Date: [Total Marks: 30]

 Time:

**Q.1: Answer the following question [5-marks]**

(1)Define:-Compact Set, Bounded set, Disconnected set, State Housedroff Principal, Closed set, Interior Point.

(2) Every Compact set Of metric space is bounded.

(3) Every Singleton subset Of metric space is closed.

(4)Define: Open Cover and prove that A and B be two separated sets of metric space X.

(5)Define: Normal Subgroup and prove the necessary and sufficient condition any group become and subgroup.

(6) Prove that any subset of discrete m.s. is connected. If and only if it is singleton.

(7)Define : Upper bound and Greatest Lower bound with an example.

(8) Find all homomorphism from (z,+) onto are onto?

(9)A homomorphism $∅:\left(G,\*\right)\rightarrow (G^{'},∆)$ is one- one iff $K\_{φ}=\left\{e\right\}.$

(10)Define : Homomorphism and give and example of it.

(11) If $H\leq G, then φ\left(H\right)\leq G^{'}.$

(12) If N is a normal subgroup of G then , $φ(N)$ is a normal subgroup of $φ\left(G\right).$

(13) If $N^{'}$ is a normal subgroup of $φ\left(G\right)$ then $φ^{-1}(N^{'})$ is a normal subgroup of G.

(14)State and prove First fundamental theorem of homomorphism.

(15)Define : Ring and write any four properties with proof.

(16)State and prove Factor theorem

(17) State and prove Reminder theorem.

(18)Define: Irreducible Polynomial and solve $ f,g\in Q\left[x\right],$

$ where f\left(x\right)=6x^{3}+5x^{2}-2x+25 and g\left(x\right)=2x^{2}-3x+25$ the find fg.

(19) Show that $x^{3}+3x^{2}-8$ is irreducible over Q.

(20) Show that Integral domain is not field.