DBJ-CMT-3001 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

June – 2022

Mathematics - 3001

(Programming in C & Numerical Methods)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are **ten** questions.

(2) Answer any **five** of them.

(3) Each question carries **14** marks.

1 Answer following short questions : **7×2=14**

(i) Define terms: Program and Lower Level Language.

(ii) Write down at least four names of C-Tokens.

(iii) Write a program which can print 1 to 30 integers in three lines.

(iv) Write a program, which can read two integers and it can find smallest integer from given two integers.

(v) Give definition of flow-chart.

(vi) Determine the value of following, when $a = 10$, $b = 20$ and $c = -12$:

(1) $a * b - 6 - 15$

(2) $b > 25 \; \&\& \; c < 0 \; || \; a > 0$.

(vii) Write down format for jump in a loop statement by break.

2 Answer following short questions : **7×2=14**

(1) Write down at least four reserved identifiers.

(2) Write a program which can print 1 to 40 integers in four lines.

(3) Write down name of Relational Operators.

(4) Express following mathematical functions in C - Language :

(i) $\cos x$, (ii) $\log_e x$, (iii) $\sqrt{x}$ and (iv) e^x.

(5) Write a program, which can read two integers and it can find the largest integer from given two integers.

(6) Give definitions: Identifier and Variable.

(7) Draw flow chart, so that one can write a program which can print small letters 'a' to 'z'.

3 Attempt following **two** : **7×2=14**

(a) Write a note about development of C - Language.

(b) Explain about Basic Structure of a C program.

4 Attempt following **one** : **1×14=14**

Discuss about Newton Raphson's Method and write down the program for the same Method.

5 Attempt following **one** : **1×14=14**

Explain about Gauss Seidel Method to solve a system of linear equations.

6 Attempt following **one** : **1×14=14**

Explain about Lagrange interpolation polynomial and derive its formula. Using it write a program for Lagrange interpolation polynomial.

7 Attempt following **one** : **1×14=14**

Explain about N-G forward polynomial and derive its formula. Using it write a program for N-G forward interpolation polynomial.

8 Attempt following **two** : **2×7=14**

(a) Explain about Switch Statement with its format or syntax and appropriate example.

(b) Write a program, which can read two integers a and b and, it can find (a, b), the GCD of a and b as well as $[a, b]$, the LCM of a and b.

9 Attempt following **two** : **2×7=14**

(1) Find a root of $f(x) = x^3 - 7$, using Bisection Method and take initial values $a = 1.5$, $b = 2$.

(2) Write a program which can read any date of 21^{st} century and it can find day of corresponding date, assuming 1^{st} Jan 2001 is Monday.

10 Attempt following **two** : **2×7=14**

(1) Discuss about False Position Method and write flowchart or program for the same method.

(2) Explain about Gauss Elimination Method.

Seat No.________________

FM-CMT-3001

M. Sc. (Sem. - III) Examination

November - 2022

Mathematics - 3001

(Programming in C and Numerical Methods)

00070

Time : 2.30 Hours / Total Marks : 70

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each questions carries 14 marks.

1 Answer any seven short questions. **[7×2=14]**

(i) Define terms : program and lower level language.
(ii) Give definition of flow-chart.
(iii) Write a program which can print 1 to 40 integers in four lines.
(iv) Write down name of Relational Operators.
(v) Give definitions; Identifier and Variable.
(vi) Write down at least four names of C-Tokens.
(vii) Draw flow chart, so that one can write a program which can print letters 'A' to 'Z'.
(viii) Write down ASCII code for 'A', '1' and 'Z'.
(ix) Write down general form for Assignment Statement.
(x) Write down general format for one dimensional array. Also write down one example for this.

2 Attempt any two. **[2×7=14]**

(a) Write a note about development of C- Language.
(b) Explain about input and output operations by their format and suitable examples.
(c) Write a program which can read two square matrices A, B of order n and it can print the matrices A - B, A * B.

3 Attempt any one. **[1×14=14]**

(a) Discuss about Newton Raphson's Method, write down the program for the same method and find the order of convergence for the N-R Method.

(b) Explain about Gauss - Elimination Method and using it solve following system of four linearly independent equations of four variables.

(i) $x_1 + x_2 + x_3 + x_4 = 25$

(ii) $2x_1 + 3x_2 + 4x_3 + 5x_4 = 75$

(iii) $x_1 + x_2 + 4x_3 + 5x_4 = 50$

(iv) $x_1 + 4x_2 + 16x_3 + 64x_4 = -35$.

4 Attempt following two. **[2×7=14]**

(a) Write a program, which can read date, month and year of 21st Century and give it day which associate with the given date. Assuming 1st Jan 2001 is Monday.

(b) Find the root of $f(x) = x^3 - 7$, using Bisection Method and take initial values $a = 1.5$, $b = 2$.

5 Attempt any two. **[2×7=14]**

(1) Write down a Note about C-Tokens.

(2) Write a program to read n integers and arrange them in ascending order.

(3) Discuss about While loop Statement.

(4) Explain about following Function Subprogram with suitable example : No Argument and No Return Value.

SAA-CMT-3001 Seat No. 003022

M. Sc. (Sem. III) (CBCS) Examination

November - 2021

CMT-3001 : Mathematics

(Programming in C & Numerical Methods)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :
(1) There are Ten questions.
(2) Answer any five questions.
(3) Each question carries 14 marks.

1 Answer following short questions [7 X 2 = 14]

(i) Write down at least four reserved identifiers.

(ii) Write a program which can print 1 to 40 integers in four lines.

(iii) Write down name of Relational Operators.

(iv) Express following mathematical functions in C - Language.

(i) cos x, (ii) $log_e x$, (iii) $\sqrt{x}$ and (iv) e^x.

(v) Write a program, which can read two integers and it can find the smallest integer from given two integers.

(vi) Give definitions: Identifier and Variable.

(vii) Draw flow chart, so that one can write a program which can print small letters 'a' to 'z'.

2 Answer following short questions [7 X 2 = 14]

(1) Define terms: Program and Lower Level Language.

(2) Write down at least four names of C-Tokens.

(3) Give definition of flow-chart.

(4) Write a program which can print 1 to 30 integers in three lines.

(5) Express following mathematical functions in C - Language.

(i) sin x, (ii) |x|, (iii) $\sqrt{x+1}$ and (iv) e^{2x+1}.

(6) Draw flow chart, so that one can write a program which can print letters 'A' to 'Z'.

(7) Write down format for jump in a loop statement by break.

3 Attempt following two [2 X 7 = 14]

(a) Write a note about importance of C – Language.

(b) Write a program, which can read date, month and year of 21 st Century and give it day corresponding to the given date. Assuming 1 st Jan 2001 is Monday.

4 Attempt following one [1 X 14 = 14]

(a) Discuss about Newton Raphson's Method, write down the program for the same method and find order of convergence for the N-R Method.

5 Attempt following one [1 X 14 = 14]

(1) Explain about Gauss – Elimination Method and using it solve following system of four linearly independent equations of four variables.

(1) $x_1 + x_2 + x_3 + x_4 = 25$

(2) $2x_1 + 3x_2 + 4x_3 + 5x_4 = 75$

(3) $x_1 + x_2 + 4x_3 + 5x_4 = 50$

(4) $x_1 + 4x_2 + 16x_3 + 64x_4 = -35$.

6 Attempt following one [1 X 14 = 14]

(a) Explain about Lagrange interpolation polynomial and derive its formula. Using it write a program for Lagrange interpolation polynomial.

7 Attempt following two [2 X 7 = 14]

(a) Explain about input and output operations by their format and suitable examples.

(b) Write a program, which can read two integers a and b and it can find (a,b), the GCD of a and b as well as [a,b], the LCM of a and b.

8 Attempt following two [2 X 7 = 14]

(a) Discuss about simple if statement as well as if else statement.

(b) Find at least two roots of f(x) = $x^3 - 4x + 1$, using any iterative method.

9 Attempt following two [2 X 7 = 14]

(1) Write a program, which can solve the linearly independent equations ax+by+c = 0 and px+qy+r = 0, using Cramer's Method.

(2) Write a program, which can read an integer n and it can check whether n is a prime or not.

10 Attempt following two [2 X 7 = 14]

(a) A function Subprogram can recourses or invoke in itself, explain with suitable examples.

(b) Discuss about Secant Method and also compute the order of convergence for the same Method.

MV-CMT-3001 Seat No. 035035

M. Sc. (Sem. III) Examination

November / December - 2020

Mathematics : CMT-3001

(Programming in C & Numerical Methods)

(New Course)

46

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer any five questions.

(2) Each question carries 14 marks.

1 Answer following seven questions : 7x2=14

10

2 (i) Define terms: Compiler and Higher Level Language.

2 (ii) Write down at least four reserved identifiers.

1 (iii) Write a program which can print 1 to 80 integers in four lines.

2 (iv) Express following mathematical functions in C - Language :

(i) cosx, (ii) $\log_e x$, (iii) $\sqrt{x}$ and (iv) e^x.

(v) Write a program, which can read two integers and it can. find smallest integer from given two integers.

(vi) Give definitions: Identifier and Variable.

2 (vii) Draw flow chart, so that one can write a program which can print letters 'A' to 'Z'.

1

2 Answer following both : 2x7=14

10

7 (a) Write a note about development of C - Language.

(b) Write a program, which can read two integers a and b. Also it can find the value of GCD (a, b) and LCM [a, b] of given two integers.

3 Discuss about Newton Raphson's Method and write down the program for the same method. 14

4 Explain about Gauss - Elimination Method and using it solve following system of four linearly independent equations of four variables : 14

(1) $x_1 + x_2 + x_3 + x_4 = 5$

(2) $2x_1 + 3x_2 + 4x_3 + 5x_4 = 15$

(3) $x_1 + x_2 + 4x_3 + 5x_4 = 10$

(4) $x_1 + 4x_2 + 16x_3 + 64x_4 = -7$

5 Attempt following both : **2x7=14**

(a) Write down a program which can display all the primes which are less than 1000.

(b) Find a root of equation $f(x) = x \cdot e^x - 2$, using Bisection Method.

6 Attempt following both : **2x7=14**

(1) Explain about Secant Method and also compute the order of convergence for this method.

(2) Explain about Basic Structure of a C program.

7 Attempt following both : **2x7=14**

(1) Write a program which can display tables of 11 to 20.

(2) Write a program which can read two square matrices A, B of order n and it can print the matrices A – B, A*B.

8 Attempt following both : **2x7=14**

(a) Find out at least two roots of $f(x) = x^3 - 4x + 1$, using any iterative method.

(b) Write a note about Argument with return value in user defined function with suitable example.

9 Discuss about False Position Method, write down the program for the same method and also compute order of convergence for this iterative method. 14

10 Attempt following **seven** : 7x2=14

(1) Write down at least four names of C-Tokens.

(2) Give definition of flow-chart.

(3) Write down format for jump in a loop statement by break.

(4) Write down order of convergence of Bisection Method and Newton Raphson Method.

(5) Write down name of Relational Operators.

(6) Write down short keys to compiling and run a C-Program.

(7) Write down all Logical Operators with their appropriate notation in C language.

* J A X - C M T - 3 0 0 1 *

JAX-CMT-3001 Seat No. ________

M. Sc. (Mathematics) (Sem. III)
(CBCS) Examination
December – 2019
CMT - 3001 : Programing in C & Numerical Methods

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are **compulsory**.
(2) Each question carries **equal** marks.

1 Answer following short questions (any seven) : **7×2=14**

(i) Define terms: Machine language and Lower level language.

(ii) Write down all the sections of Basic Structure of C Program.

(iii) Write down at least four names of C-Tokens.

(iv) Write a program which can print A to Z (Capital letters) in one line.

(v) Write down name of Relational Operators.

(vi) Write down short keys to compiling and run a C-Program.

(vii) Draw flow chart, so that one can write a program which can print integers 1 to 25.

(viii) Give names of three logical operators.

(ix) Write down all Logical Operators with their appropriate notation in C language.

(x) Remove unnecessary parentheses from following and rewrite them :

(1) ((x – (y / 5) + z) % 8) +25
(2) (x * y) +(– a/b) + (c – d).

2 Attempt any two : **2×7=14**

(a) Write a program which can read two rectangular matrices of size 4×3 and it can find the sum of given two matrices.

(b) Write a note about Development of C Language.

(c) Explain about for loop with its format.

3 Attempt any one **1×14=14**

(a) Discuss about False Position Method and write down the program for the same method.

(b) Explain about Gauss-Seidel method and write down the program for the same method.

(c) Explain about Gauss Elimination Method and write down the program for the same method.

4 Attempt any two : **2×7=14**

(a) Write down a program which can display first 200 primes.

(b) Explain about N-G Backward interpolation polynomial.

(c) Find the value of f(3) for the following unknown function f, using following table and Lagrange interpolation polynomial :

x	–1	1	4	5	3
$F(x)$	8	–2	–2	2	?

5 Attempt any two : **2×7=14**

(1) Write a program which can display tables of 1 to 5 and 6 to 10.

(2) Write a program which can read two square matrices A, B of order n and it can print the matrices A + 2B and A * B.

(3) Find out at least two roots of $f(x) = x^3 - 4x + 1$, using N-R method.

(4) Explain about User Defined Functions. Also write about one user defined function with its format and a suitable program in which the user defined function has used.

PCC-CMT-3001 Seat No. 35063

M. Sc. (Sem. III) (CBCS) Examination

December - 2018

Mathematics : CMT - 3001

(Programming in C & Numerical Methods)

(New Course)

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Answer all the five questions.

(2) Each question carries 14 marks.

1 Answer any seven short questions : 7×2=14

(1) Define term : Compiler.

(2) Write down six sections of basic structure of a C program.

(3) Write down names of all C-Tokens.

(4) Write down a list of reserved identifiers. Which contains at least six names of the reserved identifiers.

(5) Give name of all three logical operators with their symbols in C.

(6) Write a program which can print 1 to 20 integers in column form.

(7) Give definition of flow chart.

(8) Remove unnecessary paranthesis from following expressions and rewrite them :

(i) ((a – (y / 15) + z + w)%2) – (x + 25);

(ii) (a * b) – (c / d);

2 Attempt any two : 2×7=14

(a) Explain about input and output operations by their format and suitable examples :

getchar, scanf, putchar and printf.

(b) Discuss about recursion of a function in itself by appropriate programs.

(c) Explain about for loop and nesting loops by appropriate programs.

[Contd....

1×14=14

3 Attempt any one :

(a) Discuss about Bisection method and write down the program for the Bisection method.

(b) Explain about Gauss Elimination method and write the program for this method.

(c) Explain Lagrange interpolation polynomial and derive its formula. Using it, find the value of f(3), for the following unknown function f :

x	-1	1	4	5	3
$f(x)$	8	-2	-2	2	$f(3) = ?$

2×7=14

4 Attempt any two :

(a) Write a program which can display first 150 or more primed.

(b) Write the program for the Gauss-Seidel method.

(c) Write a program which can read n integers and it can arrange them in arcending order.

2×7=14

5 Attempt any two :

(a) Explain about switch statement with its format/syntax and appropriate example.

(b) Write a program which can read two square matrices A,B of same order and it can find sum and product of these matrices.

(c) Write a program which can display tables of 11 to 15 and 16 to 20.

(d) Find out at least two approximate roots of $f(x)=x^3-4x+1$, using N-R method. Take initial root x_0 from {0.25, 2 or −2}

HDU-CMT-3001 Seat No. 035064

M. Sc. (Maths) (Sem. III) (CBCS) Examination

November / December – 2017

Mathematics : CMT-3001

(Progra. in C & Numerical Methods) (New Course)

Time : 2 ½ Hours] [Total Marks : 70

Instructions : (i) Answer all the questions.
(ii) Each question carries 14 marks.

1 Attempt any seven : 7×2=14

(1) Give definition of flow-chart and draw flow-chart of a program which can display A to Z letters.

(2) Write down ASCII code for 'f' and 'B' letters.

(3) Write a program which can display 'a' to 'j' letters in column form.

(4) Give definitions of compiler and lower level language.

(5) Write down four names of reserved identifiers (keywords).

(6) Write down four name of relational operators.

(7) Write down four mathematical functions for a C program.

(8) Give definitions of integer constant and real constant.

2 Attempt any two : 2×7=14

(1) Write a note about importance of C language.

(2) Discuss about bisection method.

(3) Discuss about recursion of a function in itself by an appropriate program.

(4) Write a note about basic structure of a C program.

(5) Explain about switch statement and using it write a program which can read date of Jan 2018 and it can find associate day of the date (assuming 1st Jan 2018 is Monday).

3 Attempt any one : **1×14=14**

(a) Explain about Lagrange interpolation polynomial, write down program for Lagrange interpolation polynomial and using it solve followings :

x	−1	1	4	5	3
$f(x)$	8	−2	−2	2	?

(b) Explain Newton-Raphson's method, find order of convergence for N-R method and find an approximate root of $f(x) = x^3 - 4x + 1$ by N-R method.

(c) Explain Gauss-elimination method and write a program for Gauss elimination method to solve a system $AX=B$ of order n.

4 Attempt any two : **2×7=14**

(a) Write a program which can read two integers a and b and it can display *gcd* as well as lcm of a and b.

(b) Write a program which can print first 50 primes 2, 3, 5, 7, ..., 229.

(c) Write a program which can read two square matrices A, B of order n and it can find value of AB matrix.

5 Attempt any two : **2×7=14**

(a) Discuss about false position method.

(b) Write a program which can display tables of 1 to 10.

(c) Write a program for secant method.

(d) Solve the following system of three linearly independent equations, using Gauss-Seidel method :

$$16x_1 + 10x_2 + 2x_3 = 42$$
$$5x_1 + 10x_2 + 5x_3 = 40$$
$$x_1 + 4x_2 + 9x_3 = 36$$

MBR-003-016301 Seat No.

M. Sc. (Sem. III) (Maths.) (CBCS) Examination

December - 2016

Mathematics : CMT - 3001

(Progra. in C & Num. Methods) (Old Course)

Faculty Code : 003
Subject Code : 016301

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the five questions.
(2) Each question carries 14 marks.

1 Answer any seven : 7×2=14

(1) Give definition of flow - chart.

(2) Write a program which can print 1 to 10 integers in column form.

(3) Write down two names of higher level languages and name of associates (developers) with these languages.

(4) Write down ASCII code for the characters : 'D', 'd' and 'z'.

(5) Write a program which can print first letters Z to A.

(6) Give definition of identifier and write down two reserved identifiers (key words).

(7) Give name of following special characters : ^ and ".

(8) Determine value of followings (when a = 6, b = 8 and c = –9).

(i) $a > b \&\& a < c$

(ii) $a * b + b \% a + c$

(9) Write a program which can read a string variable and it can print it 15 times in column form.

(10) Write a program which can read four integers a, b, c, d and it can print the values of $a + b + c + d$ and $ab + cd$.

[Contd...

2 Answer any two : 2×7=14

(a) Write down a note about importance of C language.

(b) Write a note about basic structure of a C program.

(c) Explain about if....else statement and using it write a program which can find a largest number from given three numbers.

3 Answer any one : 1×14=14

(a) Explain N-G forward interpolation polynomial and derive the formula

$$P(x)=f_1+\frac{\Delta f_1}{h}(x-x_1)+\frac{\Delta^2 f_1}{2\cdot h^2}(x-x_1)(x-x_2)$$

$$+\dots+\frac{\Delta^{n-1} f_1}{(n-1)!\,h^{n-1}}(x-x_1)(x-x_2)\dots(x-x_{n-1})$$

Using this find the formula for an unknown function f given by

x	0	1	2	3	4	5
$f(x)$	−2	−3	−2	1	6	13

(b) Write the program for Gauss - Elimination method.

(c) Discuss about N-R method and using its formula find the approximate value of $\sqrt[3]{7}$ by taking initial $x_0 = 2$.

4 Answer any two : 2×7=14

(a) Write a program which can find gcd of four integers and it can use to find gcd of two integers x and y as a sub - program.

(b) Write a program which can print first 100 primes (2, 3, 5,...,541).

(c) Write a program of false position method.

5 Attempt any two : 2×7=14

(a) Explain about for loop statement with its format and syntax. Also write a program which includes loop in a loop to print 1 to 100 integers in column form.

(b) Write a program which can read two matrices A and B of the size m×n and n×p. Also it can find the product AB of these two matrices.

(c) Write a program which can display tables of 11 to 20.

(d) Discuss about Gauss-Seidel method to solve a system of linear equations :

$$a_{11} x_1 + a_{12} x_2 = b_1$$

$$a_{21} x_1 + a_{22} x_2 = b_2$$

(67)

BBG-003-016301

Seat No. ________

M. Sc. (Maths) (Sem. III) (CBCS) Examination

December – 2015

Programming in C & Numerical Methods

Faculty Code : 003

Subject Code : 016301

Time : 2.30 Hours] [Total Marks : 70

Instructions : (i) Attempt all five questions.

(ii) All questions carry equal (14) marks.

1 Answer seven MCQ questions : 14

(1) C language was developed by Dennis Ritchie in the year ..... ?

(a) 1970

(b) 1962

(c) 1960

(d) 1972

(2) Order of convergence for false position method is the positive root of ________ .

(a) $x^2 + x - 2$

(b) $x^2 - x - 2$

(c) $x^2 - x - 1$

(d) $x^3 - 7$

(3) In control string ________ format specifies for integers.

(a) %f

(b) %d

(c) %s

(d) None of these

[Contd...

(4) In C-language program sign of colon is ______.

(a) :

(b) ;

(c) %

(d) #

(5) $C = \left(a\,f(b) - b\,f(a)\right) / \left(f(b) - f(a)\right)$ can be consider as a formula for ________ iterative method.

(a) false position method

(b) secant method

(c) (a) and (b) both

(d) bisection method

(6) Iterative formula for New-Raphson's method is ________.

(a) $x_i = \dfrac{x_{i-1} + x_{i-2}}{2}$

(b) $x_i = x_{i-1} - \dfrac{f(x_{i-1})}{f'(x_{i-1})}$

(c) $x_i = \dfrac{x_{i-1} f(x_{i-2}) - x_{i-2} f(x_{i-1})}{f(x_{i-2}) - f(x_{i-1})}$

(d) None of these

(7) An integer constant has value ________ when it is assigned by 'd'+1.

(a) 100

(b) 121

(c) 69

(d) 101

2 Answer any two : 2×7=14

(a) Write a note about importance of C-language.

(b) Discuss about recursion of a function in itself by an appropriate program.

(c) Write a program which can read two integers m and n. Also it can print all the divisors of these given integers.

(d) Write a program which can display tables of 1 to 10 integer.

3 Answer any one : 1×14=14

(a) Explain Lagrange interpolation polynomial and derive the formula $p(x)=\sum_{k=1}^{n}\left[f_k \prod_{\substack{i=1 \\ i\neq k}}^{n}\left(\frac{x-x_i}{x_k-x_i}\right)\right]$. Using this find the unknown value for the following function :

x	-1	1	4	5	3
$f(x)$	8	-2	-2	2	$f(3)=?$

(b) Discuss about bisection method. Also write the program for this method.

(c) Write the program for Gauss Elimination method.

4 Answer any two : 2×7=14

(a) Solve the following system of equations :

$x_1+x_2+x_3+x_4=5;\quad x_1+x_2+4x_3+5x_4=10;$

$2x_1+3x_2+4x_3+5x_4=15;\quad x_1+4x_2+16x_3+64x_4=-7.$

using the Gauss Elimination method. Also show the triangular form for above system.

(b) Write program about false position method.

(c) Write a program which can print all the primes more than 540 and less than 1225.

(d) Explain about for loop statement with its format, syntax and an example.

5 Answer any seven : 7×2=14

(i) Give definition of single character constant and string constant.

(ii) Give definition of identifier and variable.

[Contd...

(iii) Identify unneccessary paranthesis in following statements and rewrite them.

(a) $\left(\left(x-\left(y/5\right)+z\right)\%8\right)+25$

(b) $x/\left(9*y\right)$

(iv) Give name of special characters ^, #.

(v) Give name of scientist who develope or associate with BCPL language.

(vi) Write a program which can print a to z small letters.

(vii) Write characters whose ASCII codes are 68 and 122.

(viii) Write a program which can print 200 to 101 integers in decreasing form.

(ix) Give definition of flow-chart.

(x) Write a program which can read four integers x, y, z and w. Also it will print the values of $x+y+z+w$ and $xy+zw$.

(71)

WL-152

003-016301

M.Sc. Maths (CBCS) (Sem.-III) Examination

November-2014

Programming in C and Numerical Methods

Faculty Code : 003

Subject Code : 016301

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) Answer all five questions.

(2) Each question carries 14 marks.

1. Answer MCQ type questions : 14

(1) In a C program sign of semicolon is _____ ?

(a) : (b) &

(c) ; (d) %

(2) There are _____ reserved identifiers in C-language.

(a) 29 (b) 27

(c) 127 (d) 128

(3) Name of program or programmer is written in _____ section of a program,

(a) link (b) global declaration

(c) documentation (d) none of these

(4) How many types of user defined functions are there ?

(a) 4 (b) 3

(c) 2 (d) 1

(5) Formula for Newton Raphson's method is _______ ?

(a) $x_i = x_{i-1} - \frac{f'(x_{i-1})}{f(x_{i-1})}$ (b) $x_i = x_{i-1} - \frac{f'(x_{i-1})}{f''(x_{i-1})}$

(c) $x_i = x_{i-1} + \frac{f(x_i)}{f'(x_i)}$ (d) None of these

(6) Arrange following basic structure for a C-programme in proper order.

(1) definition section (2) main function

(3) link section (4) global declaration sec.

(a) $4 \to 1 \to 2 \to 3$ (b) $1 \to 4 \to 2 \to 3$

(c) $3 \to 4 \to 1 \to 2$ (d) None of these

(7) Process to get triangular form is associate with _____ method.

(a) Gaus-Seidel (b) Gauss-Elimination

(c) Newton-Raphson (d) None of these

P.T.O.

2. Answer any two : $7 \times 2 = 14$

(a) Write a note about development of C language. 7

(b) Write a program which can find GCD and LCM of given two integers. 7

(c) Write a program which can print 200 to 101 integers in desending order. 7

(d) Explain about if .... else .... statement with a suitable example. 7

3. Answer any one. $1 \times 14 = 14$

(a) Discuss about bisection method and write the program for this method. 14

(b) Discuss about Newton-Raphson's method and using it find a root of equation $f(x) = x^3 - 7 = 0$ by taking $x_0 = 2$. 14

(c) Write a program which can print first 100 primes. 14

4. Answer any two : $2 \times 7 = 14$

(a) Write a program about false-position method to solve the equation $f(x) = 0$. 7

(b) Write a note about secant method to solve the equation $f(x) = 0$. 7

(c) Explain about while loop statement with its format and syntax. 7

(d) Write a program which can read an integer and it can print all the divisors of the given integer. 7

(e) Discuss about one-dimensional array and initialization for one-dimensional array. 7

5. Answer any seven : $7 \times 2 = 14$

(i) Give definition of high level language. 2

(ii) Write basic structure of a C Program. 2

(iii) State arithmetic operators with their sign and meaning. 2

(iv) Determine value of followings (when a = 5, b = 10 & c = – 6) 2

(a) a > b && a < c

(b) a == c || b > a

(v) Write a program which can read three integers and it can print sum and product of given integers. 2

(vi) Write a program which can print 1 to 100 integers. 2

(vii) Write a program which can read p, r and n. Also it can find simple interest for this data. 2

(viii) Give all the names of C tokens. 2

(ix) Write ASCII code for the characters 'A', 'z' and 'd'. 2

(x) Write order of convergence for N-R method and false position method. 2

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73

128/A

003-016301

M.Sc. (Sem.-III) (Maths) (CBCS) Examination

November-2013

Programming in C & Numerical Methods

Faculty Code : 003
Subject Code : 016301

Time : 2 ½ hours] [Total Marks : 70

Instructions : (1) Answer all the questions.

(2) Each question carries 14 marks.

1. Answer any seven MCQ. 14

(1) In 1972 C language developed by _______ at AT & T's Bell Laboratories.

(a) Martin Richards (b) Ken Thomson

(c) Dennis Ritchie (d) None of these

(2) To give increment to the variable x, which of following use in C language ?

(a) x++; or ++x; (b) x = x + 1;

(c) x + = 1; (d) All above three

(3) There are _______ reserved identifiers in C language.

(a) 27 (b) 127

(c) 128 (d) 29

(4) Before 1960 which language was used for commercial applications ?

(a) ForTran (b) COBOL

(c) C-language (d) None of these

P.T.O.

(5) Flow-Chart is ________ form for a program.

(a) Pictorial

(b) Chart type

(c) Table

(d) None of these

(6) Which of the following used to specify the format for integer type vari/const. ?

(a) %d

(b) %c

(c) %s

(d) %f

(7) Order of convergence for any iterative method is ________.

(a) always zero

(b) positive or negative

(c) always positive

(d) none of these

(8) Formula for Newton-Raphson's method is ________.

(a) $x_i = x_{i-1} - \frac{f(x_{i-1})}{f'(x_i)}$

(b) $x_i = x_{i-1} - f(x_{i-1})/f'(x_{i-1})$

(c) $x_i = x_{i-1} - f'(x_{i-1})/f''(x_{i-1})$

(d) $x_i = x_i - f(x_i)/f'(x_i)$

(9) Which of following is a short key, which use to compilation for a C programme ?

(a) Altr + F5

(b) Ctrl + F9

(c) Altr + F9

(d) None of these

(10) What is output of following ?

printf ("\" Wel come !");

(a) WEL COME

(b) "Wel come"

(c) "Wel Come !"

(d) "Wel Come !

2. Answer any two :

(a) Write a note about development of C language. 7

(b) Explain about arithmetic operators. and precedence of

(c) Write a program which can find gcd and lcm of given two integers. 7

(d) Write a program which can print tables of 11 to 20. 7

3. Write a note about user-defined functions. 14

OR

Discuss bisection method and also write the program for bisection method.

4. Answer any two :

(a) Discuss about Gauss-Elimination Method. 7

(b) Discuss about Gauss-Seidel Method. 7

(c) Explain false position method. 7

(d) Write a program which can give a list of first 100 or more primes. 7

5. Answer any one of following :

(a) Explain Lagrange interpolation polynomial and derive its formula

$$p(x) = \sum_{k=1}^{n}\left[f_k \cdot \prod_{\substack{i=1 \\ i \neq k}}^{n}\left(\frac{x - x_i}{x_k - x_i}\right)\right]$$

Also write the program for this method. 14

P.T.O.

(b) Write a note about N-R method and also write the program for N-R method. 14

openctor(c) (i) Write a program which can read an integer and it can check whether given integer is a prime number or not ? 7

(ii) Write a program which can two matrices of same size and it can find the sum of these two matrices. 7

$53+29=82$

78 14
56 18
14
70

77

F-13

003-016301

M.Sc. Maths (CBCS) (Sem. III) Examination
November-2012
Programming in C and Numerical Methods

Faculty Code : 003
Subject Code : 016301

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) Answer all five questions.
(2) Each question carries 14 marks.

1. Answer any seven MCQ. (14)

(1) #include<stdio.h> has used in a C program at ________.

(a) declaration section (b) documatation section
(c) top of the program (d) none of these

(2) Order of convergence for false position method is ________.

(a) positive root of $x^2 + x - 2$
(b) positive root of $x^2 - x - 2$
(c) positive root of $x^2 - x - 1$
(d) none of these

(3) ________ is a finite sequence of characters that is treated as a single data item.

(a) An array (b) A string
(c) An identifier (d) A data type

(4) C-language was developed by Dennis Ritchie in the year ________.

(a) 1972 (b) 1962
(c) 1960 (d) 1970

(5) There are ________ reserved identifiers in C-language.

(a) 27 (b) 29
(c) 127 (d) 128

(6) $C = \dfrac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$ can be consider as a formula for ________ iterative method.

(a) secant method
(b) false position method
(c) (a) and (b) both
(d) bisection method

P.T.O.

(7) In control string format ________ specifies for long integer.

(a) % wc (b) % d

(c) % s (d) % ld

(8) General equation for variable in Gauss Elimination method is ________.

(a) $x_i = \left[b_i - \sum_{i=j+1}^{n} a_{ij} x_j\right] / a_{ii}$

(b) $x_i = \left[b_i - a_{n-1\,n} x_n\right] / a_{n-1\,n-1}$

(c) $x_n = \left[b_n - a_{n-1\,n} x_n\right] / a_{nn}$

(d) none of these

(9) General form of the variable formal % wd, where w denotes ________.

(a) weight of the variable

(b) order of the variable

(c) data type for the variable

(d) none of these

(10) A program written in a higher level language can be transfer into machine language by ________.

(a) compiler (b) link

(c) declaration (d) none of these

2. Answer any two :

(a) Write a note about development of C language. (7)

(b) Explain about following statements with a suitable example and their format – if ________ else statement and for loop. (7)

(c) Write a program which can read two integers and it can print all the divisors of given integers. (7)

3. Write a note about user defined functions with examples. (14)

OR

Discuss about Gauss Elimination method and also write a program for this method.

4. Answer any two :

(a) Explain bisection method. (7)

(b) Write a program to solve $f(x) = x^3 - 7$ by secant method. (7)

(c) Write a program which can read three integers and it can find the smallest integer, using if ______ else statement. (7)

5. Answer any two :

(a) Explain Langrange interpolation polynomial and derive its formula (14)

$$P(x) = \sum_{k=1}^{n}\left[f_k \prod_{\substack{i=1 \\ i \neq k}}^{n}\left(\frac{x - x_i}{x_k - x_i}\right)\right]$$

(b) Write a program which can give a list of all primes less than 5000.

(c) Write a program which can read coordinates of three points of a triangle in $\mathbb{R}^2$ and it can check the given triangle is a right angled triangle or not.

(d) Write a program which can solve a system of linearly independent equations :

$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1$

$a_{n1} x_1 + a_{n2} x_2 + \dots + a_{nn} x_n = b_n$

by Gauss-Seidel method.

003-016301

M. Sc. (Sem. III) (Maths) Examination

December - 2011

Programming in C and Numerical Methods

Faculty Code : 003
Subject Code : 016301

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.

(2) Each question carries 14 marks.

1 Answer any seven objective type questions : 14

(1) Variable are declared in .... ?

(a) Global declaration section

(b) Definition section

(c) Declaration part of main ( ) section

(d) (a) and (c) both

(2) A language which can be understood by computer is .... ?

(a) compiler

(b) higher level language

(c) C language

(d) none of these

(3) How many types of user defined functions are there ?

(a) 1

(b) 2

(c) 3

(d) 4

[Contd...

(4) A/An .... is a sequence of characters that is treated as a single data item.

(a) array

(b) string

(c) data type

(d) none of these

(5) Which of the following is used to specify the format of character data type ?

(a) %c

(b) $\%w_1w_2f$

(c) %d

(d) none of these

(6) In a general form of the variable format %ld, where l denotes .... ?

(a) Order of the variable

(b) Weight of the variable length

(c) Data type of the variable

(d) None of these

(7) The order of convergence for the false position method is same as .... ?

(a) bisection method

(b) newton raphson method

(c) secant method

(d) none of these

(8) The formula $x_n = \dfrac{x_{n-2}f(x_{n-1}) - x_{n-1}f(x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$ is used in .... iterative method.

(a) secant method

(b) bisection method

(c) (a) and (b) both

(d) none of these

(9) Which of following use in C language to give an increment for a variable x ?

(a) x++; or ++x;

(b) x=x+1;

(c) x+=1;

(d) all these three

(10) The general equation of gauss elimination method is .... ?

(a) $x_i = \left[b_i - \sum_{j=i+1}^{n} a_{ij} x_j \right] / a_{ii}$

(b) $x_i = \left[b_i + \sum_{j=i+1}^{n} a_{ij} x_j \right] / a_{ii}$

(c) $x_n = \left[b_n - \sum_{j=1}^{n-1} a_{nj} x_j \right] / a_{nn}$

(d) none of these

2 Answer any two :

(a) Explain about arithmetic operators. 7

(b) Discuss about recursion of a function in it self. 7

(c) Write a program which can find gcd and lcm of given two integers. 7

3 Describe about importance of C language and development of C language. 14

OR

3 Write a note about user-defined functions. 14

4 Answer any two :

(a) Write a program to solve $f(x)=0$ by bisection method. 7

(b) Explain false position method. 7

(c) Solve following system by gauss elimination method : 7

$$x_1+x_2+x_3+x_4=5$$
$$x_1+x_2+4x_3+5x_4=10$$
$$2x_1+3x_2+4x_3+5x_4=15$$
$$x_1+4x_2+16x_3+64x_4=-7$$

5 Answer any two :

(a) Explain langrange interpolation polynomial and derive its formula 7

$$P(x)=\sum_{k=1}^{n}\left[f_k \prod_{\substack{i=1 \\ i\neq k}}^{n}\left(\frac{x-x_i}{x_k-x_i}\right)\right]$$

(b) Write formula for the secant method and using it compute order of convergence of this method. 7

(c) Write a program for the polynomial of an unknown function by newton gregory backward interpolation polynomial. 7

(d) Write a program which can give a list of first 100 or more primes. 7

DBK-003-1163002 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

June – 2022

Mathematics : CMT - 3002
(Functional Analysis)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any **five** questions from the following.

(2) There are total **ten** questions.

(3) Each question carries **equal** marks.

1 Answer the following : **7×2=14**

(1) Let $T : X \to X$ be a linear transformation. Justify whether R (T) is a vector space or not?

(2) Define with example: Continuous Linear transformation.

(3) Define with example: Banach Space.

(4) Justify whether a real valued function $f : [0, 1] \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 0, & when\ x \in [0, 1] \cap \mathbb{Q} \\ x, & when\ x \in [0, 1] \cap \mathbb{Q}^c \end{cases}$$

is essentially bounded or not?

(5) State Parseval's identity.

(6) Define with example: Weak* –Convergence.

(7) Define with example: Algebraic Dual Space.

2 Answer the following : **7×2=14**

(1) Justify whether dual space of l^{∞} is l^{1} or not?

(2) Define with example : Sub-linear functional.

(3) Define with example: Direct Sum.

(4) Define with example: Hilbert space.

(5) Define with example: Orthogonal elements.

(6) Justify whether two Orthonormal elements of an Inner Product Space X are linearly independent or not?

(7) Define nowhere dense set. Give an example of uncountable set which is nowhere dense set.

3 Answer the following : **2×7=14**

(1) State and prove, Minkowski's Inequality.

(2) Let X and Y be two normed spaces.
Let $T : X \to Y$ be a linear transformation. Prove that, the following are equivalent :

a) T is continuous on X.

b) The null space $N(T)$ is closed in X and the linear transformation $\tilde{T} : X|_{N(T)} \to Y$ defined by

$$\tilde{T}\left(x+N(T)\right)=T(x),\ \forall x+N(T)\in X|_{N(T)}$$ is continuous.

4 Answer the following : **2×7=14**

(1) Prove that, every finite dimensional subspace of a normed space X is complete.

(2) Let $p \in [1, \infty)$. Prove that, l^{p} is a complete metric space.

5 Answer the following : **2×7=14**

(1) Prove that, on a finite dimensional vector space X, any norm $\|\cdot\|_a$ is equivalent to any other norm $\|\cdot\|_b$.

(2) Let X and Y be Normed linear space and let $B(X, Y)$ be the space of all bounded linear transformations from X into Y. If Y is a Banach space, prove that, $B(X, Y)$ is also a Banach space

6 Answer the following : **2×7=14**

(1) State and prove, Uniform Boundedness theorem.

(2) State Baire's Category theorem. Prove that, a Banach space does not have a countably infinite Hamel Basis.

7 Answer the following : **2×7=14**

(1) State and prove, closed graph theorem.

(2) State Hahn-Banach Theorem. Prove that, if X is any normed linear space over K then

$$\|x\| = \sup_{0 \neq f \in X'} \frac{|f(x)|}{\|f\|}, \forall x \in X.$$

8 Answer the following : **2×7=14**

(1) State and Prove, Projection Theorem.

(2) Let X be an Inner Product Space. Let $x_n \to x$ in X and $y_n \to y$ in X. Prove that, $\langle x_n, y_n \rangle \to \langle x, y \rangle$

9 Answer the following : **2×7=14**

(1) State and prove, Riesz-Representation Theorem.

(2) Prove that, every Hilbert space H is reflexive.

10 Answer the following : **2×7=14**

(1) State and prove, Parallelogram law as well as Pythagorean Relation.

(2) State and prove, Polarization identity.

Seat No. ________________

FN-003-1163002

M. Sc. (Sem. III) Examination

November - 2022

Mathematics : CMT-3002

(Functional Analysis)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries equal marks.

1 Answer any seven of the following : **7×2=14**

(1) Define with example: Norm linear Space.

(2) Define with example: Schauder Basis.

(3) State Minkowski's Inequality

(4) Define with example: Dual Space.

(5) Define with example: Separable norm linear space.

(6) True or false? Justify $\left(l^{\infty}, \|\cdot\|_{\infty}\right)$ has a Schauder basis.

(7) Define :

(i) Cauchy sequence in normed space.

(ii) Compactness of a subset of a metric space.

(8) Define with example: Inner product space.

(9) Define with example: Orthonormal Set.

(10) Define with example : Weak* - Convergence.

2 Answer any two of the following : 2×7=14

(1) State and prove Holder's inequality.

(2) Let X be a norm linear space over K and $\{x_1, ..., x_n\}$ is linearly independent in X then prove that $\exists c > 0$ such that

$$\|\alpha_1 x_1 + ... + \alpha_n x_n\| \geq c(|\alpha_1| + ... + |\alpha_n|), \forall \alpha_1, ..., \alpha_n \in K$$

(3) Show that every finite dimensional vector subspace Y of normed linear space X over $\mathbb{K}$ is a Banach space.

3 Answer the following : 2×7=14

(1) State and prove Riesz lemma.

(2) State and prove closed graph theorem.

OR

3 Answer the following : 2×7=14

(1) State and prove Uniform Boundedness Principle.

(2) If Y is a closed and bounded subset of a finite dimensional normed linear space X over $\mathbb{K}$ then prove that Y is compact.

4 Answer the following : 2×7=14

(1) State and prove Bessel's inequality for an orthonormal sequence.

(2) State and prove Pythagorean inequality.

5 Answer any two of the following : 2×7=14

(1) Define reflexive space and prove every Hilbert space is reflexive.

(2) State and prove Riesz representation theorem for bounded linear functional on Hilbert spaces.

(3) Show that $l^{\infty} = \{(x_1, ..., x_n, ...) ; x_n \in K,$

$\forall n = 1, 2, ... \& |x_n| < M, \forall n$ for some $M > 0\}$ with

$\|\cdot\|_{\infty} : l^{\infty} \to \mathbb{R}$ defined by $\|(x_1,...., x_n, ...)\|_{\infty} = \underset{n}{Sup}|x_n|,$

$\forall (x_1, ..., x_n, ...) \in l^{\infty}$ is norm on l^{∞} and $(l^{\infty}, \|\cdot\|_{\infty})$ is a Banach space over K.

(4) Let $X_1, ..., X_n$ be a norm linear space over K. Then show that $(X_1 \times ..... \times X_n, \|\cdot\|)$ is a Banach space over K iff X_i is a Banach space over K, $\forall i = 1, ..., n$.

Where $\|(x_1, ..., x_n)\| = \max_{1 \le i \le n} \|x_i\|,$

$\forall (x_1, ..., x_n) \in X_1 \times ..... \times X_n$

MW-003-1163002 Seat No. 031035

M. Sc. (Sem. III) Examination

December - 2020

CMT-3002 : Mathematics

(Functional Analysis)

Faculty Code : 003

Subject Code : 1163002

28

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Answer the following: 14

(1) Define Taxicab Space.

(2) Justify: Whether every metric in a metric space is complete.

(3) Define with example Cauchy sequence in a metric space.

(4) Give an example of a space that is not a Banach space over K.

(5) Define with example complete metric space.

(6) Define with example Annihilator.

(7) Define with example Algebraic Dual Space.

2 Answer the following: 14

(1) Define with example accumulation point in a metric space.

(2) Define with example bounded linear operator.

(3) Define Weak convergence and Strong convergence in norm linear space.

(4) Justify: $\left(l^{\infty}, ||\cdot||_{\infty}\right)$ has a schaunder basis or not?

(5) Define Quotient space and Dense space in metric space.

(6) State Parseval's Inequality.

(7) Justify: Whether every finite dimensional vector subspace of a norm linear space over K is closed? **14**

3 Answer the following:

(a) State and prove Holder's Inequality.

(b) State and prove Riesz Lemma.

14

4 Answer the following:

(a) Define Banach space over K and prove that $(C[0,1], ||\cdot||_1)$ is not a Banach space over K.

(b) State without proof Baire's Theorem. Prove that a Banach space cannot have a countably infinite hamel basis.

14

5 Answer the following:

(a) State and Prove Schwartz Inequality in inner product space.

(b) Prove that a finite dimensional vector space is algebraically reflexive.

14

6 Answer the following:

(a) Prove that dual of $\mathbb{R}^n$ is $\mathbb{R}^n$.

(b) For a norm linear space X over K, prove that the dual space X^l is separable $\Rightarrow$ X is separable.

14

7 Answer the following:

(a) State and prove parallelogram law in inner product space. Also give an example of norm which does not satisfy parallelogram law. Justify your answer.

(b) In an inner product space X, $if\ x_n \to x\ and\ y_n \to y$ then prove that $< x_n, y_n > x, y >$.

8 Answer the following: 14

(a) State and prove Hanh- Banach Theorem.

(b) State and prove Polarization Identity and prove that $<,>: X \times X \to K$ is continuous for (X, <,>) be an inner product space.

9 Answer the following: 14

(a) State and Prove Projection Theorem and Pythagorean Relation in an inner product space.

(b) Prove that a Hilbert space H is separable if and only if H has a countable orthogonal basis.

10 Answer the following: 14

(a) If X is a finite dimensional norm linear space with dimX = n then prove that X' is also a finite dimensional norm linear space and dimX' = n

(b) State and prove Closed Graph Theorem.

JAY-003-1163002 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

December – 2019

Maths : Functional Analysis : CMT - 3002

(Old & New Course)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer the following questions : (Any seven) **7×2=14**

1. Define :
   i) Taxicab metric
   ii) Accumulation point in a metric space.
2. Write in brief about the function space sequence space s.
3. Is every metric in a metric space complete? Justify.
4. Prove that every convergent sequence in a metic space is a Cauchy sequence.
5. Define :
   i) Cauchy sequence in a metric space
   ii) Cauchy sequence in a normed linear space.
6. Define Complete metric space with example.
7. State and prove Translation invariance of metric d in a metric space?
8. Define :
   i) Dual Space
   ii) Algebraic Dual Space.
9. Define :
   i) Linear Functional
   ii) Bounded Linear Operator.
10. Define Banach space with example.

2 Answer the following questions : (Any two) **2×7=14**

1. State and prove Holder inequality for sums.
2. State and prove Minkowski inequality for sums.
3. Prove that A subspace M of a complete metric space X is itself complete if and only id the set M is closed in X.
4. Prove the completeness of the space IR".

3 Answer the following questions : **2×7=14**

a) Let X = (X,d) be a metric space. Then, prove that a convergent sequence in X is bounded and its limit is unique.

b) Let X = (X, d) be a metric space. Then, prove that if $x_n \rightarrow x$ and $y_n \rightarrow y$, then $d(x_n, y_n) \rightarrow d(x, Y)$.

OR

a) Let T be a linear operaot. Then, prove that the range of T, R(T), is a vector space.

b) Let T be a linear operator. Then, prove that the null space of T, N(T), is a vector space.

4 Answer the following questions : (Any two) **2×7=14**

1. Prove that if in an inner product space, $x_n \rightarrow x$ and $y_n \rightarrow y$, then $<x_n, y_n> \rightarrow <x, y>$.
2. State and prove Parseval relation.
3. Let X and Y be the inner product spaces and Q : X $\rightarrow$ Y be a bounded linear operator. Then, prove that :
   a) Q = 0 if and only if $<Qx, y> = 0$ for all $x \in X$ and $y \in Y$.
   b) If Q : X $\rightarrow$ X, where X is complex and $< Qx, x> = 0$ for all $x \in X$, then Q = 0.

5 Answer the following questions : (Any two) **2×7=14**

1. Define Hilbert space and Hilbert adjoint operator. Let H_1 and H_2 be the Hilbert spaces, S : $H_1 \rightarrow H_2$ and T : $H_1 \rightarrow H_2$ be the bounded linear operators and x be any scalar. Then, prove that

i) $< T^*y, x> \_ <y, Tx>$

ii) $(S + T)^* = S^* + T^*$.

iii) $(T^*)^* = T$.

iv) $T^*T = 0$ if and only if $T= 0$.

v) $(ST)^* =T^*S^*$.

2. Define Self ad joint operator and unitary operator. Let the operators $U : H \rightarrow H$ and $V : H \rightarrow H$ be unitary, where h is the Hilbert space. Then, prove that

i) U is isometric.

ii) $|| U || = 1$; provided $H \neq \{0\}$.

iii) $U^{-1} = (U^*)$ is unitary.

iv) U is normal.

v) UV is unitary.

3. State and prove Hann Banach Theorem (Normed linear spaces).

4. State and prove Riesz lemma.

5. State and prove Cauchy Schwarz inequality for an inner product space.

PCD-003-1163002 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

December - 2018

MATH CMT - 3002 : Functional Analysis

(Old and New Course)

Faculty Code : 003

Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all questions.

(2) Each question carries 14 marks.

(3) The figures to the right indicate the marks allotted to the question.

1 Answer any **seven** : (Each question carries 2 marks) **14**

(1) Define :

(i) Normed space

(ii) Banach space.

(2) Explain Translation invariance and give its example.

(3) Define :

(i) Cauchy sequence in normed space

(ii) Compactness of a subset of a metric space.

(4) State only Minkowski's Inequality.

(5) If T is a linear operator, then prove that $R(T)$ is a vector space.

(6) If T is a linear operator, then prove that $N(T)$ is a vector space.

(7) Let X and Y be the vector spaces, both real or both complex. $T : D(t) \to Y$ be a linear operator with $D(T)$ contained in X and $R(T)$ contained in Y. Let inverse of T exist. Then, prove that inverse of T is also a linear operator.

(8) Define :

(i) Algebraic Dual space

(ii) Dual Space.

(9) Prove that for an inner product space X, $\|x+y\| \leq \|x\|+\|y\|$, where x, y are contained in X.

(10) Can every metric be obtained from norm ? Justify.

2 Answer any **two** : **14**

(1) State and prove Riesz's lemma.

(2) Let T be a linear operator. Then, prove that if $\dim(D(T)) = n < \infty$, then $\dim(R(T)) \leq n$.

(3) If a normed space X is finite dimensional, then prove that every linear operator on X is bounded.

3 Attempt the following : **14**

(1) Let $T : D(T) \to Y$ be a linear operator and $D(T)$ is contained in X, where X and Y are the normed spaces. Then, prove that T is continuous if it is bounded.

(2) State and prove Bessel's inequality.

OR

(1) State and prove Pythagorean relation.

(2) In an inner product space X, if $x_n \to x$, and $y_n \to y$, then prove that $< x_n, y_n > \to < x, y >$.

4 All are compulsory : **14**

(1) State and prove Parseval relation.

(2) State and prove Hann Banach theorem for normed spaces.

14

5 Answer any **two** :

(1) Define Reflexive Space. Prove that for every fixed x in a normed space X, the functional $g_x(f) = f(x)$, $f \in X'$ is a bounded linear functional on X' and g_x preserves the norm.

(2) State and prove Zero operator theorem.

(3) Obtain an alternative definition of the norm of a linear operator T.

(4) Prove that any orthonormal set on X is linearly independent.

HDV-003-1163002 Seat No. 035064

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November / December – 2017

MATH CMT - 3002 : Functional Analysis

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all questions.
(2) Each question carries 14 marks.
(3) The figures to the right indicate marks allotted to the question.

1 All are compulsory : (Each question carries 2 marks) **14**

(a) True or false? Justify $\left(l^{\infty}, \|\cdot\|_{\infty}\right)$ has a Schauder basis.

(b) Define weak convergence, strong convergence in a n.l. Space.

(c) Define Banach Space.

(d) True or false? Justify Dual of a Hilbert space is a Hilbert space.

(e) Give an example of a space that is not Banach space over IK.

(f) True or false? Justify Every Separable Hilbert space is isomorphic to l^2.

(g) Define equivalent norms on a n.l. space.

2 Answer Any **Two** : **14**

(A) State and Prove the necessary and sufficient condition for a vector subspace of a Banach space to be a Banach space. True or false? Justify. $\left(C_0, \|\cdot\|_{\infty}\right)$ is a Banach space. **7**

(B) State, without proof, Baire's theorem. Prove that a Banach space cannot have a countably infinite Hamel basis. 7

(C) State and prove Riesz lemma. 7

3 All are compulsory : 14

(A) For a n.l. space X over IK, prove that the dual space X^{l} is separable $\Rightarrow X$ is separable. 7

(B) Give an example to show that a metric on a vector space x need not be induced by a norm on x, with justification. 7

OR

3 All are compulsory : 14

(A) Let X, Y be a n.l. space over IK and $\|\cdot\|$ be the norm on $B\ (X.Y)$ defined by 7

$$\|T\| = \inf\{c > 0 / \|Tx\| \le c\|x\|, \forall\ x \in X\}.$$

Prove that $\|T\| = \sup\left\{\frac{\|T_x\|}{\|x\|}, 0 \ne x \in X\right\} = \sup\{\|Tx\|, \|x\| = 1\}$.

(B) Define Canonical mapping C from a n.l. space X to X''. Prove that C : $X \to X''$ is an isometry. 7

4 Answer any two : 14

(A) State, without proof, projection theorem. If H is a Hilbert space and M is a non empty subset of H then prove that $\overline{span\ M} = M^{\perp\perp}$. 7

(B) State and prove characterization of the Hyperspace in a n.l. Space. 7

(C) State and prove closed graph theorem. 7

5 All are compulsory : (Each question carries 2 marks) 14

(A) State Hahn Banach Theorem.

(B) Define Hyper plane and Hyperspace and with an example.

(C) Give an example of a n.l. space which is not complete.

(D) If Y is closed subspace of a n.l. space X then give the definition of the induced norm on the quotient space X/Y.

(E) What is the meaning of the statement that the dual X' of X separates the points of X.

(F) Give the definitions of (1) Convergent series and (2) The absolute convergent series.

(G) Write the statement of Zorn's lemma and define sub linear functional.

MBU-003-016302 Seat No. ______

M. Sc. (Sem. III) (Mathematics) (CBCS) Examination

December - 2016

CMT-3002 : Functional Analysis

(Old Course)

Faculty Code : 003
Subject Code : 016302

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all questions.
(2) Each question carries 14 marks.
(3) The figures on the right indicate marks allotted to the question.

1 Answer any seven questions. 2×7=14

(i) Define C_0, C_{00}.

(ii) True or false ? Justify.

$\left(l^{\infty}, \|\cdot\|_{\infty}\right)$ has a Schauder basis.

(iii) If $f:[a, b] \to \mathbb{R}$ is defined by $f(t) = \frac{t-a}{b-a}$ then find $\|f\| = \text{Max}\{|f(t)| : t \in [a, b]\}$.

(iv) If $1 \le p < \infty$ and $e_n = (0, 0, ..., 0, 1, 0, ...)$ then find $\|e_n - e_m\|$, $\forall n \ne m$.

(v) Does $\|\cdot\|_p$ on l_p, $1 \le p < \infty$ satisfy the parallelogram law ? Justify.

(vi) If X is an inner product space over $\mathbb{K}$ and $A, B \subset X$, $A \subset B$ then prove that $B^{\perp} \subset A^{\perp}$.

(vii) Define weak convergence, strong convergence in a *n.l.* space.

(viii) Give an example of an inner product space X and $x, y \in X$ $s.t \|x+y\|^2 = \|x\|^2 + \|y\|^2$ and x is not orthogonal to y in X.

(ix) Prove that $\|x\| = \sup\{|f(x)| \mid f \in X', \|f\| = 1\}$, $\forall x \in X$, where X is a $n.l.$ space over $\mathbb{K}$.

(x) Prove that $\|x\|_2 \le \|x\|_1$, $\forall x \in \mathbb{K}^n$.

2 Answer any two questions : **2×7=14**

(a) Define Banach space over $\mathbb{K}$ and prove that $(C[0, 1], \|\cdot\|_1)$ is not a Banach space over $\mathbb{K}$.

(b) In a finite-dimensional $n.l.$ space X over $\mathbb{K}$, prove that $Y \subset X$ is compact iff Y is closed and bdd.

(c) State and prove Riesz lemma.

3 (a) Prove that $(C_0, \|\cdot\|_\infty)' \cong (l^1, \|\cdot\|_1)$. 7

(b) For a $n.l.$ space X over $\mathbb{K}$, prove that X' is separable $\Rightarrow X$ is separable. 7

OR

3 (c) Prove that every finite-dimensional vector subspace Y of a $n.l.$ space X is a Banach space. Deduce that every finite-dimensional vector subspace Y of a $n.l.$ space X is closed in X. 7

(d) Let X, Y be $n.l.$ space over $\mathbb{K}$ and $\|\cdot\|$ be the norm on $B(X, Y)$ defined by $\|T\| = \inf\{c>0 \mid \|Tx\| \le c\|x\|, \forall x \in X\}$. 7

Then prove that $\|T\| = Sup\left\{\frac{\|Tx\|}{\|x\|} 0 \ne x \in X\right\}$.

4 Answer any two questions : 2×7=14

(a) Define inner product space and Hilbert space over $\mathbb{K}$ and give an example of a Hilbert space with justification.

(b) State, without proof, projection theorem. If H is a Hilbert space and M is a non-empty subset of H then prove that $\overline{span\, M} = M^{\perp\perp}$.

(c) If $\{e_n\}_{n=1}^{\infty}$ is an orthonormal sequence in a Hilbert space H then prove that $\sum_{n=1}^{\infty} \alpha_n e_n$ converges in H iff $\sum_{n=1}^{\infty} |\alpha_n|^2$ converges in $\mathbb{R}$.

5 Answer any two questions : 2×7=14

(a) State, without proof, Hahn-Banach theorem for *n.l.* spaces. Given $0 \neq x_0$ in a *n.l.* space X over $\mathbb{K}$, prove that $\ni F \in X'$ s.t. $\|F\| = 1$ and $F(x_0) = \|x_0\|$.

(b) State, without proof, Baire category theorem. Deduce that Banach space can not have a countably infinite Hamel basis.

(c) Define closed linear transformation between two *n.l.* spaces over $\mathbb{K}$. State and prove closed graph theorem.

(d) Define total orthonormal set in an inner product space over $\mathbb{K}$. If $\{x_\alpha \mid \alpha \in \wedge\}$ is an orthonormal set in a Hilbert space H and $\|x\|^2 = \sum_{\alpha \in \wedge} |\langle x, e_\alpha \rangle|^2,\ \forall x \in H$ then prove that $\{x_\alpha \mid \alpha \in \wedge\}$ is total in H.

BBH-003-016302 Seat No. ______

M. Sc. (CBCS) (Sem. III) Examination

December - 2015

Mathematics : CMT-3002

(Functional Analysis)

Faculty Code : 003

Subject Code : 016302

Time : 2.30 Hours] [Total Marks : 70

Instructions : (i) Answer all questions. Each question carries 14 marks.

(ii) The figures on the right indicate the marks allotted to the question.

1 Choose the correct answer. 2×7=14

(1) c_{00} = ______ .

(A) $\left\{(x_1, x_2, \ldots\ldots x_n, \ldots\ldots) \mid x_n \in I\!K, \forall n = 1, 2, \ldots\right\}$

(B) $\left\{(x_1, x_2, \ldots, x_n, \ldots\ldots) \mid x_n \in I\!K, \forall n \text{ and } x_n = 0, \forall n \geq N \text{ for none } N \in I\!N\right\}$

(C) $\left\{(x_1, x_2, \ldots\ldots, x_n, \ldots\ldots) \mid x_n \in I\!K, \forall n \text{ and } x_n \to 0 \text{ as } n \to \infty \text{ in } I\!K\right\}$

(D) $\left\{(x_1, x_2, \ldots\ldots, x_n, \ldots\ldots) \mid x_n \in I\!K, \forall n \text{ and } \{x_n\} \text{ converges in } I\!K\right\}$

(2) ______ is not a Banach space over $I\!K$.

(A) $\left(C_{00}, \|\cdot\|_\infty\right)$

(B) $\left(C[a, b], \text{max norm}\right)$

(C) $\left(C_0, \|\cdot\|_\infty\right)$

(D) $\left(l^2, \|\cdot\|_2\right)$

(3) ______ is not reflexive.

(A) every finite dimensional $n \cdot l$ space

(B) $\left(l^2, \|\cdot\|_2\right)$

(C) $\left(l^1, \|\cdot\|_1\right)$

(D) every Hilbert space over $I\!K$

(4) $\|x\|_\infty \leq$ ____, $\forall x \in IK^n$

(A) $\|x\|_2$

(B) $n\|x\|_1$

(C) $\sqrt{n}\,\|x\|_1$

(D) $\sqrt{n}\,\|x\|_2$

(5) If $e_n = (0, 0, \ldots, 0, 1, 0, \ldots)$ (n^{th} term 1), $\forall n = 1, 2, \ldots$ then $\{e_n\}$ is not a Schauder basis of ____.

(A) $(l^p, \|\cdot\|_p), 1 \leq p < \infty$

(B) $(l^2, \|\cdot\|_2)$

(C) $(l^1, \|\cdot\|_1)$

(D) $(l^\infty, \|\cdot\|_\infty)$

(6) A vector subspace Y of a Hilbert space H is closed iff ____.

(A) $\bar{Y} = H$

(B) Y is finite-dimensional

(C) $Y = Y^{\perp\perp}$

(D) $Y^\perp = \{0\}$

(7) ____ is not separable.

(A) $(l^2, \|\cdot\|_2)$

(B) Every *n.l.* space with a Schauder basis

(C) $(l^\infty, \|\cdot\|_\infty)$

(D) Every inner product space with a countable orthonormal basis

(8) ____ is true.

(A) Weak convergence $\Rightarrow$ strong convergence in any *n.l.* space

(B) $\|x + y\|^2 = \|x\|^2 + \|y\|^2 \Rightarrow x \perp y$ in any inner product space

(C) For an orthonormal set M in an inner product space, $M^\perp = \{0\} \Rightarrow M$ is total

(D) For a *n.l.* space X, X' is separable $\Rightarrow X$ is separable

(9) ______ is not true.

(A) Differential operator is not bdd linear

(B) Hahn-Banach extension is not unique

(C) Weakly convergent sequence in a $n \cdot l.$ space is bdd

(D) Every norm on a vector space X is induced by an innter product on X.

(10) ______ is not a true statement.

(A) Every separable Banach space has a Schauder basis

(B) $\left(C_{00}, \|\cdot\|_\infty\right)' \equiv \left(l^\infty, \|\cdot\|_\infty\right)$

(C) $\left(C[0,1], \|\cdot\|_2\right)$ is not a Banach space

(D) Every separable Hilbert space is isomorphic to l^2

2 Answer any two : **2×7=14**

(a) State and prove the necessary and sufficient condition for a vector subspace of a Banach space to be a Banach space. True or false ? Justify. $\left(C_0, \|\cdot\|_\infty\right)$ is a Banach space.

(b) State, without proof, Baire's theorem. Prove that a Banach space can not have a countably infinite Hamel basis.

(c) Prove that a closed and bdd set in a finite-dimensional $n.l.$ space is compact.

3 (a) Define equivalent norms on a $n.l.$ space. Prove that any two norms on a finite-dimensional $n.l.$ space are equivalent. **14**

(b) Define weak convergence, strong convergence of sequences in a $n.l.$ space. True or false ? Justify. weak convergence $\Rightarrow$ strong convergence in a $n.l.$ space

OR

(c) Define Canonical mapping C from a $n.l.$ space X to X''. Prove that $C : X \rightarrow X''$ is an isometry and isomorphism.

(d) Define reflexive spaces. Prove that every Hilbert space is reflexive.

4 Answer any two : 2×7=14

(a) Define open mapping between two topological spaces. State and prove open mapping theorem.

(b) State, without proof, uniform bddness theorem. Give an application of uniform bddness theorem with proof.

(c) State, without proof, Hahn-Banach theorem. For a $n.l.$ space X prove that $\|x\| = \sup_{0 \neq f \in x'} \frac{|f(x)|}{\|f\|}, \forall x \in X$.

5 Answer any two : 2×7=14

(a) Define orthonormal set in an inner product space. State and prove Bessel's inequality for an orthonormal sequence in an inner product space.

(b) If $\{e_\alpha \mid \alpha \in \wedge\}$ is a total orthonormal set in a Hilbert space H then prove that $x = \sum_{\alpha \in \wedge} \langle x, e_\alpha \rangle e_\alpha, \forall x \in H$.

(c) State, without proof, Riesz representation theorem for bdd linear functionals on a Hilbert spaces. Prove or disprove that the dual of a Hilbert space is a Hilbert space.

(d) Given two Hilbert spaces H_1, H_2 and a bdd linear transformation $T : H_1 \to H_2$, prove that $\ni$ a unique bdd linear transformation $T^* : H_2 \to H_1$ s.t. $\langle Tx, y \rangle = \langle x, T^* y \rangle, \forall x, y \in H_1$ and $\|T^*\| = \|T\|$.

WM-134

003-016302

M.Sc. (Maths) (CBCS) (Sem.-III) Examination

November-2014

MATH CMT-3002 : Functional Analysis

(Set-2)

Faculty Code : 003

Subject Code : 016302

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) Answer all questions. Each question carries 1 mark.

(2) The figures on the right indicate the marks allotted to each question.

1. Answer any seven questions : $2 \times 7 = 14$

(1) ______ is not a Banach space over. K

(a) $((C[0, 1], \|\cdot\|_2)$

(b) $(C_o, \|\cdot\|_\infty)$

(c) $(l^p, \|\cdot\|_p)$, $1 \leq p \leq \infty$

(d) (C[0, 1], max. norm)

(2) ______ is not induced by any inner product.

(a) (C[0, 1], max. norm)

(b) $(C[0, 1], \|\cdot\|_2)$

(c) $\|\cdot\|_2$ on l^2

(d) $\|\cdot\|_2$ on K^n

(3) ______ is the polarization identity.

(a) $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$

(b) $4 < x,y > = (\|x + y\|^2 - \|x - y\|^2) + i(\|x + iy\|^2 - \|x - iy\|^2)$

(c) $4 < x, y > = (\|x + y\|^2 + \|x - y\|^2) + i(\|x + iy\|^2 + \|x - iy\|^2)$

(d) $4 < x, y > = (\|x + y\|^2 - \|x - y\|^2) - i(\|x + iy\|^2 - \|x - iy\|^2)$

(4) _____ of the following is correct.

(a) $\| x + y \| = \| x \| + \| y \|$ in $(x, \| \cdot \|) \Rightarrow x$, y are linear dependent in x.

(b) x, y are n.l. spaces over K $\Rightarrow$ B(x, y) with operator norm is a Banch space over K.

(c) x' is separable $\Rightarrow x$ is separable.

(d) $x \cong x'' \Rightarrow x$ is reflexive.

(5) _______ implication is correct.

(a) Weak convergence $\Rightarrow$ strong convergence in a finite-dimensional n.l. space over K.

(b) x is separable $\Rightarrow x'$ is separable.

(c) $\{x_n\}_{n=1}^{\infty}$ is not a Schauder basis of $(x, \| \cdot \|) \Rightarrow \{x_n\}_{n=1}^{\infty}$ is not a Schauder basis of (y, $\| \cdot \|$) for every vector subspace of x.

(d) Weak convergence $\Rightarrow$ strong convergence in any n.l. space over K.

(6) _______ is the statement of Baire's theorem.

(a) Every bold linear transformation is continuous linear.

(b) Every linear transformation from a finite-dimensional n.l. space to any n.l. space is bold linear.

(c) Every non-empty complete metric space is second category in itself.

(d) Any two norms on a finite dimensional space are equivalent.

(7) _______ is not bold linear.

(a) Differential operator

(b) Integral operator

(c) A continuous linear transformation

(d) Closed linear transformation between Banach spaces

(8) _______ is a true statement.

(a) x' separates points of x

(b) bold linear transformation is closed linear

(c) every closed bold set in a metric space is compact

(d) every poset has a unique maximal element

(9) ________ is not a true statement.

(a) Every n.l. space over K with a Schauder basis is separable.

(b) $(l^{\infty}, \| \cdot \|_{\infty})$ is separable

(c) Strong convergence $\Rightarrow$ weak convergence

(d) Hahn-Banach extension is not unique

(10) ________ of the following inequalities is correct in an inner product space.

(a) $\| x \| \| y \| \leq | < x, y > |$

(b) $\| x + y \|^2 + \| x - y \|^2 < 2 [\| x \|^2 + \| y \|^2]$

(c) $2[\| x \|^2 + \| y \|^2] < \| x + y \|^2 + \| x - y \|^2$

(d) $| < x, y > | \leq \| x \| \| y \|$

2. Answer any two questions : $2 \times 7 = 14$

(a) Define l^p, $1 \leq p \leq \infty$ and prove that $(l^{\infty}, \| \cdot \|_{\infty})$ is a Banach space over K (prove only completeness !)

(b) When do you say that two norms on a vector space x are equivalent ? Prove that equivalent norms on x generate the same topology.

(c) True or false ? Justify. Every closed and bold set in a n.l. space x over K is compact.

3. (a) Define bold linear transformation between n.l. spaces over K. True or false ? Justify. Every linear transformation from a finite-dimensional n.l. space to any n.l. space is bold. 7

(b) Prove that a n.l. space is finite-dimensional iff its closed unit disc is compact. 7

OR

(c) If x is a Banach space and y is a closed vector subspace of x then prove that the quotient n.l. space x/y is a Banach space. 7

(d) Prove that a vector subspace y of a Banach space $(x, \| \cdot \|)$ is a Banach space w.r.t. $\| \cdot \|$ iff y is closed in $(x, \| \cdot \|)$. 7

4. Answer any two questions : $2 \times 7 = 14$

(a) If X is a Banach space w.r.t. two norms $\| \cdot \|_1$, $\| \cdot \|_2$ and $\| x \|_1 \leq C \| x \|_2$, $\forall x \in X$ for some $C > 0$ then prove that $\ni k > 0$ s.t. $\| x \|_2 \leq k \| x \|_1$, $\forall x \in X$.

(b) Prove that $(C_0, \| \cdot \|_\infty)' \cong (l^1, \| \cdot \|_1)$

(c) State, without proof, Hahn-Banach theorem for n.l. spaces over K. Given a n.l. space X and $0 \neq x_0 \in X$, prove that, $\ni f \in x'$ s.t. $\| f \| = 1$ and $f(x_0) = \| x_0 \|$.

5. Answer any two questions : $2 \times 7 = 14$

(a) Prove that the canonical map $C : X \to X''$ is linear isometry.

(b) Give an example of an inner product space which is not a Hilbert space. Justify.

(c) State and prove parallelogram law. Give an example of a norm which does not satisfy the parallelogram law with justification.

(d) Given an inner product space X, a complete vector subspace Y of X and $x \in X$, prove that $\exists$ a unique $y_0 \in Y$ s.t. $\| x - y_0 \| = d(x, y)$ and $x - y_0$ is orthogonal to Y.

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003-016302

M.Sc. (MATHS) (CBCS) (Sem.-III) Examination

November-2013

Maths

CMT – 3002 : Functional Analysis

Faculty Code : 003

Subject Code : 016302

Time : 2½ Hours] **[Total Marks : 70**

1. Answer any seven questions : $7 \times 2 = 14$

(1) Any two norms on ________ are equivalent.

(a) any Banach space

(b) any n.l. space

(c) any infinite-dimensional n.l. space

(d) any finite-dimensional n.l. space

(2) A weak convergent sequence on a n.l. space is a

(a) Cauchy sequence (b) Convergent Sequence

(c) Bounded sequence (d) Unbounded sequence

(3) An orthonormal sequence in an inner product space is

(a) a cauchy sequence (b) an unbounded sequence

(c) a convergent sequence (d) linearly independent

(4) ________ is the Bessel's inequality in an inner product space

(a) $|\langle x, y\rangle| \leq \|x\| \|y\|, \forall x, y \in X$

(b) $\sum_{n=1}^{\infty} |\langle x, e_n\rangle|^2 \leq \|x\|^2, \forall x \in X$ and orthonormal sequence $\{e_n\}_{n=1}^{\infty}$ in X

(c) $\|x + y\| \leq \|x\| + \|y\|, \forall x, y \in X$

(d) $\sum_{n=1}^{\infty} |\langle x, e_n\rangle| \leq \|x\|, \forall x \in X$ and orthonormal sequence $\{e_n\}_{n=1}^{\infty}$ in X

(5) ________ is a true statement.

(a) Every closed and bounded set in a metric space is compact

(b) $\{f \in c[0, 1] \mid \max_{x \in [0, 1]} | f(x) | \le 1\}$ is compact in (c[0, 1], max. norm)

(c) Hahn-Banach extension is not unique

(d) Every norm on a vector space x is induced by an inner product on x

(6) ________ is not a true statement.

(a) $(c_{00}, \| \cdot \|_\infty)' \equiv (l^1, \| \cdot \|_1)$ (b) $(c_0, \| \cdot \|_\infty)' \equiv (l^1, \| \cdot \|_1)$

(c) The canonical mapping $c = x \to x''$ is an into isometry.

(d) Every norm on a vector space satisfies parallelogram law.

(7) A subspace y of a n.l. space x is a hyperspace iff

(a) y = kerf, for some linear functional f on x

(b) y = kerf, for some $0 \ne f \in x'$

(c) y = kerf, for some $f \in x'$

(d) y is finite dimensional

(8) By open mapping theorem

(a) every Banach space is of second category

(b) every onto bounded linear transformation between two Banach spaces is an open mapping

(c) Hilbert spaces reflexive

(d) a closed linear transformation between two Banach spaces is bounded linear

(9) ________ is the Parseval's identity in a Hilbert space H.

(a) $\| x + y \|^2 + \| x - y \|^2 = 2 [\| x \|^2 + \| y \|^2]$, $\forall x, y \in H$

(b) $\sum_{\alpha \in \Lambda} | \langle x, e_\alpha \rangle |^2 = \| x \|^2$, $\forall x \in H$ and total orthonormal set $\{e_\alpha\}_{\alpha \in \Lambda}$

(c) $\| x + y \|^2 = \| x \|^2 + \| y \|^2$, $\forall x, y \in H, x \perp y$

(d) $\| x + y \| = \| x \| + \| y \|$, $\forall x, y \in H$ s.t. either $y = 0$ or $x = \alpha y$ for some $\alpha \in K$

(10) _______ is bounded linear.

(a) Every linear transformation from a finite dimensional n.l. space to n.l. space

(b) Differential operator

(c) Every linear transformation between two Banach spaces

(d) Every linear transformation between two n.l. spaces

2. Answer any two questions : $2 \times 7 = 14$

(a) Define Banach space and give an example of a n.l. space which is not a Banach space with justification.

(b) If x is a n.l. space and y is a Banach space then with usual notation, prove that B(x, y) is a Banach space w.r.t. the operator norm (prove only completeness).

(c) Give an example to show that a metric on a vector space x need not be induced by a norm on x with justification.

3. (a) Define equivalent norms on a n.l. space. Prove that $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent on $\mathbb{R}^n$. 7

(b) Find the dual space of $(C_0, \|\cdot\|_\infty)$ 7

OR

(a) Prove that any n.l. space with a Schauder basis is separable. 7

(b) For an n.l. space x, prove that x' is separable => x is separable. 7

4. Answer any two questions : $2 \times 7 = 14$

(a) State and prove closed graph theorem.

(b) Give an application of uniform boundedness theorem with proof.

(c) State, without proof, Hahn-Banach theorem for n.l. spaces prove that $\forall x \in x$ s.t. $0 \neq x$, $f \in X'$ s.t. $\|f\| = 1$ and $f(x) = \|x\|$.

003-016302

P.T.O.

5. Answer any two questions : $2 \times 7 = 14$

(a) State and prove Pythagoreas theorem in an inner product space. Is the converse of this theorem true ? Justify.

(b) If X is an inner product space and y is a complete subspace of X, then prove that given $x \in X$, $\exists$ a unique $Y_0 \in Y$ s.t. $d(x, Y) = \| x - y_0\|$ and $x - y_0$ is orthogonal to Y.

(c) State, without proof, Riesz representation theorem for bdd linear functional on a Hilbert space. Deduce that every Hilbert space is reflexive.

(d) If an orthonormal set $\{e_\alpha\}_{\alpha \in \pi}$ in a Hilbert space H satisfies Parseval's identity, then prove that $\{e_\alpha\}_{\alpha \in \pi}$ is total in H.

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003-016302

M.Sc. (CBCS) (Sem-III) Examination
November-2012
MATHS CMT – 3002 : FUNCTIONAL ANALYSIS

Faculty Code : 003
Subject Code : 016302

Time : 2½ Hours] [Total Marks : 70

1. Answer any seven questions : $2 \times 7 = 14$

(i) _______ has no Schauder basis.

(a) every normed linear space over $\mathbb{K}$
(b) $(l^p, \|\cdot\|_p)$, $1 \le p < \infty$
(c) $(C_{00}, \|\cdot\|_\infty)$
(d) $(l^\infty, \|\cdot\|_\infty)$

(ii) _______ is a true statement.

(a) A Schauder basis is linearly dependent
(b) $(C_{00}, \|\cdot\|_\infty)$ is a Banach space over $\mathbb{K}$
(c) A normed linear space with a Schauder basis is separable
(d) Every metric on a vector space X over $\mathbb{K}$ is induced by a norm on X.

(iii) _______ is not a true statement.

(a) Every closed bounded set in $(\mathbb{R}^n, \|\cdot\|_2)$ is compact
(b) Every compact subset of a metric space is closed and bounded
(c) The closed unit disc in an infinite dimensional normed linear space over $\mathbb{K}$ is compact
(d) $(C[a, b], \|.\|)$ where $\|\cdot\|$ = max. norm, is a Banach space over $\mathbb{K}$

(iv) With usual notations $(C_{00}, \|\cdot\|_\infty)' =$ _______

(a) $(l^1, \|\cdot\|_1)$
(b) $(l^2, \|\cdot\|_2)$
(c) $(l^p, \|\cdot\|_p)$, $1 < p < \infty$
(d) $(l^\infty, \|\cdot\|_\infty)$

(v) $\|x + y\|_2 = \|x\|_2 + \|y\|_2$ for some $x, y \in \mathbb{R}^2$ iff

(a) x, y are linearly independent
(b) $x = y$
(c) $x = -y$
(d) either $y = 0$ or $x = \alpha y$ for some $\alpha \ge 0$.

(vi) _______ is a Hilbert space.

(a) $(l^p, \|\cdot\|_p)$, $1 < p < \infty$
(b) $(l^2, \|\cdot\|_2)$
(c) (c[a, b], max. norm)
(d) $(l^\infty, \|\cdot\|_\infty)$

(vii) ______ is not reflexive

(a) $(l^1, \|\cdot\|_1)$

(b) every finite dimensional normed linear space over K

(c) $(K^n, \|\cdot\|_2)$

(d) $(l^p, \|\cdot\|_p)$ $1 < p < \infty$

(viii) A bounded linear operator T on a Hilbert space is normal if

(a) $TT^* = T^*T$

(b) $T^* = T$

(c) $TT^* = I = T^*T$

(d) T is in isometry

(ix) In a normed linear space X over K, weak convergence implies strong convergence if

(a) X is an infinite-dimensional normed linear space over K.

(b) X is a finite-dimensional normed linear space over K.

(c) X is a Banach space.

(d) X is a Hilbert space.

(x) ______ is an orthonormal set $\mathbb{R}^2$

(a) $\{(1, 0), (1, 1)\}$

(b) $\left\{\left(0, \frac{1}{2}\right), (1, 1)\right\}$

(c) $\left\{\left(\frac{1}{2}, 1\right), \left(1, -\frac{1}{2}\right)\right\}$

(d) $\{(1, 0), (0, 1)\}$

2. Answer any two questions : $2 \times 7 = 14$

(a) Prove that a vector subspace y of a Banach space X is a Banach space iff y is closed in X. Assuming that $(l^\infty, \|\cdot\|_\infty)$ is a Banach space, deduce that $(C_0, \|\cdot\|_\infty)$ is a Banach space over $\mathbb{K}$.

(b) Define bounded linear transformation between normed linear spaces. Is every linear transformation between any two normed linear spaces over K bounded ? Justify.

(c) When do you say that two norms on a vector space over K are equivalent ? Prove that any two norms on a finite-dimensional vector space over K are equivalent.

3. (a) Define weak convergence and strong convergence in a normed linear space over K. Give an example to show that weak convergence does not imply strong convergence.

(b) Prove that a closed and bounded set in a finite-dimensional normed linear space over $\mathbb{K}$ is compact. $2 \times 7 = 14$

OR

(c) Prove that the dual of $(l^1, \|\cdot\|_1)$ is isomorphic to $(l^\infty, \|\cdot\|_\infty)$.

(d) Prove that in a finite dimensional normed linear space over K strong convergence implies weak convergence.

4. Answer any two questions: $2 \times 7 = 14$

(a) State and prove closed graph theorem.

(b) State, without proof, open mapping theorem. Give an example to show that the hypothesis "X, Y are Banach spaces" in open mapping theorem cannot be dropped and justify.

(c) State uniform boundedness theorem and give an example with justification to show that " X is Banach" in uniform-boundedness theorem cannot be dropped.

5. Answer any two questions : $2 \times 7 = 14$

(a) State Hahn Banach theorem for normed linear spaces X over $\mathbb{K}$. Deduce that given non-zero $x_0 \in X$, $\exists$ F $\in X'$ s.t $F(x_0) = \| x_0 \|$ and $\|F\| = 1$.

(b) Let X be an inner product space and M be non-empty complete convex set in X. Prove that $\exists$ a unique $y_0 \in M$ s.t $\|x - y_0\| = d(x, M)$.

(c) State and prove polarization identity in an inner product space.

(d) Define self-adjoint operator on a Hilbert space over $\mathbb{K}$. If H is a Hilbert space over $\mathbb{K}$ then prove that a ldd linear operator "T" on H is self adjoint iff $< Tx, x >$ is real, $\forall x \in H$.

003-016302 Seat No.________

M. Sc. (CBCS) (Sem. III) Examination

December - 2011

Maths. : CMT-3002

(Functional Analysis)

Faculty Code : 003
Subject Code : 016302

Time : 3 Hours] [Total Marks : 70

1 Answer any seven questions : 2×7=14

(1) The metric induced by the norm $\|\cdot\|_2$ on IK^n is $d(x, y) =$ ________, $\forall\ x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in IK^n$.

(a) $\sum_{i=1}^{n} |x_i - y_i|$ (b) $\underset{1 \le i \le n}{\text{mix}} |x_i - y_i|$

(c) $\left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{\frac{1}{2}}$ (d) $\left(\sum_{i=1}^{n} |x_i - y_i|^p\right)^{\frac{1}{p}}$, $p \ne 2, kp < \infty$

(2) The Holder's inequality is ________, $\forall\ x = (x_1, x_2, \ldots, x_n, \ldots) \in l^p$, $y = (y_1, y_2, \ldots, y_n, \ldots) \in l^q$, $1 \le p \le \infty$.

(a) $\|x + y\|_p \le \|x\|_p + \|y\|_p$

(b) $\sum_{n=1}^{\infty} |x_n y_n| \le \left(\sum_{n=1}^{\infty} |x_n|^p\right)^{\frac{1}{p}} \left(\sum_{n=1}^{\infty} |y_n|^q\right)^{\frac{1}{q}}$, $\frac{1}{p} + \frac{1}{q} = 1$

(c) $\sum_{n=1}^{\infty} |x_n y_n| \le \left(\sum_{n=1}^{\infty} |x_n|^1\right)^{\frac{1}{p}} \left(\sum_{n=1}^{\infty} |y_n|^q\right)^{\frac{1}{q}}$

(d) $\sum_{n=1}^{\infty} |x_n y_n|^p \le \left(\sum_{n=1}^{\infty} |x_n|^p\right) \left(\sum_{n=1}^{\infty} |y_n|^q\right)^{\frac{1}{q}}$

(3) With usual notations, $(l^2, \|\cdot\|_2)' \cong$ ________.

(a) $(l^1, \|\cdot\|_1)$ (b) $(l^2, \|\cdot\|_2)$

(c) $(l^\infty, \|\cdot\|_\infty)$ (d) $(c_0, \|\cdot\|_\infty)$

(4) ________ is a true statement.

(a) $\|\cdot\|_1$, max. norm are equivalent on $c[0, 1]$

(b) $\|\cdot\|_2$, max-norm are equivalent on $c[0, 1]$

(c) max-norm on $c[0, 1]$ is induced by an inner product on $c[0, 1]$

(d) $\|\cdot\|_2, \|\cdot\|_p$, $1 \le p \le \infty$, $p \ne 2$ are equivalent on IK^n.

(5) ________ is not a true statement.

(a) a closed and bounded set is compact in a metric space

(b) $\{e_n\}_{n=1}^{\infty}$ is compact in $(l^p, \|\cdot\|_p)$, $1 \le p < \infty$

(c) A Banach space is of first category

(d) $(l^p, \|\cdot\|_p)$, $1 < p < \infty$ is reflexive

(6) The uniform bddness theorem says that ________

(a) every bijective bdd linear transformation between two Banach spaces over IK has a bdd inverse

(b) every closed linear transformation between two Banach spaces is bdd

(c) Complete metric space is of second category in itself

(d) every sequence of bdd linear transformation between a Banach space and a n.l. space is uniformly bdd.

(7) ________ is an orthogonal set in $[0, 2\pi]$

(a) $\{\cos nt\}_{n=0}^{\infty}$ (b) $\{e^{int}\}_{n \in Z}$

(c) $\{\sin nt\}_{n=1}^{\infty}$ (d) $\{\cos nt\}_{n=0}^{\infty} \cup \{\sin nt\}_{n=1}^{\infty}$

(8) A subspace Y of a n.l. space X is a hyperspace in X iff Y = ________

(a) ker f

(b) ker f, for some $f \in X'$

(c) ker f, for some $0 \ne f \in X'$

(d) finite diml.

(9) If X is an inner product space over IK then the Bessels inequality is

(a) $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|, \forall x, y \in X$

(b) $\|x\| \leq \sum|\langle x, e_\alpha\rangle|^2, \forall x \in X$ where $\{e_\alpha\}_{\alpha \in X}$ is an orthonormal set in X

(10) A bdd linear operator "T" an a Hilbert space is unitary if

(a) $T^* = T$ (b) $TT^* = T^*T$

(c) $T^*T = I = TT^*$ (d) T is invertible

2 Answer any two questions :

(a) Define Banach space and prove that ($[a b]$, max-norm $\|\cdot\|$) is a Banach space 7

(b) Define bdd linear transformation and prove that every linear transformation from a finite-dimensional n.l. space to any n.l. space is bdd. 7

(c) Define weak convergence and strong convergence of a sequence in a n.l. space X over IK. Does weak convergence imply strong convergence ? Justify. 7

3 (a) When do you say that two norms on a n. l space X over IK are equivalent ? Prove that equivalent norms on a vector space generate the same topology. 7

(b) State and prove Riesz lemma. 7

(c) Prove that a n. l. X is finite dimensional iff $\bar{B}(0, 1) = \{x \in X \mid \|x\| \leq 1\}$ is compact. 7

OR

3 (a) Define the dual space X' of a n.l. space X over IK. Show that $(C_0, \|\cdot\|_\infty)' \cong (l^1, \|\cdot\|_1)$. 7

(b) If $x_n \xrightarrow{w} x$ in a n.l. space X prove that $\{\|x_n\|\}_{n=1}^{\infty}$ is bdd in IR. 7

(c) Define reflexive space. If X is reflexive then prove that X' is separable iff X is separable. 7

4 (a) State and prove open mapping theorem. Can we drop the condition "X, Y are Banach spaces" in the open mapping theorem ? Justify. 7

(b) State, without proof, Baire's theorem. Deduce the uniform bddness theorem from it. 7

(c) Give an application of uniform bddness theorem with proof. 7

5 (a) State, without proof, Hahn-Banach theorem for n.l. space X over IK. If X is a n.l. space over IK and Y is a proper closed vector subspace of X and $x_0 \in X \sim Y$ then prove that $\exists F \in X'$ set $\|F\| = 1$, $F(y) = 0, \forall y \in y$ and $F(x_0) = \inf\{\|x_0 - y\| / y \in y\}$ 7

(b) State, without proof, projection theorem. If M is a non-empty subset of a Hilbert space H then prove that $\overline{span\ M} = M^{\perp\perp}$. 7

(c) State, without proof, Riesz representation theorem for bdd linear functional on a Hilbert space. Prove that every Hilbert space is reflexive. 7

(d) Define unitary operator. Prove that a bdd linear operator on a Hilbert space is unitary iff it is an isometry and onto. 7

DBL-003-1163003 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

June – 2022

Mathematics

(3003 - Number Theory 1)

Faculty Code : 003
Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1 Answer the following : **14**

(a) Find the number of solutions of $x^{48} \equiv 9 (\text{mod } 17)$ if exists.

(b) Find $\sigma(307)$ and $\tau(19610)$.

(c) Prove that, for any two non-zero integers x and $y \exists a$ and b such that $ax + by = 1$.

(d) Define Euler's function for a positive integer m and write down the value of $\phi(139)$.

(e) State, Euclid's Algorithm and verify it by an example.

(f) Define Prime numbers and also give at least four prime numbers more than 155.

(g) For three integers a, b and $n \in \mathbb{N}$, prove that, if $a \mid b$ then $a^n \mid b^n$.

2 Answer the following : **14**

(a) Define L.c.m. with an example and prove that for $a, b \neq 0$ and $m > 0 m[a,b] = [ma, mb]$.

(b) Using standard notation prove that, $\left[\frac{x}{m}\right] = \left[\frac{[x]}{m}\right]$ for any $x \in R$ and $m \geq 1$ be any integer.

(c) Find the number of solutions of $x^{12} \equiv 16 (\bmod 17)$.

(d) Define : (i) Reduced Residue System and (ii) Solution of Congruence Equation.

(e) Is it always true that if $x \mid y$ then $x \mid ty$ for any $t \in Z$. Justify your answer.

(f) Show that, if $a \equiv b (\bmod m) \Rightarrow (a,m) = (b,m)$.

(g) Find the highest power of 61 which divide 38401!.

3 Answer the following : **14**

(a) Prove that, if p is a prime number then p^2 has exactly $(p-1)\phi(p-1)$ primitive roots in (mod p^2). 7

(b) Find the solutions of the congruence equation $x^4 - 1 \equiv 0$ $(\bmod 15)$ using Chinese Remainder Theorem. 7

4 Answer the following : **14**

(a) For any odd number g prove that 2^{α} has no primitive roots for $\alpha \geq 3$. 7

(b) (i) If p is a prime number of the form $4k+3$ and $p \mid a^2 + b^2$ then $p \mid a$ and $p \mid b$ for some $a, b \in Z$. **4**

(ii) Show that, for a prime number p of the form $4k+3, p$ cannot be expressed as a sum of squares of two integers. **3**

5 Answer the following : **14**

(a) (i) State, Fermat's Theorem. **2**

(ii) Find a solution of $x^{11} \equiv 5 (\mathrm{mod}\, 2^5)$ if exists. **5**

(b) (i) State and prove, Mobius Inversion Formulae. **5**

(ii) Prove that, $\sigma(n)$ is a multiplicative function. **2**

6 Answer the following : **14**

(a) State and Prove, Fundamental Theorem of Arithmetic. **7**

(b) Let, $a,b \in Z - \{0\}$ and $m \geq 1$ If $g = \gcd(a,m)$ then the congruence equation $ax \equiv b (\mathrm{mod}\, m)$ has a solution if and only if $g \mid b$. **7**

7 Answer the following : **14**

(a) State, Wilson's Theorem and also verify the theorem for prime number 13. **7**

(b) Prove that, there are infinitely many prime numbers. **7**

8 Answer the following : **14**

(a) State and prove, Hansel's Lemma. **7**

(b) If $\alpha \geq 3$ be any integer then prove that the set **7**

$S = \{5, 5^2, 5^3, ...5^{2^{\alpha-2}}\} \cup \{-5, -5^2, -5^3, ..., -5^{2^{\alpha-2}}\}$ is a reduced residue system (mod 2^{α}).

9 Answer the following : **14**

(a) (i) If g is a primitive root of m then show that the set $S = \{1, g, g^2, ..., g^{\phi(m)-1}\}$ is a reduced residue system (mod m). **5**

(ii) Prove that, for any odd number $a, 8 \mid a^2 - 1$. **2**

(b) For a prime number p and $n \geq 1$ with $p \nmid a$ then show that either $x^n \equiv a (\mathrm{mod}\, p)$ has no solution or there are $(n, p-1)$ solutions in any C.R.S. (mod p). **7**

10 Answer the following : **14**

(a) Suppose $f(x) \equiv 0 (\mathrm{mod}\, p)$ has degree n then prove that the n number of solutions in any C.R.S. (mod m) is $\leq n$. **7**

(b) If $m, m_1, m_2, ..., m_k \geq 1$ are integers with $m = m_1 + m_2 + \cdots .... + m_k$ then prove that **7**

$\frac{m!}{m_1! \cdot m_2! \cdot ..... \cdot m_k!}$ is an integer.

Seat No. ________

FO-003-1163003

M. Sc. (Sem. III) (CBCS) Examination

November – 2022

Mathematics : Course No.-3003

[Number Theory 1]

Faculty Code : 003
Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Do as directed : (answer any seven) 7×2=14

(a) Define :
 (i) Congruence euqation and
 (ii) Square free function.

(b) If we consider any integer a in modulo 2 system then show that $\frac{a^2\left(a^2+3\right)}{4}$ is an integer.

(c) Prove that g.c.d. of any two integers is always unique.

(d) Find the g.c.d. of $(2a+1, 9a+4); \forall a \in \square$.

(e) Prove that $[a,b]=[a,-b]; \forall a,b \in \square - \{0\}$.

(f) Define complete residue system in modulo m with an example.

(g) Define :
 (i) Euler's Phi function and
 (ii) Multiplicative function.

(h) Find all solutions of $4x \equiv 12 \pmod 8$ if exists.

(i) Prove that if p be a prime number and p does not divides a then $a^p \equiv a \pmod p$.

(j) Find $\phi(63*625*49)$ and $\tau(19610)$.

2 Answer any two of the following : 2×7=14

(a) State and prove Wilson's theorem.

(b) Prove that every g.c.d. can be expressed as a linear combination of given two integers a and b vice-versa.

(c) State Hansel's Lemma and find all the solutions of the conghruence equation of $x^2 \equiv 1 \pmod{25}$ using the congruence equation $x^2 \equiv 1 \pmod{5}$.

3 Answer the following : 2×7=14

(a) State and prove unique factorization theorem.

(b) State chinese remainder theorem and explain it with an example.

OR

3 Answer the following : 2×7=14

(a) If $m, m_1, m_2 \geq 1$ with $1 = (m_1, m_2)$, $2 \leq (\phi(m_1), \phi(m_2))$ and $m = m_1 \cdot m_2$ then prove that m does not have a primitive root.

(b) Show that the number of primes are infinite.

4 Answer the following : 2×7=14

(a) State and prove any five properties of congruence.

(b) Suppose $f(x) \equiv 0 \pmod{p}$ has degree n then prove that number of solutions in any C.R.S. $(\text{mod } p)$ is at most n.

5 Answer any two of the following : 2×7=14

(a) State Euclid's algorithm and justify it for (306, -657).

(b) (i) Prove that product of any two integers is always same as the product of its L.C.M. and G.C.D.

(ii) If $m|st$ and $(m, s) = 1$ then show that $m|t$.

(c) Prove that for a prime number p there is a solution of $f(x) \equiv 0 \pmod{p}$ where $f(x) = x^2 + 1$ if and only if $p = 2$ or $p = 4k + 1$, for some k.

(d) Prove that if p is a prime number then p has exactly $\varnothing(p-1)$ primitive roots in $(\text{mod } p)$.

SAC-003-1163003 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

November – 2021

Mathematics

(Number Theory 1)

Faculty Code : 003
Subject Code : 1163003

Time : **2:30** Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1) Answer the following : 7×2=14

(a) Define order of an element a modulo m. Also find order of 3 in (mod 5) system.

(b) Prove that, 3^{50} cannot divide 100!.

(c) Define Euler's function for a positive integer m and write down the value of $\phi(20200)$.

(d) Define : i) Mobius Function and ii) Square Free Function.

(e) Find the number of solutions of $x^{12} \equiv 16 \pmod{17}$ if exists.

(f) Find $\omega(105)$ and $\tau(150)$.

(g) Define prime numbers and also give at least four prime numbers more than 125.

2 Answer the following : 7×2=14

(a) Define g.c.d. with an example and prove that g.c.d. is always unique for any two real numbers.

(b) For three integers a, b and $n \in \mathbb{N}$, prove that, if $a | b$ then $a | b + n$ and $a | b - n$.

(c) State, Euclid's Algorithm and verify it by an example.

(d) Let $f(n)=\sum_{d|n}\mu(d)$ then the value of $f(p)$ is ? Justify your answer.

(e) Find the number of solutions of $x^2 \equiv 1$ (mod 101).

(f) Prove that, for any two non-zero integers x and y $\exists$ a and b such that $ax+by=1$.

(g) If g a primitive root of (mod m) and $g \equiv g'$ (mod m) then show that, g' is a primitive root of (mod m).

3 Answer the following :

(a) Prove that, if p is a prime number then p has exactly $\phi(p-1)$ primitive roots in (mod p). 7

(b) State, Wilson's Theorem and also verify the theorem for prime number 7. 7

4 Answer the following :

(a) In standard notations prove that : 7

(i) If $a \equiv b(\text{mod}\, m) \Rightarrow (a,m)=(b,m)$.

(ii) If $(a,m)=1$ then $\exists x_0$ such that $ax_0 \equiv 1(\text{mod}\, m)$.

(iii) If $(a,m)=1=(b,m) \Rightarrow (ab,m)=1$.

(b) If $n>1$ is an even integer with $\alpha \geq 3$ and a is any odd integer, prove that, the congruence equation $x^n \equiv a(\text{mod}\, 2^\alpha)$ has $2^{\beta+1}$ solutions if $a \equiv 1(\text{mod}\, 2^{\beta+2})$ otherwise no solutions, where $2^\beta=(n,2^{\alpha-2})$. 7

5 Answer the following : 14

(a) Prove that, for a prime number p there is a solution of $f(x) \equiv 0(\text{mod}\, p)$ where $f(x)=x^2+1$ if and only if $p=2$ or $p=4k+1$, for some k. 7

(b) (i) Prove that, if $m \geq 1$ has a primitive root with $(a,m)=1$ then $x^n \equiv a(\text{mod}\, m)$ has no solutions or $(n,\phi(m))$ solutions (mod m) for $n \geq 1$. 5

(ii) Prove that, for any odd number $a, 8 \mid a^2-1$. 2

6 Answer the following : 14

(a) State and prove, Hansel's Lemma. 7

(b) (i) If p is a prime number and $d \geq 1$ such that $d \mid p-1$ then show that, $x^d \equiv 1 (\mod p)$ has exactly d solutions in any C.R.S. (modm). 4

(ii) Prove that, for g.c.d. (a,b) if d is a positive integer of a and b such that $d \mid a$ and $d \mid b$ then $\frac{g}{d} = \left(\frac{a}{d}, \frac{b}{d}\right)$. 3

7 Answer the following :

(a) State and prove, Euler's Theorem. 7

(b) Prove that, there are infinitely many prime numbers. 7

8 Answer the following : 14

(a) For any odd integer a and $n > 2$, show that, $x^n \equiv a (\mod 2^\alpha)$ has a unique solution in any C.R.S. (or R.R.S) (mod 2^α) system. 7

(b) (i) State, Fermat's Theorem. 2

(ii) Find a solution of $x^3 \equiv 7 (\mod 2^4)$ if exists. 5

9 Answer the following :

(a) State and Prove, Chinese Remainder Theorem. 7

(b) If $m, m_1, m_2, \ldots\ldots, m_k \geq 1$ are integers with $m = m_1 m_2 + \ldots \ldots\ldots\ldots + mk$ then prove that, 7

$$\frac{m!}{m_1! \cdot m_2! \cdot \ldots\ldots \cdot m_k!}$$ is an integer.

10 Answer the following : 14

(a) (i) For $n \geq 1$ and p be a prime number 5

then $\left[\frac{n}{p^j}\right] = \left[\frac{n-1}{p^j}\right] + 1$ or 0.

(ii) Prove that, for any two integers a and b, $(a,b) = (a,b+ka)$ where k is any positive integer. 2

(b) Suppose $f(x) \equiv 0 \pmod{p}$ has degree n, prove that, number of solutions in any C.R.S. (mod p) is at most n. 7

MX-003-1163003 Seat No. 035035

M. Sc. (Sem. III) Examination

December - 2020

Mathematics

Number Theory-I

(New Course)

Faculty Code : 003

Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

58

Instructions : (i) Answer any five questions.

(ii) Each question carries 14 marks.

12

1 Answer following seven short questions **(7 X 2 = 14)**

(i) For three integers a, b and c, prove that a divide to b and a divide to c $\Rightarrow$ a divide to b + c and b - c.

2

(ii) Let g = (m, n), GCD of m and n. Prove that g must be positive integer.

(iii) Define term: GCD of two integers m and n. Also find GCD of 120 and 135.

2

(iv) When we say two integers m and n both are relatively primes. Also give two integers other than primes such that they are relativeiy primes.

(v) Write down values of (1) $\phi(31)$ and (2) $\phi(p)$, where p is a prime and ϕ is the Euler's function for a positive integer.

2

(vi) Prove that the equation $x^2 \equiv 3 \pmod 5$ has no solutions.

2

(vii) Prove that 2^{98} can't divide to the value of 100! (factorial of 100), while 2^{97} divide to the value 100!

2

2 Attempt following both (2x7 = 14)

(a) State and Prove Euler's Theorem.

(b) State and Prove Wilson's Theorem.

3 Let m, m_1, m_2 be positive integers such that m = M_1-M_2 14
and (m_1, m_2). i.e. m_1 and m_2 both are relatively primes. Prove that the number of solutions of the equation $f(x) \equiv 0$ (mod m) = the product of the number of solutions of the equation $f(x) \equiv 0$ (mod m_1) and the number of solutions of the equation, $f(x) \equiv 0$ (mod m_2).

4 Answer following seven short questions : (7 X 2 = 14)

(i) For two integers a, b, if a divide to b and b divide to a, then prove that, a = ±b.

(ii) For three integers a, b and n, where n $\in \mathbb{N}$, prove that a divide to b $\Rightarrow$ a^n divide to b^n, for all n $\in \mathbb{N}$.

(iii) Let g = (a, b), GCD of a and b. Prove that g must be unique.

(iv) Define term : Prime Number. Also give at least three primes more than 100.

(v) Define term : Composite Number. Also check 2047 is a composite number or not.

(vi) Define term : GCD of three integers. Verify that (12, (30, 40)) = ((12, 30), 40) in standard notation.

(vii) Find a solution of $x^{11} \equiv 5 \pmod{2^5}$.

5 Attempt following both (2 X 7 = 14)

(a) State and Prove Fundamental Theorem in Arithmetic for product of primes.

(b) State Euclid's Algorithm to find GCD of two non-zero integers m and n. Using this algorithm, find GCD of 3135 and 2310.

6 Attempt following both (2 X 7 = 14)

(a) Let h = order of an integer a in mod m and k = order of an integer b in mod m, for some positive integer m. Let (h, k) = 1. Prove that order of ab in mod m is hk.

(b) Suppose m > 1, (a, m) = 1 and h = order of an integer a in mod m. If $a^k \equiv 1 \pmod m$, for some integer k, then prove that h divide to k.

7 Attempt following both (2 X 7 = 14)

(1) Suppose m, m_1, m_2 be positive integers, $m = m_1 m_2$, m_1 and m_2 are relatively primes and $\phi(m_1, m_2) \geq 2$. Prove that m does not have any primitive root.

(2) Prove that, there are infinitely many primes.

8 Attempt following both (2 X 7 = 14)

(1) Let a, b, c, d are non-zero integers and m, n, q are positive integers. Let $a \equiv b \pmod m$ and $\equiv d$ (mod m). Prove that (i) $qa \equiv qb \pmod m$ and (ii) $ac \equiv bd$ (mod m).

(2) Let p be a prime and (a, p) = 1. Prove that

(i) $a^{p-1} \equiv 1 \pmod p$ and (ii) $a^p \equiv a \pmod p$.

9 Attempt following both (2 X 7 = 14) 10

(1) Let a, b be two non-zero integers. Define term a congruent to b in modulo m, $(m \in \mathbb{N})$. Also prove that the congruence relation in modulo m is an equivalence relation.

(2) For a prime p and a positive integer n, if e is the highest power of p, which divide to the value of factorial n (n!), then prove that e = $\sum_{j=1}^{\infty}\left[\frac{n}{p^j}\right]$, where [x] = integral part of x.

10 Attempt following both **(2 X 7 = 14)**

(a) For a prime p, prove that p has precisely $\phi(p-1)$ primitive roots, where ϕ is the Euler's function for a positive integer.

(b) In standard notation prove that $[x] + [y] \leq [x + y] \leq [x] + [y] + 1$, where [z] = the largest value of an integer, which is less than or equal to z. i.e. [z] = integral part of z.

JBZ-003-1163003 Seat No. ______

M. Sc. (Sem. III) Examination

December - 2019

Mathematics - 3003

(Number Theory - I)

Faculty Code : 003

Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Do as directed : (Answer any **seven**) **14**

(a) Find the number of solution of $x^2+1\equiv 0(\text{mod } p)$ for p is of the form $4k+3$.

(b) Define Totally Multiplicative Function with example.

(c) Find all solutions of $6x\equiv 2(\text{mod } 8)$.

(d) Prove that if p be a prime number and p does not divides a then $a^p\equiv a(\text{mod } p)$.

(e) State Euclid's Algorithm.

(f) Prove that g.c.d is always unique for any two real numbers.

(g) State Division Algorithm.

(h) State De-Poignac's Formulae.

(i) Find $\phi(63*625*49)$.

2 Answer any **two** of the following : **14**

(a) State and prove Chinese Remainder Theorem. **7**

(b) (i) Prove that for g = g.c.d (a, b) if d is a positive integer of a and b such that $d|a$ and $d|b$ then $\frac{g}{d}=\left(\frac{a}{d},\frac{b}{d}\right)$. **3**

(ii) For $n\geq 1$ and p be a prime number then $\left[\frac{n}{p^j}\right]=\left[\frac{n-1}{p^j}\right]+1$ or 0. **4**

(c) Let a and b are non-zero integers then prove that g.c.d of a and b exists and if g = g.c.d (a, b) then g = $ax + by$ for some integers x and y. **7**

3 Answer the following : **14**

(a) If $m_1, m_2, m_3, ...m_k \geq 1$ with the condition $m = m_1 + m_2 + m_3 + ... + m_n$ then prove that **7**

$\dfrac{m!}{m_1!\, m_2!\, m_3! .... m_n!}$ is an integer.

(b) State and prove Hansel's Lemma. **7**

OR

(b) Prove that if p is a prime number then p^2 has exactly $(p-1)\phi(p-1)$ primitive roots in (mod p^2). **7**

4 Answer the following : **14**

(a) State and Prove Euler's Theorem.

OR

(a) State and Prove Fundamental Theorem of Arithmetic.

(b) Suppose $f(x)$ is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 (\text{mod } p)$ has degree n. Prove that $f(x) \equiv 0 (\text{mod } p)$ has atmost n solutions in any completer residue system (mod p).

5 Answer the following : **14**

(a) Prove that $\sigma(n)$ and $\Im(n)$ is a multiplicative function. **4**

(b) If $m_1, m_2 \geq 1$ and $m = m_1 \cdot m_2$ provided m_1 and m_2 are relatively prime with $(\phi(m_1), \phi(m_2)) \geq 2$ then show that m does not have a primitive root. **5**

(c) Prove that if order of a(mod m) = h and order of b(mod m) = j with (h, j) = 1 then order of ab (mod m) = hj. **5**

OR

(c) Prove that for a prime number $p = 2$ or $4k + 1$ there is a solution of $f(x) \equiv 0 (\text{mod } p)$ where $f(x) = x^2 + 1$ for some k. **5**

PCE-003-1163003 Seat No. 35079

M. Sc. (Sem. III) (CBCS) Examination

December - 2018

CMT - 3003 : Mathematics

(Number Theory - I)

Faculty Code : 003

Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Do as directed : (answer any **seven**) **7×2=14**

(a) Find all solutions of $x^4 \equiv 5 \pmod 7$

(b) Prove that for any $x, y \in \mathbb{R}, [x]+[y] \le [x+y] \le [x]+[y]+1$.

(c) State Chinese Remainder Theorem.

(d) Find all solutions of $12x \equiv 18 \pmod{15}$.

(e) Prove that if p be a prime number and p does not divides a then $a^p \equiv a \pmod p$.

(f) State Euclid's Algorithm.

(g) Define :

(i) Complete Residue System (mod m).

(ii) Reduced Residue System (mod m).

(h) Prove that for $m \ne 0$, $ax \equiv ay \pmod m$ if $x \equiv y\left(\frac{m}{a, m}\right)$.

(i) Find the highest power of 2 which divides 100!

(j) Find $\phi\left(47^2\right)$.

2 Answer any two of the following : 7×2=14

(a) State and prove Hansel's Lemma.

(b) (i) For $n \geq 1$ and p be a prime number then 4

$$\left[\frac{n}{p^j}\right] = \left[\frac{n-1}{p^j}\right] + 1 \; or \; 0.$$

(ii) Find the solution $x^4 \equiv 1 (\bmod 3)$ in $C.R.S (\bmod 3)$.

(c) If $\alpha \geq 3$ then prove that the set $\left\{5, 5^2, 5^3, \ldots\ldots\ldots 5^{2^{\alpha-2}}\right\}$ $\cup \left\{-5, -5^2, -5^3, \ldots\ldots -5^{2^{\alpha-2}}\right\}$ is a reduced residue system $(\bmod 2^{\alpha})$.

3 Answer the following : 14

(a) If $f(n) = \sum_{\substack{d \mid n \\ d \geq 1}} \phi(d)$ then $f(n) = n; \forall n \in N$. 4

OR

(a) Solve $x^3 + x\,57 \equiv 0 \left(\bmod 5^3\right)$. 4

(b) Prove that 2^{α} has no primitive roots for $\alpha \geq 3$. 3

(c) Prove that for any prime number p there are exactly $\phi(p-1)$ primitive roots in $(\bmod p)$. 7

4 Answer the following : 7×2=14

(a) State and Prove Wilson's Theorem.

(b) Suppose $f(x)$ is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 (\bmod p)$ has degree n. Prove that $f(x) \equiv 0 (\bmod p)$ has atmost n solutions in any completer residue system $(\bmod p)$.

OR

(b) State and Prove Fundamental Theorem of Calculus.

5 Answer the following : 14

(a) Prove that if $m \geq 1$ has a primitive root with $(a, m) = 1$ then $x^n \equiv a \pmod{m}$ has no solutions or $(n, \phi(m))$ solutions $\pmod{m}$ for $n \geq 1$ 4

(b) Prove that if p is a prime number of the form $p = 4k + 3$ with $p / a^2 + b^2$ then for some integers a and b p divides a and b divides b. 5

(c) Prove that if order of $a \pmod{m} = h$ and order of $b \pmod{m} = j$ with $(h, j) = 1$ then order of $ab \pmod{m} = hj$. 5

OR

(c) Prove that there are infinitely many prime numbers. 5

HDY-003-1163003 Seat No. ______

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November / December – 2017

3003 : Number Theory - I

(Old & New Course)

Faculty Code : 003
Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.
(4) Figures to the right indicate full marks.

1 Fill in the blanks : (each question carries 2 marks) **14**

(i) If p is a prime number and n is a positive integer then the number of positive divisors of p^n is ________.

(ii) If p and q are distinct primes then $p^m q^n$ has ________ positive divisors. $(m, n \in \mathbb{N})$

(iii) If p is a prime of the form _____ then $x^2 + 1 \equiv 0 \pmod{p}$ has no solutions.

(iv) If p is a prime number and n a positive integer then the number of positive integers relatively prime to p^n = ________

(v) If n = 1001 × 59 then $\varnothing(n)$ = ________

(vi) If p is a prime number and p does not divide a then $a^{p-1} \equiv 1 (\text{mod } p)$. This theorem is called ______ theorem.

(vii) If m divides ab then $\frac{m}{(a, m)}$ divides ________

2 Attempt any **two** :

(i) Suppose $p_1, p_2, \ldots\ldots\ldots p_k$ are the first k primes then prove that $p_1, p_2, \ldots\ldots\ldots p_k + 1$ is a prime number and hence deduce that there infinitely many primes. **7**

(ii) State and prove Division Algorithm. **7**

(iii) State and prove Wilson's theorem. **7**

3 All are compulsory :

(i) State and prove Hensel's Lemma. **6**

(ii) Find the smallest positive integer x such that the remainder is 10 when it is divided by 11, the remainder is 12 when it divided by 13 and the remainder is 6 when it is divided by 7. **4**

(iii) Suppose $(a, m) = 1$. Prove that there is a unique integer x in the complete residue system (mod m) such that $ax \equiv 1 \pmod{m}$. **4**

OR

3 All are compulsory :

(i) Suppose $f(x)$ is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 \pmod{p}$ has degree n. Prove that $f(x) \equiv 0 \pmod{p}$ has atmost n solutions in any complete residue system (mod p). **7**

(ii) First find the solutions of $f(x) \equiv 0 \pmod{3}$, $f(x) \equiv 0 \pmod{5}$, $f(x) \equiv 0 \pmod{7}$ and use them to find all solutions of $f(x) \equiv 0 \pmod{105}$. Here $f(x) = x^2 - 1$. **7**

4 Attempt any **two** :

(i) Determine which of the following have primitive roots and if an integer has a primitive root then find atleast two primitive roots : 5, 5^2 , 82, 12 and 35. **7**

(ii) If $\alpha \geq 3$ then prove that the set **7**

$\left\{5, 5^2, 5^3, ...5^{2\alpha-2}\right\} \cup \left\{-5, -5^2, -5^3, ... -5^{2^{\alpha-2}}\right\}$ is a

reduced residue system $\left(\text{mod } 2^{\alpha}\right)$.

(iii) Suppose f is a multiplicative function then prove that **7** the function F defined by $F(n) = \sum_{d/n} f(d)$ is a multiplicative function.

5 Do as directed : (Each carries 2 marks) **14**

(i) Write the statement of mobius inversion formula.

(ii) Find the value of $\varnothing(101)$ using mobius inversion formula.

(iii) Find the highest power of 31 which divides 47321 !

(iv) Find the number of positive divisors of 2016.

(v) Find the values of $w(n)$ for n = 49, 55, 101 × 83, 105.

(vi) Give an example of a multiplicative function which is not totally multiplicative.

(vii) Find $\sigma(n)$ for n = 150, 307.

JBZ-003-1163003 Seat No. ______

M. Sc. (Sem. III) Examination

December - 2019

Mathematics - 3003

(Number Theory - I)

Faculty Code : 003
Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Do as directed : (Answer any **seven**) **14**

(a) Find the number of solution of $x^2 + 1 \equiv 0 \pmod p$ for p is of the form $4k+3$.

(b) Define Totally Multiplicative Function with example.

(c) Find all solutions of $6x \equiv 2 \pmod 8$.

(d) Prove that if p be a prime number and p does not divides a then $a^p \equiv a \pmod p$.

(e) State Euclid's Algorithm.

(f) Prove that g.c.d is always unique for any two real numbers.

(g) State Division Algorithm.

(h) State De-Poignac's Formulae.

(i) Find $\phi(63 * 625 * 49)$.

2 Answer any **two** of the following : **14**

(a) State and prove Chinese Remainder Theorem. **7**

(b) (i) Prove that for g = g.c.d (a, b) if d is a positive integer of a and b such that $d|a$ and $d|b$ then $\frac{g}{d} = \left(\frac{a}{d}, \frac{b}{d}\right)$. **3**

(ii) For $n \geq 1$ and p be a prime number then **4**

$$\left[\frac{n}{p^j}\right] = \left[\frac{n-1}{p^j}\right] + 1 \text{ or } 0.$$

(c) Let a and b are non-zero integers then prove that g.c.d of a and b exists and if g = g.c.d (a, b) then g $= ax + by$ for some integers x and y. **7**

3 Answer the following : **14**

(a) If $m_1, m_2, m_3, \ldots\ldots\ldots\ldots m_k \geq 1$ with the condition $m = m_1 + m_2 + m_3 + \ldots\ldots\ldots + m_n$ then prove that $\dfrac{m!}{m_1!\,m_2!\,m_3!\ldots\ldots\ldots m_n!}$ is an integer. **7**

(b) State and prove Hansel's Lemma. **7**

OR

(b) Prove that if p is a prime number then p^2 has exactly $(p-1)\phi(p-1)$ primitive roots in (mod p^2). **7**

4 Answer the following : **14**

(a) State and Prove Euler's Theorem.

OR

(a) State and Prove Fundamental Theorem of Arithmetic.

(b) Suppose $f(x)$ is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 (\text{mod } p)$ has degree n. Prove that $f(x) \equiv 0 (\text{mod } p)$ has atmost n solutions in any completer residue system (mod p).

5 Answer the following : **14**

(a) Prove that $\sigma(n)$ and $\Im(n)$ is a multiplicative function. **4**

(b) If $m_1, m_2 \geq 1$ and $m = m_1 \cdot m_2$ provided m_1 and m_2 are relatively prime with $(\phi(m_1), \phi(m_2)) \geq 2$ then show that m does not have a primitive root. **5**

(c) Prove that if order of a(mod m) = h and order of b(mod m) = j with (h, j) = 1 then order of ab (mod m) = hj. **5**

OR

(c) Prove that for a prime number $p = 2$ or $4k+1$ there is a solution of $f(x) \equiv 0 (\text{mod } p)$ where $f(x) = x^2 + 1$ for some k. **5**

WN-127

003-016303

M.Sc. (Maths) (CBCS) (Sem-3) Examination
December-2014
Mathematics : Paper No – 3003
Number Theory 1
(New Course)

Faculty Code : 003
Subject Code : 016303

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) There are **five** questions.
(2) **All** questions are compulsory.

1. Fill in the blanks : (each question carries **2** marks) **14**

(i) If there are integers x and y such that $ax + by = -1$ then the g.c.d. of a and b is __________

(ii) If p is a prime number and n a positive integer then the number of positive integers relatively prime to p^n is __________.

(iii) If p is a prime of the form $4k + 3$ then $x^2 + 1 \equiv 0 \pmod p$ has __________ solutions.

(iv) If p, q and r are distinct primes then $p^4 q^2 r^5$ has __________ positive divisors.

(v) If $n = 100 \times 202$ then $\phi(n) =$ __________.

(vi) If p is a prime number and x is any positive integer then $x^p - x$ is divisible by __________.

(vii) If m divides ab then __________ divides b.

2. Attempt any **two** :

(i) Prove that any integer $n > 1$ is a prime or it can be uniquely expressed as a product of primes. **7**

(ii) Prove that the Euclidean algorithm can be used to find the greatest common divisor of two integers. **7**

(iii) Define the Euler function $\phi(m)$. If $m > 0$, $(a, m) = 1$, then prove that $a^{\phi(m)} \equiv 1 \pmod m$ **7**

3. **All** are compulsory :

(i) If m = ab, (a, b) = 1, then prove that $\phi(m) = \phi(a)\,\phi(b)$. **6**

(ii) Find the smallest positive integer x such that $x \equiv 2 (\text{mod } 3)$, $x \equiv 3 (\text{mod } 5)$, $x \equiv 5 (\text{mod } 7)$. **4**

(iii) If $m \neq 0$, a, x, y are integers then prove that $ax \equiv ay \ (\text{mod } m)$ if and only if $x \equiv y\left(\text{mod } \frac{m}{(a, m)}\right)$. **4**

OR

All are compulsory :

(i) Suppose d is a positive integer, p is a prime number and d/(p – 1). Prove that $x^d \equiv 1$ has exactly d solutions. **7**

(ii) First find the solutions of $f(x) \equiv 0 (\text{mod } 3)$, $f(x) \equiv 0 (\text{mod } 5)$, $f(x) \equiv 0 (\text{mod } 7)$ and use them to find all solutions of $f(x) \equiv 0 (\text{mod } 105)$. Here $f(x) = x^2 - 1$. **7**

4. Attempt any **two** :

(i) State and prove Hansel Lemma. **7**

(ii) If m is a non-zero integer and a is relatively prime to m then define the order of a(mod m).

If order of a(mod m) = h and k is positive integer such that $a^k \equiv 1 (\text{mod } m)$ then prove that h devides k. **7**

(iii) Suppose p is a prime number then prove that p has $\phi(p - 1)$ primitive roots. **7**

5. Do as Directed : (each carries **2** marks) **14**

(i) Define the concept of primitive roots.

(ii) Find the value of $\phi(63 \times 625 \times 49)$

(iii) Find the highest power of 101 which divides 27321 !.

(iv) Find the number of positive integers relatively prime to 10100.

(v) Find primitive roots of each of the following …..11, 49, 25.

(vi) Give the definition of the congruence equation $f(x) \equiv 0 \ (\text{mod } m)$.

(vii) Find $\sigma(n)$ for n = 15, 37 and 1961.

DBM-003-1163004 Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

June - 2022

Mathematics : Course No. 3004

(Discrete Mathematics)

Faculty Code: 003

Subject Code: 1163004

Time : **2.30** Hours] [Total Marks : **70**

Instructions:

(1) There are ten questions.

(2) Answer any five of them.

(3) Each question carries 14 marks.

1 Answer the following : **7×2=14**

(a) Define: Minterm and Complemented lattice.

(b) Define with example: Isomorphism of Monoids.

(c) Draw: Hasse diagram for (D_{30}, R).

(d) Let $M_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $M_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. Find $M_1 \odot M_2$

(e) Define: Lattice, with example.

(f) Define: (1) Sub-Lattice (2) Distributive lattice.

(g) Define a Congruence relation on a Semigroup.

(h) Define: Poset with example.

2 Answer the following : **7×2=14**

(1) Define: Machine congruence on a finite state machine.

(2) Define: Phrase structure grammar.

(3) State: Kleen's Theorem.

(4) Define: Language of a Moore machine.

(5) Define the term: Proposition with example.

(6) Make a truth table for the statement: $(p \wedge q) \vee (\sim p)$.

(7) Make truth tables for the statements: (i) $p \wedge q$ (ii) $p \vee q$.

3 Answer the following questions: **2×7=14**

(a) Let R be a relation defined on A and $|A| = n$. Prove that, $R^{\infty} = R \cup R^2 \cup R^3 \cup .... \cup R^n$.

(b) Define a Modular lattice. Let $(L, \leq)$ be lattice. Then $(L, \leq)$ is Modular lattice if and only if the following holds: "If M is any Sublattice of $(L, \leq)$. Then M is not isomorphic to the Pentagon lattice."

4 Answer the following questions: **2×7=14**

(a) Let G be a group and S be a normal subgroup of G. Let R be a relation on G by aRb if and only if $ab^{-1} \in S$. Prove that, R is a congruence relation on G.

(b) Let $p(x,y,z) = (x \wedge y) \vee (y \wedge z')$. Determine the function $f : B_3 \to B$ induced by $p(x,y,z)$.

5 Answer the following questions: **2×7=14**

(a) Let p, q be any proposition or statement. Prove that, each of the following compound statements are tautology:

(i) $p \wedge q \Rightarrow p$

(ii) $p \Rightarrow p \vee q$

(iii) $\sim p \Rightarrow (p \Rightarrow q)$

(iv) $\sim (p \Rightarrow q) \Rightarrow p$

(v) $[p \wedge (p \Rightarrow q)] \Rightarrow q$

(vi) $[p \wedge (p \Rightarrow q)] \Rightarrow p$

(b) For the languages given in (i) and (ii) below, construct a phrase structure grammar G such that $L(G) = L$.

(i) $L = \{a^n b^m / n \geq 1, m \geq 3\}$ and

(ii) $L = \{x^n y^m / n \geq 2, m \geq 0 \text{ and even}\}$

6 Answer the following questions: **2×7=14**

(a) Define: Lexicographic order. Let $n \geq 1$. Let (L, $\leq$) be a finite Boolean algebra. Prove that, the number of atoms of (L, $\leq$) is same as number of co-atoms of (L, $\leq$).

(b) State and prove: Pumping lemma.

7 Answer the following questions: **2×7=14**

(a) State and prove: Fundamental theorem of Homomorphism of semigroups.

(b) Describe steps of Warshall's Algorithm for finding W_k from $W_{k-1}, k \in \{1, 2, ..., n\}$. Also using them find R^{∞} for $A = \{1, 2, 3, 4\}$ with $R = \{(1, 2), (2, 3), (3, 2), (3, 4)\}$.

8 Answer the following questions: **2×7=14**

(a) Define atom. Let (L, $\leq$) be a finite Boolean algebra. Let $a \in L, a \neq 0$. Let $\{a_1, a_2, a_3, ..., a_m\}$ be the set of all atoms of (L, $\leq$) such that $a_i \leq a$ for each $i \in \{1, 2, ..., m\}$. Prove that, $a = a_1 \vee a_2 \vee a_3 \vee ... \vee a_m$

(b) Define: GLB and LUB of a subset of (P, $\leq$). Let $(L_i \leq_i)$ be lattices for each $i \in \{1, 2, ..., n\}$. Let $L = L_1 \times L_2 \times .... \times L_n$ be the Cartesian product of $L_1, L_2, ..., L_n$. Let $\leq$ be the product partial order on *L*. Prove that, $(L = L_1 \times L_2 \times .... \times L_n, \leq)$ is also a lattice.

9 Answer the following questions: **2×7=14**

(a) Let R is an equivalence relation on $A = \{1, 2, 3, 4, 5\}$ determined by the partition P_1 of *A* whose members are {1, 2}, {3, 4}, {5} and S is another equivalence relation on *A* determined by the partition P_2 of *A* whose members are {1}, {2}, {3}, {4, 5}. Find $(R \cup S)^{\infty}$ using:

(i) Graphical Method (ii) Matrix Method

(b) Let $f_1 : B_2 \to B$ be a Boolean function with $S(f_1) = \{00, 01, 10\}$ and let $f_2 : B_3 \to B$ be a Boolean function with $S(f_2) = \{000, 001, 011, 010\}$. Construct Karnaugh maps for both f_1 and f_2. Also find the Boolean expressions for both of them.

10 Answer the following questions: **2×7=14**

(a) Prove that $(\mathbb{N}, R)$ is distributive lattice, where R is divisibility relation on $\mathbb{N}$.

(b) Let p, q be propositions. Prove that the following statements hold:

(i) $(p \Rightarrow q) \equiv (\sim p) \vee q$

(ii) $(p \Rightarrow q) \equiv \sim q \Rightarrow \sim p$

(iii) $\sim (p \Rightarrow q) \equiv (p \wedge \sim q)$

(iv) $\sim (p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

(v) $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

(vi) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

(vii) $\sim (\sim p) \equiv p$

(viii) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

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Seat No. ____________

FP-003-1163004

M. Sc. (Sem. III) (CBCS) Examination

November - 2022

Mathematics : Paper - CMT - 3004

(Discrete Mathematics)

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions :

(1) There are five questions.

(2) Answer all the questions.

(3) Each question carries 14 marks.

1 Answer any seven of the following : 7×2=14

(1) Define semigroup. Prove or disprove: $\mathbb{N}$ with usual multiplication is semigroup.

(2) Define :

(a) Reflexive closure of R

(b) Symmetric closure of R.

(3) Define with example: Lattice.

(4) Draw Hasse diagram of (D_{45}, divide) POSET.

(5) Write down idempotent properties for lattice.

(6) Define bounded lattice and give an example of a lattice which is not bounded.

(7) Define: Phrase structure grammar and language of a phrase structure grammar.

(8) Prove that, a distributive lattice is always modular.

(9) Define with example: Proposition.

(10) Explain negation of a proposition.

2 Answer any two from the following questions : 2×7=14

(a) State and prove, fundamental theorem of homomorphism of semigroups.

(b) Let G be a group and H be a subgroup of G. Define a relation R on G by ${}_aR_b$ iff $ab^{-1} \in H$ for any a, b $\in$ G. Prove that, R is a congruence relation on G $\Leftrightarrow$ H is a normal subgroup of G.

(c) $f:(S,*) \to (T,*')$ be a homomorphism of semigroups. Prove that,

(1) f(S) will be a subsemigroup of $(T,*')$.

(2) If (S, *) is a monoid and if f is a monoid homomorphism, then f (S) will be a submonoid of $(T,*')$.

3 Answer following two : 2×7=14

(a) Describe the steps of warshall's algorithm for finding W_k from W_{k-1} where $k \in \{(1, 2,..,n\}$. Also using them find R^{∞} for $A = \{1, 2, 3, 4\}$ with $R = \{(1, 2), (2, 1), (2, 3), (3, 4)\}$.

(b) Find number of atoms in finite Boolean algebra. Also prove that, in finite Boolean algebra number of atoms and number of co atoms are same.

OR

3 Answer following two : 2×7=14

(a) Prove that, $(L, \le)$ is a distributive lattice if and only if for all a, b, c $\in$ L

$$(a \wedge b) \vee (b \wedge c) \vee (a \wedge c) = (a \vee b) \wedge (b \vee c) \wedge (a \vee c)$$

(b) Let $G = (V, S, v_0, \mapsto)$ be a phrase structure grammar in which $V = \{v_0, w, a, b, c\}$, $S = \{a, b, c\}$ and v_0 is the starting symbol for substitutions and the production relation $\mapsto$ is given by

(1) $v_0 \mapsto aw$

(2) $w \mapsto bbw$

(3) $w \mapsto c$. Find L (G).

4 Answer following two : 2×7=14

(a) Explain contradiction. Verify by truth table that given compound statements are contradiction or not :

(1) $(a \wedge b) \wedge (\sim a)$

(2) $(p \wedge \sim q) \wedge (\sim p \vee q)$

(3) $\sim p \wedge [(p \vee \sim q) \wedge q)]$

(b) Explain logical equivalence. Verify the following by truth table :

(1) $\sim (p \to q) \equiv (p \wedge \sim q)$

(2) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(3) $a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$

5 Answer any two from the following questions : 2×7=14

(a) Explain the following :

(1) Conditional statement

(2) Biconditional statement

(3) Inverse of a conditional statement

(4) Conjunction of propositions.

(b) State and prove, Kleene's theorem.

(c) Prove that, if $(L, \leq)$ is lattice then $(L, \leq^{-1})$ is also a lattice. Where $\leq^{-1}$ is defined by $a \leq^{-1} b$, if $b \leq a$, $\forall$ a, $b \in L$.

(d) $f : (\mathbb{R}, +) \to (\mathbb{R}^{+}, \times)$ defined by $f(x) = e^x$ for any $x \in \mathbb{R}$. Prove that, f is an isomorphism of semigroups.

MY-003-1163004

Seat No. 035035

M. Sc. (Sem. III) Examination

December - 2020

Mathematics : CMT-3004

(Discrete Maths) (New Course)

66

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer any five questions.

(2) Each question carries 14 marks.

1 Answer following seven short questions : 7×2=14

(i) Let A be non-empty set and P(A) = the power set of A. Prove that $(P(A),\cup)$ and $(P(A),\cap)$, both are semigroups.

(ii) In standard notation, prove that $(M_{m\times n}(Z), +)$ is a semigroup.

(iii) Define terms: Subsemigroup and Submonoid.

(iv) Define terms: Homomorphism of semigroups and homomorphism of monoids.

(v) Define term: Partial Order Relation.

(vi) Define term totally ordered set and prove that, (N, ≤) is a totally ordered set.

(vii) Prove that (N, divide) is a POSET and it is not a totally ordered set.

2 Let f: (S, *) → (T, *') be an onto homomorphism of semigroups. Prove that the relation R defined on S by aRb if and only if f(a) = f(b), is a congruence relation on (S, *) and $(S/R, \otimes) \simeq (T, *')$ as semigroups. 14

14.

3 Answer following seven short questions : 7×2=14

(i) Define term: Connectivity relation for a relation R on a non-empty set A.

(ii) Define term: Homomorphism of Lattices.

(iii) Define terms: Smallest element and greatest element in a Lattice (L, $\leq$).

(iv) What mean by Hasse diagram of a finite POSET?

(v) Prove that $\sim p \wedge [p \vee (p \wedge q)]$ is a contradiction.

(vi) Prove that $\sim p \vee [p \wedge (p \vee q)]$ is a tautology.

(vii) Let A = {1, 2, 3, 4} and R is a relation on A defined by R = {(1, 2), (2, 1), (2, 3), (3, 4)}. Determine, R^{∞}, the connectivity relation for R on A.

4 Attempt following both : 2×7=14

(a) Let G be a group and S subgroup of G. Define a relation R on G by a R b iff $ab^{-1} \in S$. Prove that S is a normal subgroup of G, whenever R is a congruence relation on G.

(b) Let (P, $\leq$) be a POSET. Prove that (P, ≤ -1) is also a POSET, where $\leq^{-1}: P \times P \rightarrow P$ defined by $a \leq^{-1}$ if $b \leq a$, for all $a, b \in P$.

5 Let p, q be any propositions or statements. Prove that each of following compound statements are tautologies: 14

(1) $p \wedge q \Rightarrow p$

(2) $p \Rightarrow p \vee q$

(3) $\sim p \Rightarrow (p \Rightarrow q)$

(4) $[\sim (p \Rightarrow q)] \Rightarrow p$

(5) $[\sim (p \Rightarrow q)] \Rightarrow q$

(6) $[p \wedge (p \Rightarrow q)] \Rightarrow p$

(7) $(p \wedge q) \vee p' \vee q'$.

6 Let R be a relation on a non-empty set A. Prove that, the connectivity relation R^{∞} for R is transitive relation on A and $R^{\infty} = R \cup R^2 \cup .... \cup R^n$, when |A| = n. 14

7 Define following terms : 14

(1) Syntax of a language

(2) Semantics of a language

(3) Phrase Structure Grammar

(4) Finite State Machine

(5) Language of a phrase structure grammar

(6) Moore Machine

(7) Language of a Moore Machine.

8 Attempt following both : 2×7=14

(1) Let $(A, \leq_A)$, $(B, \leq_B)$ be two POSETS and $f : A \to B$ be a bijection. Prove that f is an isomorphism of POSETS iff for any $a_1, a_2 \in A, a_1 \leq_A a_2 \Leftrightarrow f(a_1) \leq_B f(a_2)$.

(2) Let $(A_i, \leq_i)$ i=1, 2, ... n be POSETS and $A = A_1 \times A_2 \times .... \times A_n$.. Prove that A is also a POSET under the relation $\leq$ on A defined by $(a_1, a_2, ... a_n) \leq (b_1, b_2, ... b_n)$ iff $a_i \leq_i b_i$, for all $(a_1, a_2, ... a_n)\ (b_1, b_2, ... b_n) \in A$.

9 Attempt following both : 2×7=14

(1) Define relative complement of an element in an interval [a, b] of a Lattice $(L, \leq)$. For the Lattice $(L, \leq)$, prove that it is modular Lattice iff for any interval [a, b] in $(L, \leq)$, if x, y $\in [a, b]$, $x \leq y$ and x, y both admits a common relative complement in [a, b], then x = y.

(2) Let f: $f : (L_1, \leq_1) \to (L_2, \leq_2)$ be an onto homomorphism of Lattices. Prove that

(a) If $(L_1, \leq_1)$ is distributive, then so is $(L_2, \leq_2)$.

(b) If $(L_1, \leq_1)$ is complemented, then so is $(L_2, \leq_2)$.

[Contd...

10 Attempt following both : 2×7=14

(1) Let $((L, \leq)$ be a Boolean algebra. Prove that it satisfies the D'Morgon Law. i.e.

(a) $(a \vee b)' = a' \wedge b'$ and

(b) $(a \wedge b)' = a' \vee b'$.

(2) Let p, q, r be statements. In standard notations prove that $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ is a tautology.

JBB-003-1163005 Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

December - 2019

Mathematics : CMT - 3004

(Discrete Mathematics) (Old & New Course)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.
(2) There are 5 questions.
(3) All questions carry equal marks.

1 Answer any **seven** questions : **7×2=14**

(a) Define: Congruence relation on semigroups.

(b) Let $f:(S,*')\to(T,*)$ be a homomorphism of semigroups. Prove that the image of *f* is a subsemigroup of (*T*, *',).

(c) Give an example of lattice which is modular but not distributive.

(d) Prove that any finite lattice is bounded.

(e) Define : Context free grammar and Context free language.

(f) Find regular expression of language of all string of length eight or less.

(g) Define: Toutology with an example.

(h) Define: Existential quantification and Existential quantifier.

(i) Explain: Channel and Noise.

(j) Decide which codeword was transmitted if the received codeword is 1100111 from Hamming code of length seven with three parity bits.

2 Answer any **two** : **2×7=14**

(a) State and prove Fundamental theorem of homomorphism of semigroups.

(b) Let R be a relation defined on a non-empty set A. Then prove that the transitive closure of R equals $\bigcup_{n=1}^{\infty} R^n$.

(c) Let G be a group. Let R be a congruence relation on G. Prove that there exist a normal subgroup N of G such that for any $a,b\in G, aRb$ if and only if $ab^{-1}\in N$.

3 Answer the following : **2×7=14**

(a) Let $(L, \leq)$ be a lattice. Suppose that $(L, \leq)$ is modular. Prove that $(L, \leq)$ is distributive iff L does not contain any sublattice which is isomorphic to the pentagon lattice.

(b) Let $n > 1$ then prove that $(D_n, \leq_{div})$ is complemented iff n is the product of distinct primes.

OR

(a) Let $(L, \leq)$ be a lattice. Then prove that $(L, \leq)$ is distributive iff for all $a, b, c \in L$,

$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

(b) Let V be non zero vector space defined on field F then $(L(V), \subseteq)$ is distributive iff $\dim_F V = 1$.

4 Answer any **two** : **2×7=14**

(a) Find Context free grammars of following languages :

(1) Language of non-palindromes

(2) $L = \left\{x \in \{0, 1\}^* \middle| n_0(x) = n_1(x)\right\}$

(b) Show that for any NFA $M = (Q, \Sigma, q_0, A, \delta)$ accepting a language $L \subseteq \Sigma^*$, there is an FA $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$, that also accepts L.

(c) Draw FAs corresponding to following regular expressions.

(1) $(11 + 10)^*$

(2) $(0 + 1)^* (1 + 00) (0 + 1)^*$

5 Answer any **two** : **2×7=14**

(a) Using indirect proof technique show that for all $x, x^2 + 1$ is odd then x is even.

(b) Let p and q be two statements then prove that

(1) $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$

(2) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

(c) Explain Hamming code and find Hamming code of length seven with three parity bits.

(d) Show that a binary code C can correct up to k errors in any codeword if and only if $d(C) \geq 2k + 1$.

an example.

PCF-003-1163004 Seat No. 35063

M. Sc. (Sem. III) (CBCS) Examination

December - 2018

CMT - 3004 : Mathematics

(Discrete Mathematics)

(Old & New Course)

Faculty Code : 003

Subject Code : 1163004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Attempt all the questions.

(2) There are 5 questions.

(3) Figures to the right indicate full marks.

1 Answer any **seven** of the following : 14

(a) Find all complements in lattice (S_{10}, D)

(b) Find minimal and maximal element of poset (P, D) with $P = \{2,3,4,5,6,7,8,9,10\}$ and $D =$ Divisibility relation.

(c) Give an example of lattice which is modular but not distributive.

(d) State isotonicity property for lattices.

(e) Find context free grammar of language $L = \{a^n b^n \mid n \geq 0\}$

(f) Write regular expression of language of all strings that ends in 00.

(g) Define connectivity relation.

(h) Define direct product of semigroups.

(i) Define the terms : Channel, Decoder

(j) Define Hamming distance of a code.

2 Answer any **two** of the following : **14**

(a) Let $n>1$ then show that (S_n, D) is complemented iff n is square free integer.

(b) State and prove fundamental theorem of homomorphism of semi groups.

(c) Show that a code C can detect all combination of k or fewer or error if and only if $d(C) \geq k+1$.

3 (a) Let $(L,\leq)$ be a lattice then show that $(L,\leq)$ is modular if $[x,y]$ is any closed interval in $(L,\leq)$ and $a \leq c(a,c \in [x,y])$ and both a, c admit a common relative complement then $a=c$. **14**

(b) Show that for any NFA $M=(Q,\Sigma,q_0 A,\delta)$ accepting the language $L \subseteq \Sigma^*$. there is an FA $M_1=(Q_1,\Sigma,q_1,A_1,\delta_1)$ that also accepts L.

OR

3 (a) Let $(L,\leq)$ be a lattice then show that following statement are equivalents **14**

(i) For any $a,b,c \in L$ with $a \leq c$, $a \oplus (b * c) = (a \oplus b) * c$

(ii) For all $x,y,z \in L$ with $z \leq x$

$$x*(y \oplus z) = (x*y) \oplus z$$

(b) State Pumping lemma for regular languages. Using Pumping lemma show that language of all Palindromes is not regular.

4 Answer the followings : **14**

(a) Let R and S be equivalence relation defined on a non empty set A then show that $(R \cup S)^{\infty}$ is smallest equivalence relation defined on A which include $R \cup S$.

(b) Let V be a non zero vector space over field F then $(L(V),\subseteq)$ is distributive iff $\dim_F V = 1$.

5 Answer any **two** of following questions : 14

(a) Let $(L,\leq)$ be a lattice then $(L,\leq)$ is distributive if and only if for any $a,b,c \in L$

$$(a*b)\oplus(b*c)\oplus(c*a)=(a\oplus b)*(b\oplus c)*(c\oplus a)$$

(b) Show that a binary code C can correct up to k errors in any codeword if and only if $d(C)\geq 2k+1$.

(c) Let G be a group and H be a normal subgroup of G then show that a relation R defined on G by $g_1 \, R \, g_2$ if $g_1 \, g_2^{-1} \in H$ is a congruence relation of G.

(d) If L_1 and L_2 are context free languages then show that $L_1 \cup L_2$ is also context free language. Using above result find context free grammar of language $=\left\{x \in \{0,1\}^* \mid n_0(x) \neq n_1(x)\right\}$

HDZ-003-1163004 Seat No. 035064

M. Sc. (Sem. III) (CBCS) Examination

November/December – 2017

Mathematics : MATHS. CMT-3004

(Discrete Mathematics) (New Course)

Faculty Code : 003
Subject Code : 1163004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the quesitons.
(2) Each question carries 14 marks.

1 Answer any Seven : 7×2=14

(a) Let A be a nonempty set. Define the concept of the free semigroup generated by A.

(b) Let $A = \{0,1\}$. Show that the following expressions are regular expressions over A.

(i) $0^* (0 \vee 1)^*$

(ii) $(01)^*(01 \vee 1^*)$

(c) Define a complemented lattice and illustrate it with an example.

(d) Let $f : (S, *) \rightarrow (T, *')$ be a homomorphism of semigroups. If f is onto and if $(S, *)$ is a monoid, then show that $(T, *')$ is a monoid.

(e) Define a Boolean Algebra. State the reason why the diamond lattice is not a Boolean Algebra.

(f) Let $L \subseteq \{x, y\}^*$. When is L said to be a type 2 language over $\{x, y\}$?

(g) Define a (i) phrase structure grammar and a (ii) Moore machine.

(h) Define a machine congruence on a finite state machine.

(i) State Kleene's theorem.

(j) Define a modular lattice. Illustrate that a finite lattice need not be modular.

2 Answer any Two : **2×7=14**

(a) State and prove the fundamental theorem of homomorphism of semigroups.

(b) Let $(L, \leq)$ be a lattice. Show that $(L, \leq)$ is distributive if and only if for all.

$$a, b, c \in L, (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \wedge c) \wedge (c \vee a)$$

(c) Let $n \geq 1$ and let $f : B_n \rightarrow B$. Prove that f is produced by a Boolean expression.

3 (a) Let G be a group and let H be a normal subgroup of G. Let R be a relation defined on G by aRb if and only if $ab^{-1} \in H$. Prove that R is a congruence relation on G. **5**

(b) Let V be a vector space over a field F. Show that the lattice of subspaces of V is modular. **5**

(c) Let $f : A \rightarrow B$ be a bijection. If $(A, \leq_A)$ is a partially ordered set, then show that we can define a relation $\leq_B$ on B such that $(B, \leq_B)$ is a poset and $f : (A, \leq_A) \rightarrow (B, \leq_B)$ is an isomorphism of posets. **4**

OR

3 (a) Let $n \geq 1$. Prove that D_n, the lattice of positive divisors of n is distributive. **5**

(b) Let $G = (V, S, v_0, \mapsto)$ be a phrase structure grammar in which $\{v_0, x, y, z\}$, $S = \{x, y, z\}$, and the productions are given by **5**

(1) $v_0 \mapsto xv_0$,

(2) $v_0 \mapsto yv_0$, and

(3) $v_0 \mapsto z$.

Find $L(G)$

(c) Let R be a symmetric relation defined on a nonempty set A. Prove that R^{∞} is symmetric. 4

4 Answer any Two : 2×7=14

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Prove that the number of atoms of $(L, \leq)$ is equal to the number of coatoms of $(L, \leq)$.

(b) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. Prove that there exists a type 3 phrase structure grammar G with I as its set of terminal symbols such that $L(M) = L(G)$.

(c) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. If R is the w-compatibility relation defined on S, then show that R is a machine congruence on M and $L(M) = L(M/R)$.

5 Answer any Two : 2×7=14

(a) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. If $w \in L(M)$ is such that $l(w) \geq |S|$, then show that there exist $w_1, w_2, w_3 \in I^*$ such that $l(w_2) > 0$, $w = w_1 w_2 w_3$ and $w_1 w_2^k w_3 \in L(M)$ for all $k \geq 0$.

(b) For the languages given in (i) and (ii) below, construct a phrase structure grammar G such that $L(G) = L$.

(i) $L = \{a^n b^m \mid n \geq 1, m \geq 3\}$

(ii) $L = \{x^n y^m \mid n \geq 2, m \geq 0 \text{ and even}\}$

(c) Let $(L, *)$ be a commutative semigroup in which $a * a = a$ for all $a \in L$. Prove that the relation $\leq$ defined on L by $a \leq b$ if and only if $a * b = b$ is a partial order and for any $a, b \in L$, $a * b$ is the least upper bound of $\{a, b\}$ in $(L, \leq)$.

(d) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. For each $n \geq 0$, let R_n be the relation defined on S by $s_i R_n s_j$ if and only if s_i and s_j are w-compatible for all $w \in I^*$ with $l(w) \leq n$. Let $k \geq 0$ and let $s, t \in S$. Show that the following statements are equivalent :

(i) $sR_{k+1}t$

(ii) $sR_k t$ and $f_x(s)R_k f_x(t)$ for each $x \in I$.

MBS-003-016304 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

December - 2016

Mathematics : MATH.CMT-3004

(Discrete Mathematics)

Faculty Code : 003
Subject Code : 016304

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1 Answer any seven : 7×2=14

(a) Define :

(i) a semigroup and

(ii) a monoid.

(b) Define a congruence relation on a semigroup $(S,*)$.

(c) State the fundamental theorem of homomorphism of semigroups.

(d) Define :

(i) a bounded lattice and

(ii) a complemented lattice.

(e) Let $(L,\leq)$ be a lattice with greatest element 1.

Define the concept of coatom of $(L,\leq)$. Mention the set of coatoms of D_{165}.

(f) Define a phrase structure grammar and illustrate it with an example.

(g) Determine the regular set corresponding to the regular expression aa^*b^*a over $\{a,b\}$.

(h) Let $f_1, f_2 : B_3 \to B$. Verify that $S(f_1 \vee f_2) = S(f_1) \cup S(f_2)$.

(i) Show that any distributive lattice is modular.

(j) Let R be a machine congruence on a Moore machine M. Show that $L(M) \subseteq L(M/R)$.

2 Answer any two : **2×7=14**

(a) Let R be a congruence relation on a group G. Show that there exists a normal subgroup H of G such that for any $a, b \in G$, aRb if and only if $ab^{-1} \in H$.

(b) Let $n > 1$. Prove that D_n, the lattice of positive divisors of n is complemented if and only if n is the product of distinct primes.

(c) If R, S are equivalence relations defined on a nonempty set A, then prove that $(R \cup S)^{\infty}$ is the smallest equivalence relation on A containing both R and S.

3 (a) Let V be a vector space over a field F. Prove that $L(V)$, the lattice of subspaces of V is modular. 5

(b) Let $f : B_3 \to B$ be such that $S(f) = \{000, 001, 011, 100, 111\}$. Show that $p(x,y,z) = (y' \wedge z') \vee (x' \wedge y') \vee (y \wedge z)$ is a Boolean expression for f. 5

(c) Let $(L, \leq)$ be a Boolean Algebra. Let $a, b \in L$. Show that $(a \vee b)' = a' \wedge b'$. 4

OR

3 (a) Let A be a finite nonempty set containing exactly n elements. Let R be a relation defined on A. Show that $R^{\infty} = R \cup \ldots \cup R^n$. 5

(b) Prove that a lattice $(L, \leq)$ is distributive if and only if for all $a, b, c \in L$, $(a \vee b) \wedge c \leq a \vee (b \wedge c)$. 5

(c) Let $V=\{v_0, w, a, b, c\}$, $S=\{a,b,c\}$ and $\mapsto$ is a relation on $V*$ given by 4

(1) $v_0 \mapsto aw$,

(2) $w \mapsto bbw$, and

(3) $w \mapsto c$.

Let $G=(V, S, v_0, \mapsto)$. Find $L(G)$.

4 Answer any two : 2×7=14

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Prove that $(L, \leq)$ is isomorphic of $(P(A), \subseteq)$ for some nonempty finite set A.

(b) Let R be a relation defined on a nonempty finite set A. Describe Warshall's Algorithm for finding the transitive closure of R.

(c) Construct a phrase structure grammar G such that $L(G)=\{a^n b^m : n \geq 1, m \geq 1\}$. Prove that there exists no type 3 grammar G such that $L(G)=\{a^n b^n : n \geq 0\}$.

5 Answer any two : 2×7=14

(a) Let $n \geq 2$ and $(L_i, \leq_i)$ be a lattice for each $i \in \{1, 2, ..., n\}$. Let $L = L_1 \times L_2 \times ...... \times L_n$. Show that $(L, \leq)$ is distributive if and only if $(L_i, \leq_i)$ is distributive for each $i \in \{1, 2, ..., n\}$, where $\leq$ is the product partial order on L.

(b) Let M be a Moore machine with $S=\{s_0, s_1\}$. $I=\{0, 1\}$, $f_0 : S \to S$ is the identity mapping on S, $f_1(s_0)=s_1 \cdot f_1(s_1)=s_0$, and $T=\{s_1\}$. Determine $L(M)$ and find a regular expression α over $\{0,1\}$ such that $L(M)$ = the regular set corresponding to α. Also construct a type 3 grammar G such that $L(G)=L(M)$.

(c) Prove tha fundamental theorem of homomorphism of semigroups.

(d) Let $(A, \leq_A)$ be a lattice. Let $f: A \to B$ be a bijection. Prove that there exists a relation $\leq_B$ defined on B such that $(B, \leq_B)$ is a lattice and f is an isomorphism of lattices.

(g) $aa^*b^*a = \{aa, aaa, aba, aaba, aa^2a, aa^2ba, \ldots\}$

BBJ-003-016304 Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

December - 2015

Mathematics : CMT-3004
(Discrete Mathematics)

Faculty Code : 003
Subject Code : 016304

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the questions
(2) Each question carries 14 marks.

1. Answer any Seven $2 \times 7 = 14$
   (a) Let p, q, r be propositions. Show that $p \vee (q \vee r) \equiv (p \vee q) \vee r$.
   (b) Let R be a relation defined on a nonempty set A. If R is symmetric, then show that R^{∞} is symmetric.
   (c) Let H be a normal subgroup of a group G. Let R be the relation defined on G as follows: aRb if and only if $ab^{-1} \in H$. Prove that R is a congruence relation.
   (d) When is a lattice $(L, \leq)$ said to be distributive? Show that the homomorphic image of a distributive lattice is distributive.
   (e) Define type 3 phrase structure grammar. Illustrate it with an example.
   (f) Let $n \geq 1$ and $x, y, z \in B^n$. Prove that $\delta(x, z) \leq \delta(x, y) + \delta(y, z)$.
   (g) State the fundamental theorem of homomorphism of semigroups. Show that for any nonempty set A, the semigroup $(\mathbb{N} \cup \{0\}, +)$ is isomorphic to a quotient semigroup of A^*.
   (h) If $(L_1, \leq_1)$ and $(L_2, \leq_2)$ are complemented lattices, then prove that their direct product $L = L_1 \times L_2$ is also complemented.
   (i) Let A be a nonempty set. State the rules for computing the regular set corresponding to a regular expression over A.
   (j) State Kleene's theorem.

2. Answer any Two $2 \times 7 = 14$
   (a) Let $A = \{a, b, c, d\}$ and $R = \{((a, b), (b, a), (b, c), (c, d)\}$. Find the matrix of the transitive closure of R using Warshall's Algorithm.
   (b) Prove that a lattice $(L, \leq)$ is distributive if and only if for all $a, b, c \in L$, $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.
   (c) If a lattice $(L, \leq)$ is not modular, then show that $(L, \leq)$ contains a sublattice isomorphic to the pentagon lattice.

3. (a) Let $f : B_3 \to B$ be such that $S(f) = \{000, 001, 011, 100, 111\}$. Find a Boolean expression for f. 5

(b) Let $G = (V, , S, v_0, \mapsto)$ be a phrase structure grammar with $V = \{v_0, x, y, z\}$, $S = \{x, y, z\}$, and the production relation is given by $1. v_0 \mapsto xv_0$, $2. v_0 \mapsto yv_0$, and $3. v_0 \mapsto z$. Find $L(G)$. 5

(c) Let $f : (S, \star) \to (T, \star')$ be a surjective homomorphiosm of semigroups. If $(S, \star)$ is a monoid, then prove that $(T, \star')$ is a monoid. 4

OR

3. (a) Let M be a Moore machine with $S = \{s_0, s_1\}$, $I = \{0,1\}$. f_0 = Identity mapping on S, $f_1(s_0) = s_1$, $f_1(s_1) = s_0$, and $T = \{s_0\}$. Determine $L(M)$. 5

(b) Construct a phrase structure grammar G such that $L(G) = \{a^n b^m | n \geq 1, m \geq 1\}$. 5

(c) State and prove De Morgan's laws for logic. 4

4. Answer any Two $2 \times 7 = 14$

(a) State and prove Pumping lemma.

(b) Let $e : B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a nonzero code word.

(c) Let M be a Moore machine with state set S and input set I. Let $k \geq 0$ and R_k be the relation defined on S as follows: for any $s, t \in S$, $sR_k t$ if and only if s and t are w-compatible for all $w \in I^*$ with $l(w) \leq k$. Show that for any $s, t \in S$, $sR_{k+1}t$ if and only if the following conditions hold: (1) $sR_k t$ and (2) $f_x(s) R_k f_x(t)$ for each $x \in I$.

5. Answer any Two $2 \times 7 = 14$

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Let $a \in L$, $a \neq 0$. Prove that a can be expressed as a join of atoms in L.

(b) (i) Let R be a relation defined on a nonempty set A. Prove that R^∞ is the transitive closure of R. If A contains exactly n elements, then prove that $R^\infty = R \cup \cdots \cup R^n$.

(c) Let M be a nondeterministic finite state machine. Show that there exists a Moore machine M_1 such that $L(M) = L(M_1)$.

(d) lLet p, q be propositions. Show that each one of the following is a tautology. (i) $(p \wedge (p \Rightarrow q)) \Rightarrow q$ (ii) $(p \Rightarrow q) \Leftrightarrow (\tilde{q} \Rightarrow \tilde{p})$.

DBN-003-1163005 Seat No. ________

M. Sc. (Sem. III) Examination

June – 2022

Mathematics : EMT - 3011

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any **five** questions from the following.

(2) There are total **ten** questions.

(3) Each question carries **equal** marks

1 Attempt the following : **14**

(1) Define with example: Regular curve.

(2) Define: ε- neighborhood in R^2.

(3) Find curvature and torsion of the circle $2x^2 + 2y^2 - 12x - 12y - 36 = 0$.

(4) Define with examples: Functions of class k.

(5) Define: Tangent vector field.

(6) Define: Length of a regular curve segment.

(7) Define: Unit speed curve.

2 Attempt the following : **14**

(1) Find the curvature and torsion of the curves
(i) $5x + 2y = 0$ and (ii) $x^2 + y^2 = 4$.

(2) Define with example: Simple surface.

(3) Define : The Osculating plane and the Rectifying plane. Also demonstrate them on a surface of an upper Hemisphere.

(4) Is the curve $\alpha(x) = (x^{100}, 2x + 7, 5x^2 + 3)$ is regular? Justify your answer.

(5) Define: Normal curvature and Geodesic curvature.

(6) Identify the curve $x \cos \alpha + y \sin \alpha = p$ and find its curvature and torsion.

(7) Define: Velocity vector of a regular curve α.

3 Attempt the following : **14**

(a) Define tangent line to a curve. Show that the curve $\alpha(t) = (\sin 6t \cos t, \sin 6t \sin t, 0)$ is regular. Also find the equation of tangent line to α at the point $t = \frac{\pi}{6}$.

(b) Show that the curve $\alpha(S) = \left(\frac{5}{13}\cos S, \frac{8}{13} - \sin S, -\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its curvature and torsion of the given curve.

4 Attempt the following : **14**

(a) Define reparametrization of a curve. If $g : [c, d] \to [a, b]$ is a reparametrization of a curve segment $\alpha : [a, b] \to R^3$ then prove that the length of α is equal to the length of $\beta = \alpha \circ g$. Also derive the relation between their tangent planes.

(b) Define the arc length of a curve and prove that the arc length is one - one function mapping (a, b) onto (c, d) and it is a reparametrization. Is the curve reparametrized by its arc length yield a unit speed curve? Justify your answer.

5 Attempt the following : **14**

(a) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and $r^2 = \rho^2 (\rho' \sigma)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

(b) Is the curve $\alpha(t) = (\sin t,\ \cos^2 t,\ \cos t)$ regular? If so then find the equation of tangent line at $t = \frac{\pi}{4}$.

6 Attempt the following : **14**

(a) Prove that: The set of all tangent vectors to a simple surface $x : u \to R^3$ at P is a vector space. Also find the dimension of that vector space.

(b) Show that the length of the curve

$\alpha(t) = \left(2a\left(\sin^{-1} t + t\sqrt{1-t^2}\right),\ 2at^2,\ 4at\right)$ between the points $t = t_1$ to $t = t_2$ is $4a\sqrt{2}(t_2 - t_1)$. What will be the arc length between the points $t_1 = 25$ and $t_2 = 30$?

7 Attempt the following: **14**

(a) Define orthonormal vectors and prove that the set {T, N, B} is orthonormal.

(b) Find the arc lengths of the curves $\alpha(t) = (r\cos t,\ r\sin t,\ 0)$ and $\alpha(t) = (r\cos\omega s,\ r\sin\omega s,\ h\omega s)$. Also reparametrize them by their arc lengths.

8 Attempt the following : **14**

(a) Show that a simple surface remains simple even after coordinate transformation.

(b) Prove in the usual notations the relation

$$g_{ij} = \Sigma\, g_{\alpha\beta} \frac{\partial v^{\alpha}}{\partial u^{i}} \frac{\partial v^{\beta}}{\partial u^{j}}.$$

9 Attempt the following : **14**

(a) Define Monge patch and compute coefficients of first and second fundamental form. Also find Christoffel symbols for the same.

(b) State and prove Frenet - Serret theorem.

10 Attempt the following : **14**

(a) Prove in the usual notations :

$$\Gamma_{ij}^{l} = \frac{1}{2}\Sigma_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^{j}} + \frac{\partial g_{kj}}{\partial u^{i}} - \frac{\partial g_{ij}}{\partial u^{k}}\right)$$

(b) Prove that: A necessary and sufficient condition for a curve to be a straight line is that the curvature $K = 0$.

Seat No. ________________

FQ-003-1163005

M. Sc. (Sem. III) Examination

November - 2022

Mathematics : EMT-3011

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) There are total Five questions.

(2) Each question carries equal marks.

(3) All the questions are compulsory.

1 Attempt any **seven** : **14**

1. Define : Regular curve and simple surface.
2. Define : Right circular helix.
3. Is the curve $\alpha(t) = (t^{15}, t^{35}, 5+5t^2)$ regular ? Justify your answer.
4. Define : Arc length of a curve.
5. Define : Monge patch.
6. Define : Proper co-ordinate patch.
7. Define : Normal curvature and Geodesic curvature.
8. Define : Reparametrization of a curve.
9. Fine the curvature and torsion of the circle $x^2 + y^2 = 49$.
10. Define : $\in$-neighbourhood.

2 Attempt the following :

(a) Find the arc length of the curve $\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$.

(b) Reparametrize the circle $\alpha(t)=(a\cos t, a\sin t, 0)$ by its arc length and also find its curvature and torsion (where $a>0$).

OR

(b) If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\to R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$. Also establish the relation between the tangent vector spaces S and T of β and α respectively.

3 Attempt the following : 14

(a) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, show that $k=\omega^2 r$ and $\tau=\omega^2 h$ (where $\omega=(r^2+h^2)^{-\frac{1}{2}}$).

OR

(a) Is the curve $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}}\right)$ unit speed curve ? If so compute its Frenet-Serret apparatus.

(b) Prove in the usual notations :

$$\Gamma_{ij}^{l}=\frac{1}{2}\sum_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial y^{j}}+\frac{\partial g_{kj}}{\partial u^{i}}-\frac{\partial g_{ij}}{\partial u^{k}}\right).$$

4 Attempt the following : 14

(a) Define binormal vector. Prove that the set $\{T, N, B\}$ is orthonormal. Sate and prove Frenet – Serret theorem.

(b) Prove that : A unit speed cure $\alpha(s)$ with $k\neq 0$ is a right circular helix iff there is a constant c such that $\tau=ck$.

5 Attempt any **two** : **14**

(a) If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x(f(a,b))$ is equal to the tangent plane to the simple surface y at $P = y(a,b)$.

(ii) The normal to the surface x at P is same as the normal to the surface y at P except possibly it may have the opposite sign.

(b) Find the coefficient of second fundamental form and Christoffel symbols for the Monge patch.

(c) Define sphere of radius r and centre m. If $\alpha(s)$ is a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and $r^2 = \rho^2 + (\rho' \sigma)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

(d) For a simple surface $x : u \to R^3$ prove that :

(i) $x_{ij} = L_{ij} n + \sum_k \Gamma_{ij}^k x_k$

(ii) For any unit speed curve $\gamma(S) = x\left(\gamma'(S), \gamma^2(S)\right)$,

$$k_n = \sum_{i,j} L_{ij} (\gamma^i)'(\gamma^i)' \text{ and}$$

$$k_g S = \sum_k \left[\left(\gamma^k\right)'' + \sum_{i,j} \Gamma_{ij}^{\ k} \left(\gamma^i\right)' \left(\gamma^j\right)' \right] x_k .$$

MZ-003-1163005 Seat No. ______

M. Sc. (Sem. III) Examination

December - 2020

EMT-3011 : Mathematics

(Differential Geometry)

(New Course)

Faculty Code : 003

Subject Code : 1163005

48

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 10 questions.

(2) Attempt any 5 questions.

(3) Figures to the right indicate full marks.

10

1 Attempt the following : **14**

2 (1) Define Regular Curve. Is the curve $a(t)=\left(t^{15}, t^{95}, 100t\right)$ regular ? Justify your answer.

(2) Define : Velocity vector and tangent vector field of a regular curve α.

(3) Define : Analytic function.

2 (4) Define : Arc length and curvature of a curve.

2 (5) Define : Unit Speed Curve.

2 (6) Define : Simple Surface.

2 (7) Define : Tangent space to a simple surface.

2 Attempt the following : **14**

(1) Define : Normal curvature and Geodesic curvature.

(2) Define : Tangent vector to a simple surface.

(3) Define : Monge patch.

(4) Define First fundamental form.

(5) Is the surface $x\left(u^1,u^2\right)=\left(u^1,u^2,u^1u^2\right)$ simple ? Justify your answer.

(6) Find g^{11} and g^{22} for the surface considered in above question no. (5).

(7) Obtain matrix g_{ij} and g for the surface $x(u,v)=(u^3, uv, v^3)$. 14

3 Attempt the following :

(a) Find the arc length of the helix $a(t)=(r=\cos t, r\sin t, ar\tan\alpha)$.

(b) Define : Reparametrization. If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $a:[a,b]\to R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$. 14

4 Attempt the following :

(a) Identify the cruve $\alpha(\theta)=(a\cos\theta, a\sin\theta, 0)$. Also (i) Find the curvature. (ii) Reparametrize the curve by its arc length (where a > 0).

(b) Show that $\alpha(s)=\frac{1}{2}\left(\cos^{-1}s, s\sqrt{1-s^2}, 1-s^2, 0\right)$ is a unit speed curve. Also find its torsion. 14

5 Attempt the following :

(a) Is the curve $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}}\right)$ unit speed curve ? If so, compute its Fenet-Serret appartus.

(b) State and prove Fenet-Serret theorem. 14

6 Attempt the following :

(a) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k\neq 0$. Also if $\tau\neq 0$ then $\alpha-m=-\rho N-\rho'\sigma\beta$ and $r^2=\rho^2+(\rho'\sigma)^2$ $\left(\text{where } \rho=\frac{1}{k} \text{ and } \sigma=\frac{1}{\tau}\right)$.

(b) Find curvature and torsion for the circular helix $\alpha(t) = (r\cos\omega s, r\sin\omega s, h\omega s)$, $\left(\text{where } \omega = (r^2 + h^2)^{-\frac{1}{2}}\right)$.

Also prove that : A unit speed curve $\alpha(s)$ with $k \neq 0$ is a right circular helix iff there is a constant c such that $\tau = CK$.

7 Attempt the following : 14

(a) If $x:u \to R^3$ is a simple surface and $f:v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x(f(a,b))$ is equal to the tangent plane to the simple surface at $P = y(a,b)$.

(ii) The normal to the surface x at P is same as the normal to the surface y at P except possibly it may have the opposite sign.

(b) Prove in the usual notations the relation $K^2 = K_n^2 + K_g^2$.

For the surface $x(r,s) = \left(r, s, \sqrt{1 - r^r - s^2}\right)$ show that $K_n = -1$ and $K_g = 0$.

8 Attempt the following : 14

(a) Prove that : The set of all tangent vectors to a simple surface $x:u \to R^3$ at P is a vector space. Also show that the dimension of this vector space is 2.

(b) For a simple surface $x:u \to R^3$ prove that :

(i) $\chi_{ij} = L_{ij}n + \sum_k \Gamma_{ij}^k \chi_k$

(ii) For any unit speed curve $\gamma(S) = x(\gamma'(S), \gamma^2(S))$,

$$k_n = \sum_{i,j} L_{ij}(\gamma^i)(\gamma^i) \text{ and}$$

$$k_g S = \sum_k \left[(\gamma^k) + \sum_{i,j} \Gamma_{ij}^k (\gamma^i)(\gamma^j)\right] \chi_k.$$

9 Attempt the following : 14

(a) Define first fundamental and second fundamental forms. Find the Christoffel symbols for the surface

$$x\left(u^1, u^2\right) = \left(u^1, u^2, f\left(u^1 u^2\right)\right).$$

(b) Prove in the usual notations the relation : $g = \breve{g}\det\left(g_{ij}\right)$

10 Attempt the following : 14

(a) For the Christoffel symbols of second kind show that :

$$\Gamma_{ij}^{l} = \frac{1}{2}\sum_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{ki}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k}\right)$$

(b) Let $f : X \rightarrow R^3$ be a simple surface and $f : v \rightarrow u$ is a co-ordinate transformation then prove that $y = X \circ f : v \rightarrow R^3$ is also a simple surface.

JBA-003-1163004 Seat No. ______

M. Sc. (Sem. III) Examination

December - 2019

EMT-3011 : Mathematics

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Attempt any **seven** : **14**

(1) Define : Regular curve and regular curve segment.

(2) Define : Proper co-ordinate patch.

(3) Is the curve a $\alpha(t) = \left(t^3, t^2, 100t\right)$ regular ? Justify your answer.

(4) Define : Arc length.

(5) Define : Unit speed curve.

(6) Define : The tangent space and the normal space.

(7) Define : Normal curvature and Geodesic curvature.

(8) Define: Simple surface.

(9) Define : Tangent vector to a simple surface.

(10) Define : Tangent vector field.

2 Attempt the following : **14**

(a) Define right circular helix and find the arc length of the helix $\alpha(t) = \left(a\cos t, a\sin t, at\tan\alpha\right)$.

(b) Define : Reparametrization. If $g:[c,d]\rightarrow[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\rightarrow R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$.

OR

(b) Reparametrize the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ by its arc length and also find its curvature (where $r>0$).

3 Attempt the following : **14**

(a) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, compute Frenet - Serret appartus.

$\left(\text{where } \omega=\left(r^2+h^2\right)^{-\frac{1}{2}}\right)$.

OR

(a) Show that $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3},\frac{(1-s)^{\frac{3}{2}}}{3},\frac{s}{\sqrt{2}}\right)$ is a unit speed curve and compute its Frenet Serret appartus.

(b) Show that the curve $\alpha(S)=\left(\frac{5}{13}\cos S,\frac{8}{13}-\sin S,-\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its Frenet – Serret appartus.

4 Attempt the following : **14**

(a) Prove that : The set of all tangent vectors to a simple surface $x:u\rightarrow R^3$ at P is a vector space.

(b) State and prove Frenet - Serret theorem.

5 Attempt any **two** : **14**

(a) If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x\left(f\left(a, b\right)\right)$ is equal to the tangent plane to the simple surface y at $P = y\left(a, b\right)$.

(ii) The normal to the surface x at P is same as the normal to the surface y at P except possibly it may have the opposite sign.

(b) Let α(s) be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $r \neq 0$ then $\alpha - m = \rho N - \rho' \sigma \beta$ and

$r^2 = \rho^2 + \left(\rho' \sigma\right)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

(c) Find the co-efficient of second fundamental form and Christoffel symbols for the surface

$x\left(u^1, u^2\right) = \left(u^1, u^2, f\left(u^1, u^2\right)\right)$.

(d) Find the curvature of the curves

(i) $2x - 3y + 5 = 0$

(ii) $x^2 + y^2 + 6x - 8y + 64 = 0$

PCG-003-1163005 Seat No. 35063

M. Sc. (Sem. III) (CBCS) Examination

December - 2018

EMT - 3011 : Mathematics

(Differential Geometry)

(New & Old Course)

Faculty Code : 003

Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) There are 5 questions.

(3) Figures on right side indicate full marks.

1 Attempt any seven : **14**

(1) Define with examples Functions of class k and a regular curve.

(2) Define with example : An open subset of R^2

(3) Find curvature of the circle $2x^2 + 2y^2 - 4x - 4y + 4 = 0$.

(4) Define : Reparametrization of a curve.

(5) Define : Regular curve segment.

(6) Define : Length of a regular curve segment.

(7) Define : Unit speed curve.

(8) Define : Normal curvature and Geodesic curvature.

(9) Define : Simple surface.

(10) Define : The tangent plane and the normal plane.

2 Attempt the following : 14

(a) Is the curve $\alpha(t)=(\sin 6t\cos t, \sin 6t\sin t, 0)$ regular? If so then find the equation of tangent line to a at $t=\frac{\pi}{3}$.

(b) Is the curve $\alpha(t)=\left(\sin t, \cos^2 t, \cos t\right)$ regular? If so then find the equation of tangent line at $t=\frac{\pi}{4}$.

OR

(a) If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\to R^3$ then prove that the length of α is equal to the length of $\beta=\alpha\circ g$. Also derive the relation between their tangent planes.

(b) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k\neq 0$. Also if $\iota\neq 0$ then $\alpha-m=-\rho N-\rho'\sigma\beta$ and $r^2=\rho^2+(\rho'\sigma)^2$ (where $\rho=\frac{1}{k}$ and $\sigma=\frac{1}{\iota}$.)

3 Attempt the following : 14

(a) Define the arc length of a curve and prove that the arc length is one – one function mapping (a,b) onto (c,d) and it is a reparametrization.

(b) (Find the arc length of the curve $\alpha(t)=(r\cos t, r\sin, 0)$ and reparametrize the curve by its arc length) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, compute Frenet–Serret appartus $\left(\text{where } \omega=\left(r^2+h^2\right)^{-\frac{1}{2}}\right)$.

OR

(b) Show that the length of the curve $\alpha(t)=\left(2a\left(\sin^{-1}t+t\sqrt{1-t^2}\right), 2at^2, 4at\right)$ between the points $t=t_1$ to $t=t_2$ is $4\alpha\sqrt{2}(t_2-t_1)$.

4 Attempt the following : 14

(a) State and prove Frenet–Serret theorem.

(b) Show that the curve

$$\alpha(S) = \left(\frac{5}{13}\cos S, \frac{8}{13} - \sin S, \frac{12}{13}\cos S.\right)$$ is a unit speed curve. Also compute its Frenet–Serret appartus.

5 Attempt any **two** : 14

(a) Let $f : X \to R^3$ be a simple surface and $f : v \to u$ is a co-ordinate transformation then prove that $y = X \circ f : v \to R^3$ is also a simple surface.

(b) For a simple surface $x : u \to R^3$ prove that

(i) $X_{ij} = L_{ij} n + \sum_k \Gamma_{ij}^k x_k$

(ii) For any unit speed curve $\gamma(S) = x\left(\gamma'(S), \gamma^2(S)\right)$,

$$k_n = \sum_{i.j} L_{ij} \left(\gamma^i\right)' \left(\gamma^i\right)'$$ and

$$k_g S = \sum_k \left[\left(\gamma^k\right)'' + \sum_{i.j} \Gamma_{ij}{}^k \left(\gamma^i\right)'' \left(\gamma^j\right)'\right] x_k.$$

(c) Prove that : The set of all tangent vectors to simple surface $x : u \to R^3$ at P is a vector space.

(d) Define Monge patch and compute coefficients of second fundamental form and Christoffel symbols for the same.

HEA-003-1163005 Seat No. 035028

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November/December – 2017

Differential Geometry : EMT-3011

(New Course)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are five questions.
(2) Attempt all the questions.
(3) Figures to the right indicate full marks.

1 Attempt any seven : 14

(1) Define : Regular curve.

(2) Define : Tangent vector field.

(3) Is the curve $\alpha(t) = (t^3, t^2, 2t)$ regular ? Justify your answer.

(4) Define : Arc length.

(5) Define : Unit speed curve.

(6) Define : The tangent space and the normal space.

(7) Define : Normal curvature and Geodesic curvature.

(8) Define : Simple surface.

(9) Define : Tangent vector to a simple surface.

(10) Define : Proper co-ordinate patch.

[Contd...

2 Attempt the following : **14**

(a) Define : Reparametrization. If $g:[c,d]\rightarrow[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\rightarrow R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$.

(b) Reparametrize the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ by its arc length and also find its curvature (where $r>0$).

OR

(b) Find the arc length of the helix $\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$.

3 Attempt the following : **14**

(a) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, compute Frenet - Serret appartus (where $\omega=\left(r^2+h^2\right)^{-\frac{1}{2}}$)

OR

(a) Show that $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}}\right)$ is a unit speed curve and compute its Frenet - Serret appartus.

(b) Show that $\alpha(s)=\frac{1}{2}\left(\cos^{-1}s - s\sqrt{1-s^2}, 1-s^2, 0\right)$ is a unit speed curve and compute its Frenet - Serret appartus.

4 Attempt the following : 14

(a) State and prove Frenet - Serret theorem.

(b) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and

$$r^2 = \rho^2 + (\rho' \sigma)^2 \text{ (where } \rho = \frac{1}{k} \text{ and } \sigma = \frac{1}{\tau}\text{)}.$$

5 Attempt any two : 14

(a) If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x(f(a,b))$ is equal to the tangent plane to the simple surface y at $P = y(a,b)$.

(ii) The normal to the surface x at p is same as the normal to the surface y at p except possibly it may have the opposite sign.

(b) Prove that : The set of all tangent vectors to a simple surface $x : u \to R^3$ at P is a vector space.

(c) Find the co-efficient of second fundamental form and Christoffel symbols for the surface $x(u^1, u^2) = (u^1, u^2, f(u^1, u^2))$.

[80/8]

MBT-003-016305 Seat No.

M. Sc. (Sem. III) (CBCS) Examination

December – 2016

EMT-3011 : Mathematics

(Differential Geometry)

Faculty Code : 003
Subject Code : 016305

Time : $2\frac{1}{2}$ Hours] [Total Marks 70

Instructions : (1) Attempt all the questions.
(2) Each question carries equal marks.

1 Attempt the following : (any seven)

(1) Define regular curve.

(2) Define unit speed curve.

(3) Define tangent vector field to a regular curve.

(4) Is the function $g:R\rightarrow R$ defined by $g(x)=x^3$ C^1 ? Justify your answer.

(5) Find the curvature of the curve $2x+3y-5=0$.

(6) Find the curvature of the curve $x^2+y^2=9$.

(7) What is the dimension of a tangent vector space ?

(8) Is the curve $\left(t^2+t, 1, 1\right)$ is regular ? Why ?

(9) Which parameter is measured by the quantity torsion of a curve ?

(10) Define osculating plane.

2 Attempt the following : (any two)

(a) Show that the curve

$\alpha(t)=(\sin 3t\cos t, \sin 3t\sin t, 0)$ is regular. Also find the equation of a tangent line at $t=\pi/3$.

(b) Show that the curve

$$\alpha(s)=\left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\right)$$ is a unit speed curve and compute its Frenet-Serret apparatus.

(c) Is the curve $\alpha(t)=(\cos t, 1-\cos t-\sin t, -\sin t)$ regular ? If yes, then find the equation of tangent at the point $t=\pi/4$.

3 Attempt the following :

(a) Show that the arc length of the curve

$$\alpha(t)=\left(2a\left(\sin^{-1}t+t\sqrt{1-t^2}\right), 2at^2, 4at\right)$$ between the points $t=t_1$ to $t=t_2$ is $4a\sqrt{2}(t_2-t_1)$.

(b) For the helix $\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$ show that the arc length is $\sec\alpha$.

OR

(a) Find the arc length of curve $\alpha=(3\cosh 2t, 3\sinh 2t, 6t)$ between $t=0$ to $t=\pi$.

(b) Reparametrize the curve $\alpha(t)=\left(e^t\cos t, e^t\sin t, e^t\right)$ by arc length.

4 Attempt the following :

(a) State and prove Frenet-Serret theorem.

(b) If image of a unit-speed curve $\alpha(s)$ lies on a surface of a sphere with radius r and centre m, then show that $k \neq 0$. Also show that if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and $r^2 = \rho^2 + (\rho' \sigma)^2$ where notations are being usual.

5 Attempt the following : (any two)

(a) Define Normal space and Normal curvature and prove that $k^2 = k_n^2 + k_g^2$.

(b) Prove that :

$$\Gamma_{ij}^{l} = \frac{1}{2} \sum_{k=1}^{2} g^{kl} \left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k} \right)$$

where notations are being usual.

(c) Prove that the set of all tangent vectors to a simple surface $x : u \to R^3$ is a vector space.

(d) Find the coefficients of second fundamental form and Christoffel symbols for Monge patch.

BBK-003-016305 Seat No. ________

M. Sc. (Maths) (Sem. III) (CBCS) Examination

December - 2015

Mathematics : EMT-3011

(Differential Geometry)

Faculty Code : 003
Subject Code : 016305

Time : 2:30 Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

(3) There are five questions.

1 Choose the appropriate alternative/alternatives :

(1) A curve $\alpha:(a,b)\to R^3$ is regular if

(A) $\frac{d\alpha}{dt}=0$ (B) $\frac{d\alpha}{dt}\neq 0$

(C) $\frac{d\alpha}{dt}<0$ (D) None of these

(2) If $\alpha:(a,b)\to R^3$ is a unit speed curve then

(A) $\left|\frac{d\alpha}{dt}\right|=1$ (B) $\left|\frac{d\alpha}{dt}\right|=0$

(C) $\left|\frac{d\alpha}{dt}\right|=2$ (D) None of these

(3) The curvature of the curve $\frac{x}{2}+\frac{y}{4}=1$ is

(A) 2 (B) 4

(C) 1 (D) 0

(4) The curvature of circle with centre at origin and radius 4 is

(A) 0 (B) 4

(C) 1/4 (D) None of these

(5) The dimension of a tangent vector space is

(A) 2 (B) 3

(C) 1 (D) 0

(6) Which of the followings is/are Frenet-Serret apparatus

(A) T (B) N

(C) B (D) N'

(7) Which of the following is/are not Frenet-Serret apparatus

(A) k (B) k'

(C) N' (D) B'

(8) Which of the following curves is/are regular ?

(A) $(t^3+t^2,0,0)$ (B) $(t^2+t,1,1)$

(C) $(t, 2, 1)$ (D) All of these

(9) A surface $x:u \to R^3$ is simple if

(A) $\partial x/\partial u' = 0$ (B) $\frac{\partial x}{\partial u^2}=0$

(C) $\frac{\partial x}{\partial u^1} \times \frac{\partial x}{\partial u^2} \neq 0$ (D) None of these

2 Attempt any two :

(a) Is the curve $\alpha(t)=(\cos t, 1-\cos t-\sin t, -\sin t)$ regular ? If yes, then find the equation of its tangent line at $t=\pi/4$.

(b) Define arc length of the curve and find the arc length of the curve

$$\alpha(t)=(r\cos t, r\sin t, ht) \text{ for } 0\le t\le 10$$

(c) Find the arc length of the curve

$$\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$$

3 Attempt the followings :

(a) Show that the curve

$\alpha(t)=(\sin 3t\cos t, \sin 3t\sin t, 0)$ is regular. Also find the equation of its tangent line at $t=\pi/3$.

(b) Define reparmetrization of a curve and reparametrize the curve

$\alpha(u)=(a\cos u, a\sin u, cu)$ (where $0\le u<\pi$) by $t=\tan u/2$.

OR

3 (a) Show that the curve

$$\alpha(s)=\left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, s/\sqrt{2}\right)$$ is a unit-speed curve.

Also compute its Frenet-Serret apparatus.

(b) Show that the curve

$$\alpha(s)=\frac{1}{\sqrt{5}}\left(\sqrt{1+s^2}, 2s, \log\left(s+\sqrt{1+s^2}\right)\right)$$ is a unit-speed curve. Also compute its Frenet-Serret apparatus.

4 Attempt the followings :

(a) State and prove Frenet-Serret theorem.

(b) If the image of a unit speed curve $\alpha(s)$ lies on a surface of a sphere with radius r and centre m then show that $k \neq 0$. Also show that if $z \neq 0$. Then $\alpha - m = -\rho N - \rho'\sigma B$. Hence deduce that $r^2 = \rho^2 + (\rho'\sigma)^2$ where notations are being usual.

5 Attempt any two :

(a) Define Monge patch and compute first fundamental forms for the same.

(b) For a co-ordinate patch $x : u \to R^3$ with metric coefficients g_{ij} then prove that

$$\Gamma_{ij}^{l} = \frac{1}{2}\sum_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k}\right)$$

(c) Define normal space and normal curvature. Also prove that $k^2 = k_n^2 + k_g^2$ where notations are begin usual.

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003-016305

M.Sc. (Maths) (CBCS) Sem.-III Examination

December-2014

EMT-3011 : Differential Geometry

Faculty Code : 003

Subject Code : 016305

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

1. Choose the appropriate alternative/alternatives :

(1) The curvature of the curve $\frac{x}{a}+\frac{y}{b}=1$ is

(a) 1 (b) 2

(c) 0 (d) –1

(2) The curvature of the curve $x^2+y^2-8x-6y+9=0$ is

(a) 9 (b) 2

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(3) A curve $\alpha : (a, b) \to R^3$ is regular if

(a) $\frac{d\alpha}{dt}=0$ (b) $\frac{d\alpha}{dt}\neq 0$

(c) $\frac{d\alpha}{dt}=1$ (d) None of these

(4) If B is binormal to curve α then

(a) B = T/N (b) B = TN

(c) B = T × N (d) B = 0

(5) The torsion of a curve measures

(a) twist (b) curvature

(c) arc length (d) none of these

(6) Which of the following is/are Frenet-Serret apparatus ?

(a) T

(b) N

(c) N'

(d) B'

(7) Which of the following is/are not regular curves ?

(a) $(t^3, 0, 0)$.

(b) $(t^5, 1, 1)$

(c) $(t, 2, 1)$

(d) $(t^3 + 2t, 1, t)$

(8) A surface $x : u \to R^3$ is simple if

(a) $\frac{\partial x}{\partial u^1} = 0$

(b) $\frac{\partial x}{\partial u^1} \times \frac{\partial x}{\partial u^2} = 0$

(c) $\frac{\partial x}{\partial u^2} = 0$

(d) None of these

(9) The Christoffel symbols are

(a) Symmetric and measure tangential components

(b) Antisymmetric and measure tangential components

(c) Symmetric and measure normal components

(d) Antisymmetric and measure normal components

(10) For any circle $x^2 + y^2 = r^2$ larger the curvature

(a) larger the radius

(b) smaller the radius

(c) radius = curvature

(d) None of these

2. Attempt any two :

(a) Show that the curve $\alpha(t) = (\sin 3t \cos t, \sin 3t \sin t, 0)$ is regular. Also find the equation of tangent line at $t = \frac{\pi}{3}$.

(b) Is the curve $\alpha(t) = (\cos t, 1 - \cos t - \sin t, -\sin t)$ regular ? If yes, then find the equation of tangent line at $t = \frac{\pi}{4}$.

(c) Define arc length of the curve $\alpha(t) = (2a(\sin^{-1}t + t\sqrt{1 - t^2}), 2at^2, 4at)$ between the points $t = t_1$ to $t = t_2$ is $4a\sqrt{2}(t_2 - t_1)$.

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3 Attempt the following :

(a) Show that the curve $\alpha(s) = \left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\right)$ is a unit-speed curve. Also compute its Frenet-Serret apparatus.

(b) Show that the curve $\alpha(s) = \frac{1}{\sqrt{5}}(\sqrt{1+s^2}, 2s, \ln(s+\sqrt{1+s^2}))$ is a unit-speed curve and compute its Frenet-Serret apparatus.

OR

(a) State and prove Frenet-Serret theorem.

(b) Define Reparametrization of a curve and reparametrize the curve $\alpha(u) = (a\cos u, a\sin u, cu)$ (where $0 \le u < \pi$) by $t = \tan\frac{u}{2}$.

4. Attempt any two :

(a) Define :

(i) Osculating plane

(ii) Normal plane

(iii) Rectifying plane

Also prove that a unit speed curve $\alpha(s)$ with $k \neq 0$ is a helix iff there is a constant c such that $\tau = ck$.

(b) If image of a unit-speed curve $\alpha(s)$ lies on a surface of a sphere with radius r and centre m, then show that $k \neq 0$. Also show that if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho'\sigma B$ and $r^2 = \rho^2 + (\rho'\sigma)^2$ (where notations are being usual)

(c) Find the coefficients of second fundamental form and christoffel symbols for Monge patch.

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5. Attempt any two :

(a) Define normal space and normal curvature. Also prove that $k^2 = kn^2 + kg^2$.

(b) Prove that : $\Gamma^{l}_{ij} = \frac{1}{2} \sum_{k=1}^{2} g^{kl} \left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k} \right)$, where notations are being usual.

(c) Prove in the usual notations :

(i) $x_{ij} = L_{ij}\, n + \sum_{k} \Gamma^{k}_{ij}\, x_k$

(ii) $kN = k_n n + kg\, S$

(d) Prove that the set of all tangent vectors to a simple surface $x : u \rightarrow \mathbb{R}^3$ is a vector space.

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003-016305

M.Sc. (Maths) (CBCS) - (Sem.-III) Examination

November-2013

Differential Geometry

EMT - 3011

Faculty Code : 003

Subject Code : 016305

Time : 2 ½ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

1. Choose the appropriate alternative/alternatives (any seven) :

(1) The curvature of the curve $x^2 + y^2 = 16$ is

(a) 16 (b) 4

(c) 2 (d) 1/4

(2) The curvature of the curve $2x - 4y = 16$ is

(a) 16 (b) 2

(c) 1/2 (d) 0

(3) The rate of change of tangent vector is zero then

(a) k = 0 (b) k ≠ 0

(c) k = 2 (d) None of these

(4) A curve α : (a, b) → R^3 is regular if

(a) $\frac{d\alpha}{dt} = 1$ (b) $\frac{d\alpha}{dt} \neq 0$

(c) $\frac{d\alpha}{dt} = 0$ (d) None of these

(5) The dimension of tangent vector space

(a) 0 (b) 2

(c) −1 (d) 3

(6) If B is bi-normal to curve α then

(a) $B = 0$ (b) $B = T \times N$

(c) $B = TN$ (d) $B = T/N$

(7) The torsion of a curve measures

(a) twist of a curve (b) bending of a curve

(c) speed of a curve (d) none of these

(8) Which of the following is/are regular curve(s) ?

(a) $(t^3 + t, 1, 1)$ (b) $(2t^2, 0, 0)$

(c) $(t^3 + 2t, 2t, t)$ (d) $(2t, 0, 0)$

(9) Which of the following is (are) Frenet-Serret apparatus ?

(a) k (b) T'

(c) B' (d) N'

(10) A surface $x : u \to R^3$ is simple if

(a) $\frac{\partial x}{\partial u^1} \times \frac{\partial x}{\partial u^2} = 0$ (b) $\frac{\partial x}{\partial u^1} \times \frac{\partial x}{\partial u^2} \neq 0$

(c) $\frac{\partial x}{\partial u^1} = \frac{\partial x}{\partial u^2}$ (d) None of these

2. Attempt any two :

(a) Define :

(i) Function of Class k.

(ii) Reparametrization of curve.

Also show that the curve

$\alpha(t) = (\sin 3t \cos t, \sin 3t \sin t, 0)$ is regular and find the equation of tangent line to α at $t = \pi/3$.

(b) Is the curve $\alpha(t) = (\cos t, \cos^2 t, \sin t)$ regular ? If yes then find the equation of its tangent line at $t = \pi/4$.

(c) If $g: [c, d] \to [a, b]$ is a reparametrization of a curve segment $\alpha : [a, b] \to R^3$ then prove that length of α is equal to the length of $\beta = \alpha \circ g$.

3. Attempt the followings :

(a) Find the arc length of the curve
$\alpha(t) = (3 \cosh 2t, 3 \sinh 2t, 6t)$ from $t = 0$ to $t = \pi$.

(b) Show that the arc length of the curve $\alpha(t) = (a \cos t, a \sin t, at \tan \alpha)$ is at $\sec \alpha$.

OR

Attempt the following :

(a) Define : (1) Osculating plane
(2) Normal plane

Also show that

$$\alpha(S) = \left(\frac{5}{13}\cos S, \frac{8}{13} - \sin S, \frac{-12}{13}\cos S\right)$$

is a unit-speed curve and compute its Frenet-Serret apparatus.

(b) Prove that : A unit speed curve $\alpha(S)$ with $k \neq 0$ is a helix iff there is a constant C such that $\tau = ck$.

$\tau = c \cdot k$

4. Attempt the following (any two) :

(a) State and prove Frenet-Serret theorem.

(b) If the image of a unit speed curve $\alpha(S)$ lies on a surface of a sphere with radius r and centre m, then show that $k \neq 0$. Also show that if $C \neq 0$ then $\alpha\text{-}m = -\rho N - \rho' \sigma B$. Hence $r^2 = \rho^2 + (\rho'\sigma)^2$ where rotations are being usual.

(c) Define Monge patch and compute first fundamental forms for the same.

P.T.O.

5. Attempt any two :

(a) Define normal space and normal curvature. Also prove that $k^2 = k_n^2 + k_g^2$ where notations are being usual.

(b) For a simple surface $x : u \to R^3$ and a unit speed curve $\gamma(s) = x(\gamma^1(s), \gamma^2(s))$ show that

(1) $x_{ij} = L_{ij}\, n + \sum_k \Gamma^k_{ij}\, x_k$

(2) $k_n = \sum_{i,j} L_{ij} (\gamma^i)' (\gamma^j)'$

(3) $k_g S = \sum_k \left((\gamma^k)'' + \sum_{i,j} \Gamma^k_{ij} (\gamma^i)' (\gamma^j)' \right) x_k$

where notations are being usual.

(c) Find geodesic, curvature and normal curvature for the upper hemisphere.

(d) Find the coefficients of second fundamental forms and Christoffel symbols for monge patch.

J-3

003-016305

M.Sc. (CBCS) (Sem. III) Examination
December-2012
EMT-3011 : Differential Geometry
(Mathematics)

Faculty Code : 003
Subject Code : 016305

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.
(2) Each question carries equal marks.

1. Choose appropriate alternative/alternatives (any seven) :

(1) A curve $\alpha : (a, b) \rightarrow R^3$ is regular if

(a) $\frac{d\alpha}{dt} = 0$ (b) $\frac{d\alpha}{dt} = 1$

(c) $\frac{d\alpha}{dt} \neq 0$ (d) None of these

(2) The curvature of $2x + 3y = 0$ is

(a) $-2/3$ (b) $-3/2$

(c) 0 (d) 6

(3) The rate of change of tangent vector is zero, then

(a) $k = 0$ (b) $k \neq 0$

(c) $k = 1$ (d) None of these

(4) For any circle $x^2/y^2 = r^2$ smaller the radius

(a) curvature is 0

(b) curvature is half of r

(c) larger the curvature

(d) none of these

(5) Which of the following is/are not the Frenet-Serret apparatus ?

(a) k

(b) τ

(c) N

(d) T

(6) Which of the following is/are not regular curves ?

(a) $(t^3, 0, 0)$

(b) $(2t^2, 1, 1)$

(c) $(2t, t, 3t)$

(d) $(t^2 + 1, 1, 0)$

(7) If B is binormal to any curve α then

(a) $B = 0$

(b) $B = 1$

(c) $B = T.N$

(d) $B = T \times N$

(8) The dimension of tangent vector space is

(a) 3

(b) -1

(c) 2

(d) 0

(9) The torsion of a curve measures

(a) twist of a curve

(b) bending of a curve

(c) expansion of a curve

(d) none of these

(10) The curvature of a geodesic on a simple surface M is

(a) 0

(b) 2

(c) -2

(d) 1

2. Attempt any two :

(a) Show that $\alpha(t) = (\sin 3t \cos t, \sin 3t \sin t, 0)$ is regular. Also find the equation of tangent line at $t = \pi/3$.

(b) Is the curve $\alpha(t) = (\cos t, 1 - \cos t - \sin t, -\sin t)$ regular ? If yes, then find the equation of tangent line at $t = \pi/4$.

(c) Reparametrize the curve $\alpha(u) = (a \cos u, a \sin u, cu)$ where $0 \le u \le \pi$ by $t = \tan u/2$.

3. Answer the following :

(a) Find the arc length of the curve as well as reparametrize the curve by arc length where curve $\alpha(t) = (r \cos t, r \sin t, ht)$ and $0 \le t \le 10$.

(b) Show that the length of the curve $\alpha(t) = (2a(\sin^{-1} t + t\sqrt{1-t^2}), 2at^2, 4at)$ is $4a\sqrt{2}(t_2 - t_1)$ between the points $t = t_1$ to $t = t_2$.

(c) Obtain Frenet-Serret apparatus for the curve $\alpha(s) = (r \cos(s/r), r \sin(s/r), 0)$.

4. Attempt the following :

(a) Show that $\alpha(s) = \frac{1}{\sqrt{5}}\left(\sqrt{1+s^2}, 2s, \log\left(s + \sqrt{1+s^2}\right)\right)$ is a unit speed curve. Also compute its Frenet-Serret apparatus.

(b) Define :

(1) Osculating plane

(2) Normal plane

(3) Rectifying plane

Also prove that a unit speed curve $\alpha(s)$ with $k \neq 0$ is a helix iff there is a constant c such that $\tau = ck$.

OR

(a) State and prove Frenet-Serret theorem.

(b) If image of a unit speed curve $\alpha(s)$ lies on a surface of a sphere with radius r and centre m, then show that $k \neq 0$. Also show that if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho'\sigma\beta$. Hence $r^2 = \rho^2 + (\rho'\sigma)^2$. (where notations are being usual).

5. Attempt any two :

(a) Define Monge patch and compute first fundamental forms for the same.

(b) Define normal space and normal curvature. Also prove that $k^2 = k_n^2 + k_g^2$ where notations are being usual.

(c) Prove that :

If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation, then show that the tangent plane to the simple surface x at $p = x(f(a, b))$ is equal to the tangent plane to the simple surface $y = x \circ f$ at $p = y(a, b)$.

(d) Let $x : u \to R^3$ be a simple surface and $r(s) = x(r^1(s), r^2(s))$ be any unit speed curve then prove that

(1) $x_{ij} = L_{ij}n + \sum_k \Gamma^k_{ij} x_k$

and

(2) $k_n = \sum_{i,j} L_{ij}(r^i)'(r^j)'$

and

(3) $k_g s = \sum_k \left[(r^k)'' + \sum_{i,j} \Gamma^k_{ij} (r^i)'(r^j)' \right] x_k$

$2^2 \cdot 3^3$
(4)(27)
108
1, 108, 2, 3, 36, 54
4, 27, 18, 6, 9, 1

003-016305 Seat No. 33803

M.Sc. (Maths) (Sem. - III) (CBCS) Examination

December - 2011

EMT-3011 : Differential Geometry

Faculty Code : 003

Subject Code : 016305

Time : 2.30 Hours] [Total Marks : 70

Instruction : (1) Attempt all the questions.

(2) Each question carries equal marks.

(3) There are 5 questions.

1 Choose the appropriate alternatives. (any seven)

(1) A real valued function f is said to be of class K over real interval I if

(A) it is infinitely many integrable

(B) it is not integrable.

(C) its k^{th} derivative exists at each point of I and this derivative is continuous.

(D) it is exponential function.

(2) A vector valued function $R=(x,y,z)$ is said to be of c^k

(A) if x is continuous

(B) if x and z is continuous

(C) if each of its components are of c^k

(D) none of these

(3) A curve $\alpha:(a,b)\to R^3$ is regular if

(A) it is integrable

(B) it is of class k and $\frac{d\alpha}{dt}\neq 0$

(C) it is not continuous

(D) none of these

(4) The curvature of a unit speed curve $\alpha(s)$ is defined as

(A) $k(s)=|T'(s)|$ (B) $k(s)=|N'(s)|$

(C) $k(s)=|T(s)\times T'(s)|$ (D) $k(s)=T^2$

(5) Curvature will measure

(A) moment of curve (B) period of curve

(C) arc length of curve (D) bending of curve

(6) Which of the followings is not the Frenet-Serret apparatus ?

(A) $k(s)$ (B) $\tau(s)$

(C) $T'(s)$ (D) $B(s)$

(7) Vectors u and v are orthogonal to each other if

(A) $u\times v=0$ (B) $\langle u,v\rangle=0$

(C) $\langle u\times v\rangle\neq 0$ (D) $u\times v=\langle u,v\rangle=-\langle v,u\rangle$

(8) Which of the following is a Frennet-Serret equation ?

(A) $T'(s)=K(s)N(s)$ (B) $T(s)=K(s)/N(s)$

(C) $N(s)=K(s)B(s)$ (D) $\left|\frac{dT}{dS}\right|\neq 0$

(9) The curvature of any straight line is

(A) 2 (B) 0

(C) ∞ (D) 1

(10) Which of the following notations is used to denote the geodesic curvature ?

(A) K_n (B) K_g

(C) L_{ij} (D) K.

2 Attempt any two.

(a) Define regular curve and show that the curve $\alpha(t)=(\sin 3t\cos t,\sin 3t\sin t,0)$ is regular. Also find the equation of tangent line to α at $t=\pi/3$.

(b) Define reparametrization of a curve and reparametrize the curve $\alpha(u)=(a\cos u, a\sin u, 2u)$ by $t=\tan\left(\frac{u}{2}\right)$

(c) Define a regular curve segment and length of a regular curve segment moreover reparametrize the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ by arc length.

3 Attempt the followings :

(a) Find the arc length of the following curves :

(i) $\alpha(t)=(r\cos t, r\sin t, ht)$ for $0\le t\le 10$

(ii) $\alpha(t)=(2\cosh 3t, -2\sinh 3t, 6t)$ for $0\le t\le 5$.

(b) Show that the arc length of the helix $\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$ is $\sec\alpha$.

OR

3 Attempt the followings :

(a) Let $g:[c,d]\to[a,b]$ be a reparametrization of a curve segment $\alpha:[a,b]\to R^3$ then the length of α is equal to the length of $\beta=\alpha o g$.

(b) A unit speed curve $\alpha(s)$ with $k\ne 0$ is a helix iff there is a constant c such that $\tau=ck$.

4 Attempt any two.

(a) For the unit speed curve α prove that

(i) $T'(s)=k(s)N(s)$

(ii) $N'(s)=-k(s)T(s)+\tau(s)B(s)$

(iii) $B'(s)=-\tau(s)N(s)$

where notations are being usual.

(b) Compute Frenet-Serret apparatus for the curve

$$\alpha(s)=\left(\frac{5}{13}\cos S, \frac{8}{13}-\sin S, \frac{-12}{13}\cos S\right)$$

(c) Define :

(i) C^k co-ordinate patch.

(ii) Monge patch.

Moreover let $U = \{(u^1, u^2) \in R^2 / (u^1)^2 + (u^2)^2 < 1\}$ and $X(u^1, u^2) = \left(u^1, u^2, \sqrt{1-(u^1)^2-(u^2)^2}\right)$ then find unit normal and equation of tangent plane at $X\left(\frac{1}{2}, \frac{1}{2}\right)$.

5 Attempt any two :

(a) Find the coefficients of second fundamental form and Christoffel symbols for the surface $x(u^1, u^2) = (u^1, u^2, f(u^1, u^2))$.

(b) Show that for a co-ordinate patch $x : U \to R^3$ with metric coefficients g_{ij} $\Gamma^l_{ij} = \frac{1}{2}\sum_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k}\right)$.

(c) Prove in the usual notations the relation $k^2 = k_n^2 + k_g^2$.