (3), and students work with specific example.(4)

Step 4.Students work with specific examples.

Step 3.Teacher provides examples.

Step 2.Teacher clarifies key terms.

Step 1.Teacher presents abstraction or generality of lesson.

The differences between inductive and deductive approaches are as follows.

No.	Inductive approach	No	Deductive approach
1	The teaching pattern moves from specific to general e. g. data, examples etc.	1	In deductive approach it is from general to specific e.g. rules, principles.
2	Inferences and generalizations are drawn from specifics.	2	Specific learning experiences are presented to apply generalizations.
3	It involves discovery learning and is student centered and autonomous.	3	It involves verification skills. it is mostly teacher centered
4	It encourages motivation and students learn how to learn.	4	Self-learning is minimal
5	This approach consumes more time, consequently syllabus coverage is difficult.	5	It is less time consuming. Hence, syllabus coverage is easy.

Analytic-synthetic, .

Analytic Method.

It proceeds from unknown to known. Analysis means breaking up the problem in hand so that it ultimately gets connected with something already known. It is the procees of opening up or unfolding of the problem to know its hidden interior. We start with what is to be found out. Then we think of further steps or possibilities which may connect the unknown with the known.

Procedure.

Algebraic identities and geometrical proposition provide suitable examples for the illustrations of the use of this method. in these cases the problems consist of two parts. One of the parts is called known or given and the other is called unknown or to be proved. We make the unknown as our starting point analyses the statement of the problem, work out step by step requirements, connect the unknown with something known and conclude that the unknown stands proved.

Example.

If $\frac{a}{b} = \frac{c}{d}$,prove that $\frac{ac+3b^2}{b} = \frac{c^2 3bd}{d}$

We begin from the unknown part the statement.

$\frac{ac+3b^2}{B} = \frac{c^2 3bd}{d}$ is to be proved true.

What will be the first step towards the simplification of the two sides of this equation? (cross multiplication).

$\frac{ac+3b^2}{b} = \frac{c^2 3bd}{d}$ will be true

if $acd+3b^2d=bc^2+3b^2d$

what is the next possibility for further simplification?

(cancelling $3b^2d$, which exists on both sides of the equation.)

$acd+3b^2d=bc^2+3b^2d$ will be true

if $acd=bc^2$

What next?(c can be further cancelled as common on both sides)

.: $acd=bc^2$ will be true

If ad=bc

Arrange it in to a more systematic form.

ad=bc will be true.

If $\frac{a}{B} = \frac{c}{d}$, which is known and true

There fore by going back through the chain of argument we can say that

that $\frac{ac+3b^2}{b} = \frac{c^2 3bd}{d}$ is also true.

Merits.

1.it is suits the learner as it facilitates understanding and clarity. It involves his active participation.

2.it suits the subject as it is a logical method. It employs convincing arguments and justification for every step.
3.The student is throughout faced with questions. He faces the problem intelligently and with confidence
4.The step is its procedure are developed in a general manner. no cramming is Necessary

Drawbacks.
1.it is a slow method.
2.it does not ensure speed and efficiency.
3.it may not be applicable to all the topics equally well.

Synthetic Method.

It is opposite of analytic mrthod.Here we proceed from known to unknown. In practice, it is the complement of analysis. To synthsise means to place together things that are apart. It states with something already known and connects that with the unknown part of the statement.

Procedure.

We shall illustrate this procedure with the same example used for analysis.

Example.

If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{ac+3b^2}{b}=\frac{c^2 3bd}{d}$

If $\frac{a}{b}=\frac{c}{d}$, (it is known and hence the starting point)

Add $\frac{3b}{C}$ to both sides.(But why ?).

$\frac{a}{b}+\frac{3b}{c}=\frac{c}{d}+\frac{3b}{c}$ or

$\frac{ac+3b^2}{bc}=\frac{c^2+3bd}{cd}$

or

$\frac{ac+3b^2}{b} = \frac{c^2+3bd}{d}$ (canceling $\frac{1}{c}$ from both sides)

Hence the identity stands proved.

Merits.

1.it is a short and elegant method.

2.it glorifies memory as it involves cramming.

Drawbacks.

1.it leaves many doubts in the mind of the learner and offers no explanation for them.

2.it does not provide clarity and understanding.

3.There is no scope of thinking and discovery.

4.Memorisation and home work are likely to be heavy.

Unit.4.The instrument and activities in mathematic Education.

4.1.audio aids. Chart, model

CHART.

Chart are the handy teaching aids ,the teacher can prepare the chart to present different structures, drilling material, etc. it can be written or drawn on thick colored papers and can be hanged or displayed by using any of the board .

MODEL.

Models have their own special importance as aids because they can be handled and manipulated. Their additional merit lies in that they afford pleasure and educative experience in their making. They have a creative value for the maker.

Even a fractional piece of cardboard is a model. All the geometric from and figures can be cut out of cardboard and used as models. There can be working models for proving various propositions and exercises. In the area of solid geometry, the use of a model becomes necessary, when we want to present various figures There is need for models on the problems concerning area , volumes , heights and distances , graphs, etc.

A device known as a place value box, and another known as a hundred board are very simple illustrations of commercially prepared models. A circular piece of thick paper can be used to explain the angles at a point and their relation with the full circumference of the circle.

The teacher can use different models while teaching in the classroom suppose the teacher wants to present the vocabulary on diagrams , she can take fingers as models to the classroom presenting vocabulary with the help of models provides primary experiences to the learners. Some times some models are working models , they can be easily found in mathematic. They are also usefull while teaching in the classroom

Diagram.

4.1.2.projective aids.ohp, lcd

Give the importance and uses of over head projector and data projecter.

O.H.P.

OHP. Is the hardware use to project the transparency, the transparency is a plastic sheet on which teacher can develop his on material to be presented, The teacher can draw sketches ,matchstick drawing or prepare the chart on it.

L.C.D

The L.E.D is the instrument through which one can view educational progrmmes are also aired from respective L.C.D programs production centre, they are a good as radio programs which teacher cannot select.

4.1.3.audio –visual aids.t.v.,coputer,

T.V.

though the t.v.is a new corner in the field of science teaching but it hasbecome key audio visual aid to day.for the teaching of science. Though the service of good teacher can be available to large number of student at same time,video recording of good lessons.can be put Of effective use in educational institutions through t.v.sets.fitted with video cassette projectors, national and regional telecasts of science lessons can be attended by the students, film based on literary classes can be seen and practice in listening comprehension can be gained.

Computer.

The computer is electronic instrument the computer technology has brought multimedia presentation, text, graphics, audio and video are simultaneously available. The teacher can use this aid instead of all resources available prior to it the computer is V.C.D, VCR, slide of film projector the learning with the help of computer.

4.2.state four activity of mathematics fun

Method-1 : Numbers near and less than the bases of powers of 10.

Exa. 1: 9^2 Here base is 10.
The answer is separated in to two parts by a'/'
Note that deficit is 10 - 9 = 1
Multiply the deficit by itself or square it
1^2 = 1. As the deficiency is 1, subtract it from the number i.e., 9–1 = 8.
Now put 8 on the left and 1 on the right side of the vertical line or slash i.e., 8/1.
Hence 81 is answer.

Exa. 2: 96^2 Here base is 100.
Since deficit is 100-96=4 and square of it is 16 and the deficiency subtracted from the number 96 gives 96-4 = 92, we get the answer 92 / 16
Thus 96^2 = 9216.

Exa. 3: 994^2 Base is 1000
Deficit is 1000 - 994 = 6. Square of it is 36.
Deficiency subtracted from 994 gives 994 - 6 = 988
Answer is 988 / 036 [since base is 1000]

Exa. 4: 9988^2 Base is 10,000.
Deficit = 10000 - 9988 = 12.
Square of deficit = 12^2 = 144.
Deficiency subtracted from number = 9988 - 12 = 9976.
87
Answer is 9976 / 0144 [since base is 10,000].

Exa. 5: 88^2 Base is 100.
Deficit = 100 - 88 = 12.
Square of deficit = 12^2 = 144.
Deficiency subtracted from number =88 - 12 = 76.
Now answer is 76 / 144 =7744 [since base is 100]