



[Time: 2.30 Hours]

[Total Marks: 70]

1 Answer any seven

7x2=14

- (a) Every normal space is \_\_\_\_\_.
- (b) Limit point of  $(0,1) \cup (2,3)$  is \_\_\_\_\_ in  $\mathbb{R}$  with standard topology .
- (c)  $[0,1)$  is open set in \_\_\_\_\_ topology on  $\mathbb{R}$ .
- (d) Every locally compact Hausdroff space is \_\_\_\_\_.
- (e) Every closed subspace of compact space is \_\_\_\_\_.
- (f) Any compact subspace of metric space is \_\_\_\_\_.
- (g) If every infinite subset of  $X$  has limit point in  $X$  then  $X$  is \_\_\_\_\_.
- (h) Give example of compact set which is not finite. .
- (i) Any compact Hausdroff space is \_\_\_\_\_.
- (j) True or False: Any closed subset of  $\mathbb{R}$  is compact.

2 Answer any two

2x7=14

- (a) State and prove tube lemma.
- (b) Let  $X$  and  $Y$  be any topological spaces then show that  $X$  and  $Y$  are Compact if and only if  $X \times Y$  are compact. 7
- (c)  $\mathcal{T} = \{ G \subseteq \mathbb{N} \mid \mathbb{N} - G \text{ is finite} \} \cup \emptyset$  is topology on  $\mathbb{N}$ . Show that  $\mathcal{T}$  be a topology on  $\mathbb{N}$  and  $\mathcal{T}$  is Compact.

3

- (a) Prove that any compact subset of Hausdroff space is closed. 5
- (b) Show that  $\mathbb{R}$  is not compact. 4
- (c) Give example of infinite set. Which is compact. 5

4 Answer of the following questions

2x7=14

- (a)  $X$  and  $Y$  are locally compact if and only if  $X \times Y$  are locally compact.
- (b) Prove that  $\mathbb{R}^n$  is complete metric space.

5 Answer of the following questions

2x7=14

- (a)  $Y^X$  is uniform metric space then show that  $(Y, d)$  is complete metric space then  $Y^X$  is Complete.
- (b) State and prove Lebesgue covering lemma.

Best of Luck

