Shree H.N. Shukla Group of College

M.Sc. SEMESTER 4

Sub. Code: CMT-4001

Core Sub.1 : Linear Algebra

Question Bank

(1) Let V is a n- dimentional vector space over a field F , then prove that $A_F(V)$ and F_n are isomorphic as algebras over F.

(2) Let T,S ∈ $A_F(V)$.If S is regular , then show that T and STS^{-1} have the same minimal polynomial.

(3) If n_1 is the index of nilpotence of a nilpotent T∈ $A_F(V)$ and if v∈ V is such that T^{n_1-1}(v)≠0 , then prove that {v , T(v),…, T^{n_1-1}(v) } is linearly independent over F.

(4) Prove that any T∈ $A_F(V)$ satisfies its characteristic polynomial.

(5) Let A∈ $\mathbb{C}_n$ be Hermitian .Show that any characteristic root of A is real.

(6) Let A∈ F_n .show that det(A) = det(A') .

(7) Let A∈ F_n and suppose that K is the splitting field of minimal polynomial of A over F.show that there is an invertible matrix C∈ K_n such that CAC^{-1} is in Jordan form.

(8) Let n≥1 .Show that the mapping f: $F_n \rightarrow F_n$ defined by f(A) =A' is an adjoint of F_n.

(9) Let T ∈ $A_F(V)$.If V is cyclic relative to T, then prove that there exist a basis B of V over F such that the matrix of T in B is C(p(x)) , where p(x) is the minimal polynomial of T over F.

(10) Let T_1,T_2∈ $A_F(V)$ and they both have same invariants.Prove that they are similar .

(11) Let A∈ F_n .Prove that the interchanging two rows of A change the sign of its determinant.

(12) Let tr(T^k) =0 , ∀ k∈ ℕ . Prove that T is nilpotent.

(13) State and prove Jacobson lemma.

(14) Prove that the determinant of triangular matrix is equal to the product of its all the entries of the main diagonal .

(15) Solve by Cramer' s rule :

$x_1+ 2x_2+3x_3 = -5$,

$2x_1+ x_2+x_3 = -7$ and

$x_1+ x_2+x_3 = 0$.

(16) If $T\in A_F(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which tha matrix of T is triangular .

(17) If $T\in A_F(V)$, then prove that tr(T) is the sum of the characteristic roots of T(using each characteristic root as often as its multiplicity) .

(18) Let (V, < >) be a finite dimensional inner product space over ℂ .Let $T\in A_F(V)$ .Then given $v\in V$, prove that there exist an element $w\in V$, depending on v and T such that <T(u) ,v> = < u, w > , for all $v \in V$.

(19) Let (V, < >) be a finite dimensional inner product space over ℂ .Let $T,S\in A_C(V)$.Prove that $(ST)^* = T^*S^*$.

(20) Let $A,B\in F_n$. Prove that tr(AB)=tr(BA) .

(21) Let F be a field of characteristic 0.If $T,S \in A_F(V)$ are such that ST-TS commutes with S, then prove that ST-TS is nilpotent .

(22) Prove that any $A\in F_n$ satisfies its secular equation.

(23) Let V be a finite dimensional vector space over a field F and let $T\in A_F(V)$. Prove that T is invertible if and only if the constant term of the minimal polynomial for T over F is nonzero .

(24) Let (V, < >) be a finite dimensional inner product space over ℂ. If $N\in A_C$ is normal ,then prove that there exist an orthonormal basis of V consisting of characteristic vectors of N, in which the matrix of N is diagonal

(25) Let f: V x V→ F be a bilinear form on an n-dimentional vector space V over F .If B , B' are any two basis of V over F ,then prove that there exist an invertible matrix C∈ F_n such that $[f]_{B'} = C\,[f]_B C'$.