Shree H.N.Shukla College of Science Rajkot MATHEMATICS

## T.Y.B.Sc. (Sem.VI) (CBCS)

PRELIMS EXAM
PAPER- 603
Optimization \& Numerical Analysis-II

Time: 2.5 hour]
[Total Marks: 70
Instruction: (i) All questions are compulsory.
(ii) Figures to the right indicate full marks of the question.

## 1. (A) Answer the following:

1) Define: Slack variables
2) Define: Optimum solution of LPP
3) What is Surplus variable with respect to the LPP?
4) Define: Basic Feasible Solution of LPP
(B) Attempt any one:
5) Write the Matrix form of Linear Programming Problem.
6) State the Fundamental Theorem of Linear Programming.
(C) Attempt any one:
7) Obtain the dual of following;

Minimize: $Z=5 x_{1}+x_{2}-6 x_{3}$
Subject to the constraints
$-2 x_{1}+x_{2}+11 x_{3} \leq-2$
$-x_{1}+7 x_{2}+x_{3} \geq 7$
$3 x_{1}-x_{2}+4 x_{3} \leq 5$ and $x_{1}, x_{2}, x_{3} \geq 0$
2) Write summary of the general relationship between Primal \& Dual LPP.
(D) Attempt any one:

1) Explain the steps of Graphical method to solve the LPP.
2) Find only BFS and construct only first table to solve the following LPP using SIMPLEX METHOD (complete solution is not required)
Maximize: $Z=3 x_{1}+5 x_{2}+4 x_{3}$
Subject to the constraints
$2 x_{1}+3 x_{2} \leq 8$
$2 x_{2}+5 x_{3} \leq 10$
$3 x_{1}+2 x_{2}+4 x_{3} \leq 15$ and $x_{1}, x_{2}, x_{3} \geq 0$

## 2. (A) Answer the following:

1) Which three methods are used to obtain an initial solution of transportation problem?
2) What is the full form of NWCM?
3) What is the name of the method to find optimum solution of transportation problem?
4) What is the name of the method to solve Assignment problems?
(B) Attempt any one:
5) State the general mathematical form of assignment problem.
6) Write full form of LCM \& VPM?

## (C) Attempt any one:

1) Obtain the initial solution of given transportation problem using LCM method;

|  |  | TO |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ |  |
| FROM | $\mathrm{P}_{1}$ | 2 | 3 | 11 | 7 | 6 |
|  | $\mathrm{P}_{2}$ | 1 | 0 | 6 | 1 | 1 |
|  | $\mathrm{P}_{3}$ | 5 | 8 | 15 | 9 | 10 |
| Demand |  | 7 | 5 | 3 | 2 |  |

2) Solve the following Assignment problem;

|  |  | Subordinates |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | II | III | IV |  |
| Task | A | 8 | 26 | 17 | 11 |
|  | B | 13 | 28 | 4 | 26 |
|  | C | 38 | 19 | 18 | 15 |
|  | D | 19 | 26 | 24 | 10 |

(D) Attempt any one:

1) Explain the steps of Hungarian method to solve the Assignment problem.
2) Obtain the Optimum solution of given transportation problem using MODI method;

|  |  | TO |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{D}_{\mathbf{2}}$ | $\mathbf{D}_{\mathbf{3}}$ | $\mathbf{D}_{\mathbf{4}}$ |  |  |
| FROM | $\mathbf{S}_{\mathbf{1}}$ | 5 | 3 | 6 | 4 | $\mathbf{3 0}$ |
|  | $\mathbf{S}_{\mathbf{2}}$ | 3 | 4 | 7 | 8 | $\mathbf{1 5}$ |
|  | $\mathbf{S}_{\mathbf{3}}$ | 9 | 6 | 5 | 8 | $\mathbf{1 5}$ |
| Demand |  | $\mathbf{1 0}$ | $\mathbf{2 5}$ | $\mathbf{1 8}$ | 7 | 60 |

## 3. (A) Answer the following:

1) Write formula for Gauss forward interpolation formula.
2) The first order divided difference $f\left(x_{0}, x_{1}\right)=$ $\qquad$
3) If $f(x)=x^{3}$ then what is the value of $f(1,3)$ ?
4) Write Lagrange's formula for inverse interpolation.
(B) Attempt any one:
5) Derive relation between divided difference and forward difference.
6) Define inverse interpolation and write Lagrange's formula for inverse interpolation.
(C) Attempt any one:
7) Derive Stirling's formula.
8) Prove that divided difference is symmetrical in their arguments.
(D) Attempt any one:
[05]
9) Use Sterling's formula to find $f(1.63)$ given

| $x$ | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 17.609 | 20.412 | 23.045 | 25.527 | 27.875 |

2) Derive Gauss Backward interpolation formula.

## 4. (A) Answer the following:

1) To derive Simpson's $1 / 3$ rule we can take $n=$ $\qquad$ in general quadrature formula.
2) What is numerical integration?
3) General Quadrature formula is also known as $\qquad$ .
4) Write Trapezoidal rule.
(B) Attempt any one:
5) Derive Simpson's $\frac{1}{3}$ rule.
6) Write General Quadrature formula.
(C) Attempt any one:
7) Derive Trapezoidal rule.
8) Derive Simpson's $3 / 8$ rule.

## (D) Attempt any one:

1) Evaluate $\int_{0}^{10} \frac{d x}{1+x^{2}}$ by using Simpson's $3 / 8$ rule.
2) Derive General Quadrature formula.

## 5. (A) Answer the following:

1) Write Taylor formula to solve Ordinary Differential Equation.
2) Write Picard's formula to solve Ordinary Differential Equation.
3) Write Range's formula to solve Ordinary Differential Equation.
4) Write Milne's Predictor formula to solve Ordinary Differential Equation.
(B) Attempt any one:
5) Using Picard's formula to find $y(0.1)$ given that

$$
\frac{d y}{d x}=x+y, y(0)=1, h=0.1
$$

2) Find the value of $y$ at $x=0.2$ by Euler's method

$$
\frac{d y}{d x}=2 x+y, y(0)=1
$$

(C) Attempt any one:

1) Explain Euler's method to solve Ordinary Differential Equation.
2) Use Range's method to find $y(0.2)$ given that $y^{\prime}=x+y, y(0)=1$.
(D) Attempt any one:
3) Derive Milne's Predictor-Corrector formula.
4) Use Range-Kutta's method to find $\mathrm{y}(0.1), \mathrm{y}(0.2)$ and $\mathrm{y}(0.3)$ given that $y^{\prime}=x y+y^{2}, y(0)=1$
