

**Shree H.N. Shukla College of Science**

**M. Sc (Mathematics) (Sem-3)**

**Preliminary Exam**

**MATH.EMT-3011:Differential Geometry**

## [Time: 2.5 Hours] [Total Marks: 70]

## **Instructions :** (1) All questions are compulsory.

##  (2) There are 5 questions.

##  (3) Figures on right side indicate full marks .

1 Attempt any seven : 14

 (1) Is the curve α(s)= (t3 ,t2 ,2t) regular ? justify your answer.

(2) Define : Arc length.by the quantity torsion of a curve ?

(3) What is curvature of the curve x2+y2 =9 ?

(4) Define : Normal curvature and Geodesic curvature.

 (5) Which parameter is measured by the quantity torsion of a curve ?

(6) Define the term Osculating plane and Normal plane.

 (7) Define : Regular curve.

(8) What is curvature of 3x+2y=0 ? justify your answer.

(9) Define :Simple Surface.

(10) Define with example :An open subset of R2.

1. Answer any two 14
2. Let α(s) be a unit speed curve whose image lies on a sphere of radius r and center m then show that κ ≠0 and if Ʈ≠0 then

 α-m = -ρN-ρ’σB

 where ρ=1/κ and σ=1/Ʈ.

 Hence, r2=ρ2+(ρ’σ)2.

 (b) If g: [c,d] →[a,b] is a reparametrization of a curve segment α :[a,b] →R3 then prove that

 length of α is equal to the length of β = αog.

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 (c) Show that α(s)=( (1+s)3/2/3, (1-s)3/2/3, s/$\sqrt{2}) is a unit speed $curve and compute its frenet serret apparatus .

 (d) Define : (i) Osculating plane

 (ii) Rectifying plane

 Also prove that a unit speed curve α(s) with κ $\ne 0 is $a helix iff there is a constant c such that

 Ʈ = cκ.

 **3** Attempt the following :  **14**

1. Define a regular curve segment and length of a regular curve segment moreover reparametrize the curve α(t)=(rcost , rsint , 0) by arc length.
2. Define Normal space and Normal curvature and prove that κ2 = kn2 +kg2

(c ) Define reparamerization of a curve and reparametrize the curve

 α(u)=(acosu , asinu , cu) by t=tan(u/2). (where 0<= u < π)

(d)Find the coefficients of second fundamental form and Christoffel symbols for Monge patch

4 Attempt the following : **14**

**(a)**State and prove Frennet- Serret theorem.

 (b) Prove in usual notations:

 (i) xij = Lij n + $\sum\_{k}^{}┌\_{ij}^{k}$ xk

(ii) kn =$\sum\_{i,j}^{}L\_{ij }$(γi)′ (γj)′

 and

 (iii) kg S =$\sum\_{k}^{}[ $(γk)′′ + $\sum\_{i,j}^{}┌\_{ij}^{k}$ (γi)′ (γj)′ ] xk

1. Attempt the following :  **14**

 (a) Prove that :

 If x: u $\rightarrow $ R3 is a simple surface and f : v$\rightarrow u is$ a co-ordinate transformation ,then show

 Show that the tangent plane to a simple surface x at P =x(a,b)

 Is equal to the tangent plane to the simple surface y= xof at P(a,b) .

 (b) show that the curve the curve α(t)= (2a(sin-1 t+t$\sqrt{1-t2},$2at2, 4at) between the points t=t1 to t=t2 is 4a$\sqrt{2}$ (t2 –t1).

1. Prove that

 $┌\_{ij}^{l}$ = ½ $\sum\_{k=1}^{2}g^{kl}(\frac{∂g\_{ik}}{∂u^{j}}+\frac{∂g\_{kj}}{∂u^{i}}+\frac{∂g\_{ij}}{∂u^{k}})$ where notations are being usual.

 (d) Define : (i) Ck co-ordinate patch.

 (ii) Monge patch.

 Moreover let u={(u1 ,u2 ) ϵR2 / (u1)2 +(u2)2 <1 } and

 X(u1 , u2 ) = (u1 , u2 ,$\sqrt{1- (u^{1})^{2}}$-$(u^{2})^{2}$ ) then find unit normal and equation of tangent

 Plane at X(1/2 ,1/2) .

 BEST OF LUCK