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MAL-003-001616 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2018

Mathematics : BSMT - 601 (A)

(Graph Theory and Complex Analysis - 2)

(New Course)

Faculty Code : 003

Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate full marks.

1 Answer all questions : **20**

(1) Write the number of internal vertices in a binary tree with 13 vertices.

(2) Write the number of vertices in a connected graph with 2 faces and 6 edges.

(3) Write the edge connectivity of a tree.

(4) Write the chromatic number of a complete graph with 5 vertices.

(5) Write the nullity of a connected graph with 8 vertices and 4 edges.

(6) How many edges are there in K_8 graph ?

(7) How many edges are there in a tree with 5 vertices ?

(8) Write the degree of a pendant vertex.

(9) Write the maximum number edges in a simple graph with 4 vertices and 2 components.

(10) How many vertices are there in Kuratowski's first graph K_5 ?

(11) Write Maclaurin's expansion of $\frac{1}{1+z}$.

(12) Write an isolated singular point for $f(z)=\frac{1}{z-2}$.

(13) Find the Residue of $\frac{\cos z}{z}$ at $z=0$.

(14) Find the critical point of $w=\frac{1}{z-1}$.

(15) Find the fixed point of $w=-\frac{1}{z+2}$.

(16) Find : Res $\left(\frac{e^{Z}}{z(z+1)},0\right)$.

(17) Find radius of convergence for the series $\sum_{n=1}^{\infty} n!z^{n}$

(18) Find Residue of tan z at $z=\frac{\pi}{2}$.

(19) Find singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.

(20) Which function is represented by the power series $\sum_{n=0}^{\infty}\frac{z^{n}}{n!}$?

2 (A) Attempt any **three** : **6**

(1) Define : (i) Simple graph (ii) Complete graph

(2) Find the number of vertices in the complete graph K_n, if it has 45 edges.

(3) Prove that the number of vertices n in a binary tree is always odd.

(4) Define : (i) Diagraph. (ii) Spanning tree.

(5) In any simple, connected planar graph with f regions, n vertices and e edges $(e>2)$ prove that $e \leq 3n-6$.

(6) State and prove second theorem of graph theory.

(B) Attempt any **three** : **9**

(1) Find the smallest integer n such that the complete graph k_n has at least 10 edges.

(2) Prove that a graph is a tree if and only if it is minimally connected.

(3) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuit.

(4) State and prove characteristic of a disconnected graph.

(5) In usual notation prove that $(W_G, \oplus)$ is an abelian group.

(6) Prove that a covering of a graph is minimal if and only if g contains no path of length three or more.

(C) Attempt any **two** : **10**

(1) In usual notation prove that $n - e + f = k + 1$ where n is vertices, e is edges, f is faces and k is components in a graph.

(2) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even order.

(3) Prove that a graph with at least one edge is 2–chromatic if and only if it has no odd circuit.

(4) Explain Konigsberg Bridge Problem.

(5) State and prove Euler's formula.

3 (A) Attempt any **three** : **6**

(1) Define : (i) Power series. (ii) Singular point.

(2) Expand $\sin z$ in Taylor's series for $z_0 = 0$.

(3) Prove that if the series $\sum z_n$ is absolute convergent then $\sum z_n$ is also convergent.

(4) Define : (i) Mobius mapping (ii) Critical points

(5) Discuss the fixed points of bilinear transformation.

(6) Find critical point of $w = \frac{z-1}{z+1}$.

(B) Attempt any **three** : **9**

(1) State and prove Cauchy-Residue theorem.

(2) Find the residue of $f(z) = \frac{z+2}{(z-1)(z-2)}$ at simple pole.

(3) Expand $\frac{1}{z^2 - 3z + 2}$ in Laurent's series for $1 < |z| < 2$.

(4) Show that the composition of bilinear maps is again a bilinear.

(5) Find the bilinear transformation which maps $z_1 = \infty, z_2 = i, z_3 = 0$ onto $w_1 = 0, w_2 = i, w_3 = \infty$.

(6) Prove that $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} = \frac{2\pi}{\sqrt{3}}$.

(C) Attempt any **two** : **10**

(1) Evaluate : $\int_c \frac{z}{z^4 - 1} dz$ where $C : |z| = 2$.

(2) State and prove Taylor's infinite series of an analytic function.

(3) Evaluate : $\int_c \frac{e^z}{z(z-1)^2} dz$ where $C : |z| = 2$.

(4) Prove that $\int_0^{\infty} \frac{dx}{\left(x^2 + a^2\right)^{n+1}} = \frac{\pi(2n)!}{(n!)^2 . (2a)^{2n+1}}$ where $a > 0$.

(5) Using residue theorem prove that $\int_{-\infty}^{\infty} \frac{dx}{\left(1 + x^2\right)^3} = \frac{3\pi}{8}$.