Shree H.N.Shukla College of Science Rajkot

MATHEMATICS

S.Y.B.Sc. (Sem.IV) (CBCS)

PRELIMS EXAM

PAPER- 401

Linear Algebra, Real Analysis & Differential Geometry

Time: 2.5 hour] **[Total Marks: 70**

Instruction: (i) All questions are compulsory.

(ii) Figures to the right indicate full marks of the question.

1. (A) Answer the following: **[04]**

1) Define: monotonic sequence
2) State Cauchy's General Principle of Convergence.
3) Define: Subsequence of a sequence
4) Give an example of a sequence which is lower bounded.

(B) Attempt any one: **[02]**

1) Show that every convergent sequence is bounded.
2) Determine the sequence is convergent or not:

$$\left\{\frac{5n+(-1)^n}{2n+5}\right\}$$

(C) Attempt any one: **[03]**

1) Show that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{2}$ & $S_{n+1} = \sqrt{2S_n}$ converges to 2.
2) Show that

$$\lim_{n\to\infty} \frac{1+\frac{1}{3}+\frac{1}{5}+ \ldots\ldots\ldots\ldots + \frac{1}{2n-1}}{n} = 0$$

(D) Attempt any one: **[05]**

1) State and prove Cauchy's first theorem on limits.
2) Show that the sequence {S_n} defined by $S_1 = 1$ & $S_{n+1} = \frac{4+3S_n}{3+2S_n}$, $\forall n \in N$ is convergent and find its limit.

2. (A) Answer the following: **[04]**

1) If the D'Alemert's ratio test fails, then what to do?
2) Narrate the Libnitz test for convergence of an alternating series.
3) Write the condition for convergence of $1 + r + r^2 + r^3 + \dots$
4) Define: Oscillatory series

(B) Attempt any one: **[02]**

1) Show that the series $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \frac{5}{26} - \dots\dots$ is absolutely convergent.
2) Test the convergence of ;

$$\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$$

(C) Attempt any one: **[03]**

1) Test the convergence or divergence of the series:

$$\frac{1}{4*6} + \frac{\sqrt{3}}{6*8} + \frac{\sqrt{5}}{8*10} + \frac{\sqrt{7}}{10*12} + \dots\dots\dots.$$

2) Find the radius and interval of convergence of the series

$$\sum_{\text{n}=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

(D) Attempt any one: **[05]**

1) Discuss the convergence of

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots\dots\dots.$$

2) Discuss the convergence of

$$\sum \frac{1^2 * 4^2 * 7^2 * \ldots\ldots\ldots\ldots(3n-2)^2}{3^2 * 6^2 * 9^2 * \ldots\ldots\ldots\ldots(3n)^2}$$

3. (A) Answer the following: [04]

1) Define: Kernel of a linear transformation
2) Define: Linear transformation
3) Define: Idempotent linear transformation
4) Let T: U$\rightarrow$V is a linear transformation. Let θ and θ' be zero vectors for U and V respectively, Prove that T(θ)=θ'

(B) Attempt any one: [02]

1) Define: (i) Range of a linear transformation
   (ii) Non-singular linear transformation
2) Let T: $R^2 \rightarrow R^2$, T(x, y) = (x+1, y-2); $\forall$ (x, y) $\in R^2$, then show that T is not a linear transformation.

(C) Attempt any one: [03]

1) Find the linear transformation T: $R^3 \rightarrow R^2$ such that T(e_1)=(1, 1), T(e_1+e_2)=(1, 0), T($e_1+e_2+e_3$)=(1, -1). Also find T (2, 5, 7), where {e_1, e_1+e_2, $e_1+e_2+e_3$} is a basis of R^3.
2) Let T: V$\rightarrow$V be any linear transformation such that T^2-T+I=0 then prove that T is non-singular.

(D) Attempt any one: [05]

1) Prove that L (U, V) is a vector space over F=R with respect to addition & scalar multiplication of linear transformation, where L (U, V)=the set of all linear transformations from U to V.
2) State and prove Rank-nullity theorem.

4. (A) Answer the following: [04]

1) Define: Dual of a vector space

2) If dimU=m, dimV=n then what is the dimL (U, V)?
3) Define: Eigen value of a linear transformation
4) Define: Diagonalization of a linear transformation

(B) Attempt any one: [02]

1) Let T: $R^2 \rightarrow R^2$, T(x,y)=(x, -y), $\forall$ (x,y)ϵR^2 and B_1={(1, 1), (1, 0)} and B_2= {(2, 3), (4, 5)}. Then find [T; B_1, B_2].
2) Define: Matrix associated with a linear transformation

(C) Attempt any one: [03]

1) Find Eigen values of the linear transformation T:$R^3 \rightarrow R^3$;
T (a, b, c) = (a+b+c, a+b+c, a+b+c).
2) Linear transformation T: P_2(R)$\rightarrow P_3$(R) is defined by $T\big(p(x)\big) = \int_o^x p(x)dx$.
B_1 = {1, x, x^2} and B_2 = {1, x, x^2, x^3} are bases of P_2(R) and P_3(R) respectively.
Find [T: B_1, B_2].

(D) Attempt any one: [05]

1) Find the Eigen value and Eigen vector for the linear transformation
T: $R^3 \rightarrow R^3$, T(x, y, z) = (-2y-2z, -2x-3y-2z, 3x+6y+5z), $\forall$ (x, y, z) ϵ R^3 by considering the standard basis of R^3.
2) Let T: V$\rightarrow$V be a linear transformation and let B be any basis of V. Then T is singular if and only if det ([T; B]) =0.

5. (A) Answer the following: [04]

1) Define: Multiple point of a given curve
2) Write the formula to find radius of curvature of the curve given by $r = f(\theta)$.
3) Find the radius of the curvature of the curve
$s = 4a\sin\psi$
4) Define: Point of inflexion

(B) Attempt any one: [02]

1) For the curve
   $(x^2 + y^2)x - ay^2 = 0$,
   Prove that the origin is cusp.
2) Prove that $y = \log x$ is convex upward everywhere.

(C) Attempt any one: [03]

1) Show that the parabola $y^2 = 4ax$ has no asymptotes.
2) Find the radius of curvature at origin for the curve
   $x^3 + y^3 = 3axy$ using Newton's method.

(D) Attempt any one: [05]

1) Discuss double points of the curve
   $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$
2) Show that the radius of curvature of any point on the cardiod $r = a(1 + \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$. Hence prove that $\frac{\rho^2}{r}$ is constant.

****BEST OF LUCK****