

**Shree H.N. Shukla College of Science**

**M. Sc (Mathematics) (Sem-3)**

**Preliminary Exam**

**MATH.EMT-3011:Differential Geometry**

## [Time: 2.5 Hours] [Total Marks: 70]

## **Instructions :** (1) All questions are compulsory.

## (2) There are 5 questions.

## (3) Figures on right side indicate full marks .

1 Attempt any seven : 14

(1) Is the curve α(s)= (t3 ,t2 ,2t) regular ? justify your answer.

(2) Define : Arc length.by the quantity torsion of a curve ?

(3) What is curvature of the curve x2+y2 =9 ?

(4) Define : Normal curvature and Geodesic curvature.

(5) Which parameter is measured by the quantity torsion of a curve ?

(6) Define the term Osculating plane and Normal plane.

(7) Define : Regular curve.

(8) What is curvature of 3x+2y=0 ? justify your answer.

(9) Define :Simple Surface.

(10) Define with example :An open subset of R2.

1. Answer any two 14
2. Let α(s) be a unit speed curve whose image lies on a sphere of radius r and center m then show that κ ≠0 and if Ʈ≠0 then

α-m = -ρN-ρ’σB

where ρ=1/κ and σ=1/Ʈ.

Hence, r2=ρ2+(ρ’σ)2.

(b) If g: [c,d] →[a,b] is a reparametrization of a curve segment α :[a,b] →R3 then prove that

length of α is equal to the length of β = αog.

.

(c) Show that α(s)=( (1+s)3/2/3, (1-s)3/2/3, s/curve and compute its frenet serret apparatus .

(d) Define : (i) Osculating plane

(ii) Rectifying plane

Also prove that a unit speed curve α(s) with κ a helix iff there is a constant c such that

Ʈ = cκ.

**3** Attempt the following :  **14**

1. Define a regular curve segment and length of a regular curve segment moreover reparametrize the curve α(t)=(rcost , rsint , 0) by arc length.
2. Define Normal space and Normal curvature and prove that κ2 = kn2 +kg2

(c ) Define reparamerization of a curve and reparametrize the curve

α(u)=(acosu , asinu , cu) by t=tan(u/2). (where 0<= u < π)

(d)Find the coefficients of second fundamental form and Christoffel symbols for Monge patch

4 Attempt the following : **14**

**(a)**State and prove Frennet- Serret theorem.

(b) Prove in usual notations:

(i) xij = Lij n + xk

(ii) kn =(γi)′ (γj)′

and

(iii) kg S =(γk)′′ + (γi)′ (γj)′ ] xk

1. Attempt the following :  **14**

(a) Prove that :

If x: u R3 is a simple surface and f : v a co-ordinate transformation ,then show

Show that the tangent plane to a simple surface x at P =x(a,b)

Is equal to the tangent plane to the simple surface y= xof at P(a,b) .

(b) show that the curve the curve α(t)= (2a(sin-1 t+t2at2, 4at) between the points t=t1 to t=t2 is 4a (t2 –t1).

1. Prove that

= ½ where notations are being usual.

(d) Define : (i) Ck co-ordinate patch.

(ii) Monge patch.

Moreover let u={(u1 ,u2 ) ϵR2 / (u1)2 +(u2)2 <1 } and

X(u1 , u2 ) = (u1 , u2 ,- ) then find unit normal and equation of tangent

Plane at X(1/2 ,1/2) .

BEST OF LUCK