16SI-MSMA-CO-02-00001

M.Sc. sem-2 April-2025.

Seat No. ____________

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 2 APRIL 2025 (Regular) APRIL - 2025

ALGEBRA 2

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-02-00001

Time : 2:30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) **14**

1 1. Define the term with its Notation: Direct sum of a family $\{N_\lambda\}_{\lambda \in \Lambda}$ of R-sub modules of an R-module.

2. Define term: R-homomorphism of R-modules.

3. Define term: Fixed field.

4. Prove or disprove that, $x^3 + 3x + 2 \in \mathbb{Z}_7[x]$ is irreducible over $\mathbb{Z}_7$.

5 5. Define term: Monic polynomial and justify that every monic polynomial is a primitive polynomial.

6 6. Define terms: Extension of fields and finite field extension (finite extension).

7 7. Let f(x) = $x^3 + 6x^2 + 7x + 8$. Prove that, $f(x-2)$ is an irreducible polynomial. Is f(x) irreducible? (Y / N)

8. Write down the minimal polynomial of the complex number $\sqrt{2} + \sqrt{3}i$ over

9. Define algebraic element and algebraic extension.

10 10. Define term transcendental element and transcendental field extension.

Q.2 Answer the following (Any Two) **14**

1 Let p(x) $\in$ F[x] be an irreducible polynomial. Prove that, there is an extension $E|_F$ such that E contains one root of p(x).

2 Let $E|_F$ and $K|_E$ both are algebraic extensions. Prove that, $K|_F$ is also an algebraic extension.

3 State and Prove, Eisenstein Criterion.

Q.3 Answer the following

1 Let $f : M \rightarrow N$ be an R-homomorphism of R-modules. Prove that, Ker f and f(M) are R-sub modules of M and N respectively.

2 Let $f: M \rightarrow N$ be an onto R-homomorphism of R-modules M onto N. Prove that,

$\frac{M}{Ker\ f} \simeq N.$

OR

Answer the following

1 Let F be a finite field. Prove that, $F^* = F - \{0\}$ is a cyclic group under multiplication.

2 Let F be a finite field and $|F| = p^n$, for some prime p and integer n. Prove that, F has a sub field K with $|K| = p^m$ if and only if m is divisor of n.

Q.4 Answer the following questions (Any Two)

1 Prove that, $Q(\sqrt{2},\sqrt{3},\ldots\ldots,\sqrt{p},\ldots)|Q$ is an infinite algebraic extension.

2 Let $E|_F$ be an extension and $[E : F] = 2$. Prove that, $E|_F$ is a normal extension.

Q.5 Answer the following (Any Two)

1 Find the Galois group of the polynomial $x^4 - 2 \in Q[x]$ over Q.

2 Let $p(x) \in F[x]$ be an irreducible polynomial and degree of p(x) = n. Let $E|_F$ be an extension such that $\alpha \in E$ and α is a root of p(x). Prove that,

$F[\alpha] = F(\alpha)$, $[F(\alpha):F] = n$ and $\{1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}\}$ is a basis of $F[\alpha]$ over F.

3 Let R be a ring with unity and M is an R-module. Prove that, M is cyclic if and only if

$M \simeq \frac{R}{I}$, for some left ideal I of R.

4

Let A, B be R-sub modules of two R-modules $M \text{ and } N$ respectively. In standard notation, prove that, $\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$.