

**Shree H.N. Shukla College of Science**

**M.Sc. (Mathematics) Sem-3**

**Prelims Test**

**MATH.CMT-3004: Discrete Mathematics**

## [Time: 2.5 Hours] [Total Marks: 70]

1. Answer any **seven** : 7x2=14
2. Find least element and greatest element of poset (P,D) with P={2,4,8,16,32} and D=Divisibility relation
3. Give an example of lattice which is modular but not distributive.
4. Define : i) Bounded lattice ii) Complemented lattice
5. State Distributive Inequality for lattices
6. Find context free grammar of language of palindromes.
7. Construct FA(Finite Automata) of language of all string that ends in 10.
8. Define : Direct product of semi-groups
9. Define : Reflexive closure
10. Define : Massage and Channel
11. Define : Hamming Distance

## Answer any **two** : 2x7=14

* 1. Let *V* be a vector space over a field *F*. Prove that *L(V)* = the lattice of subspace of *V* is modular.
	2. State and Prove the Fundamental theorem of homomorphism of semi-group.
	3. Let A code C can detect all combination of k or fewer error iff d(C) $\geq $ k+1
1. Answer the following :
2. Let L1 and L2 are context free languages then show that $L\_{1}L\_{2}$ is also context free langualge. Using above result find context free grammar of language =$\{0^{i}1^{j}0^{k} / j>i+k\}$
3. Let $(L,\leq )$ be a lattice then *L* is distributive if and only if for any $a,b,c \in L, \left(a⨁b\right)\*c\leq a⨁(b\*c)$.

**OR**

3 Answer the following :

1. A lattice $(S\_{n}, D)$ is complemented if and only if n is square free integer.
2. State Pumping lemma for regular languages. Using Pumping lemma show that language L={0i1i / i $\geq $ 0} is not regular.

4 Answer the following :

1. Let R be any relation defined on a non-empty set A then $R^{\infty }$ is transitive closure of R.
2. Let $(L,\leq )$ be a lattice then $(L,\leq )$ is modular if and only if *M* is any sub-lattice of *L* then *M* is not isomorphic to pentagon lattice.

5 Answer any **two** :

1. $(N,D)$ is distributive lattice.
2. A code C can correct all combination of k or fewer error iff d(C) $\geq $ 2k+1.
3. Let G be a group and R be a congruence relation on G then there exist a normal subgroup H of G satisfying a condition. For any a,b $\in $ G , aRb iff ab-1 $\in $ H.
4. Let $(L,\leq )$ be a lattice then show that $(L,\leq )$ is Modular if $[x,y]$ is any closed interval in $(L,\leq )$ and $a\leq c (a,c \in \left[x,y\right])$ and both $a,c$ admit a common relative complement then $=c$ .

 **BEST OF LUCK**