SHREE H. N. SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics) Semester-2

IMP Questions of Topology-II

1. Prove that every sequentially metric space is compact.
2. Give an example of an infinite topological space which is not compact.
3. Prove that a T_1 space X is normal if and only if for each closed set A and an open set U with $A \subset U$ there is a closed set V that $A \subset V \subset \overline{V} \subset U$.
4. Prove that any compact hausdorff space is regular.
5. Prove that a T_1 space X is regular if and only if for every open set U and $x \in U$ there is an open set V such that $x \in V \subset \overline{V} \subset U$.
6. Prove that $(\mathbb{R}, d)$ is a complete metric space.
7. Prove that any open subspace of a locally compact Hausdorff space is locally compact.
8. State and prove Lebesgue's covering lemma.
9. Prove that
   (i) Every subspace of a $T_1 - space$ is $T_1 - space$.
   (ii) Suppose $X \times Y$ is hausdorff. Prove that X and Y both are hausdorff.
10. Give an open cover of the subspace $\mathbb{Q}$ of rational numbers (with usual topology) which has no finite sub cover.
11. Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.
12. Let $A = [0, 1)$. Is A a compact subset of $\mathbb{R}$? Give reasons for your answer.
13. Give an example of a uncountable disconnected space.
14. Give an example of a non compact, locally compact, Hausdorff space which is uncountable.
15. Give an example of a finite connected space.
16. Give the definition of a separation of a space X.

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17. A space X is _____ if every open cover of X has finite sub cover.

18. Every closed subspace of a compact space is _____.

19. The one point compactification of a locally compact, non - compact Hausdorff space is _____ and _____.

20. The subspace $\mathbb{R}\setminus\mathbb{Q}$ of irrationals is not connected because $\mathbb{R}\setminus\mathbb{Q}$ is not _____.

21. If X and Y are not compact then $X \times Y$ is _____.

$[0,1]\times[0,1]$ with dictionary order topology is connected but not _____.

22. A component of a space X is a maximal _____ subset of X.