MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 1 JANUARY 2025 (Regular) JANUARY - 2025

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-01-00004

Time : 2:30 Hours] **[Total Marks : 70**

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 1. Write the differential equations, $y_1' = y_2 + \sin t, y_2' = y_1 + 5y_2$ in the matrix form.

2. Check whether $u(t) = \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$ is solution of system of differential equation $y' = A \cdot y$ or not?

Where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

3. Show that, $\sin t$ and $\cos t$ are L.I. solutions of $y'' + y = 0$.

4

4. If A is $n \times n$ matrix and T is non-singular $n \times n$ matrix. Show that,

$\exp(T^{-1}AT) = T^{-1} \exp(A)\, T$.

5. Find the general solution of $y''' + 3y'' + 3y' + y = 0$ on $\mathbb{R}$.

6. For $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$, Write $\exp(tA)$.

7 7. Define: Regular singular point and Irregular singular point.

8

8. Define: Gamma function. Show that, $\Gamma(1) = 1$.

9 9. Define: Convolution. Also state only, Convolution Theorem for Laplace transform.

10 10. Find: $L^{-1}\left[\frac{1}{(s-5)(s+4)}\right]$.

Q.2 Answer the following (Any Two) 14

Let $p,q,r : I \to \mathbb{R}$ be continuous and $t_0 \in \mathbb{R}$, $\eta_1,\eta_2 \in \mathbb{R}$. Prove that, the I.V.P. of second order linear differential equation $y'' + p(t)y' + q(t)\,y = r(t)$ with $y(t_0) = \eta_1$, $y'(t_0) = \eta_2$ is equivalent to the I.V.P. of first order $y' = A(t)\,y + g(t)$ with $y(t_0) = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}$, $A(t) = \begin{bmatrix}0 & 1\\ -q(t) & -p(t)\end{bmatrix}$ and $g(t) = \begin{bmatrix}0\\ r(t)\end{bmatrix}$.

State and prove, Abel's formula.

Find the general solution of $y'' - 6y' + 9y = e^{t}$ on $\mathbb{R}$.

Q.3 Answer the following 14

If A is $n \times n$ real matrix with $\lambda_1,\lambda_2,\dots,\lambda_n$ be distinct eigen values and $v_1,v_2,\dots,v_n$ be L.I. eigen vectors corresponding to eigen values $\lambda_1,\lambda_2,\dots,\lambda_n$ respectively. Prove that, $\phi(t) = [e^{\lambda_1 t}v_1 \quad e^{\lambda_2 t}v_2 \quad \cdots \quad e^{\lambda_n t}v_n]_{n\times n}$ is fundamental matrix of $y' = A\cdot y$ on $\mathbb{R}$. Also prove that, $\exp(t\cdot A) = \phi(t)\cdot[\phi(0)]^{-1}$;$\forall t \in \mathbb{R}$

Find the fundamental matrix of $y' = A\cdot y$ on $\mathbb{R}$ where $A = \begin{bmatrix}2 & 5\\ -5 & 2\end{bmatrix}$. Also obtain, $\exp[t\cdot A]$

OR

Answer the following 14

1

a) Find the solution of I.V.P. $y' = A \cdot y + g(t)$ with $y(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ & $g(t) = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$.

b) Let A be constant 2×2 complex matrix. Show that, there exist a non-singular constant 2×2 matrix T such that, $T^{-1}AT$ having form: (i) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$ (ii) $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$.

Q.4 Answer the following questions (Any Two) 14

1

1. Find the fundamental matrix of $y' = A \cdot y$ on $\mathbb{R}$. Where $A = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$, where $\alpha, \beta \in \mathbb{R}$. Also show that, $\exp[t \cdot A] = e^{\alpha t} \begin{bmatrix} \cos\beta t & \sin\beta t \\ -\sin\beta t & \cos\beta t \end{bmatrix}$.
2. Locate and classify the singularities of $t^4(1-t^2)^3 y''' + 5t^5(1+t)y'' - 2t^2(1-t^2)y' + 2y = 0$

Q.5 Answer the following (Any Two) 14

I. Prove that, ϕ is a solution of IVP $y' = f(t, y)$ with $y(t_0) = y_0 \Leftrightarrow \phi$ is a solution of Volterra's equation $y(t) = y_0 + \int_{t_0}^{t} f(s, y(s))\, ds$.

2

II. Show that, $\cosh ct \in \mathcal{H}$ then find, $L(\cosh ct)$.

III. State and prove, First shifting theorem. Also deduce that, $L(t^n e^{ct})(z) = \frac{n!}{(z-c)^{n+1}}$, $\forall n = 1, 2, \ldots$.

IV. Using Laplace Transform Solve: $y'' - 3y' + 2y = 4e^{2t}$ with $y(0) = -3$ and $y'(0) = 5$.
