M.Sc.
Sem-2
April-2025

Seat No.

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 2 APRIL 2025 (Regular) APRIL - 2025

TOPOLOGY 2

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-02-00003

Time : 2:30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 **Define Hausdroff space with example.**

2 **Prove that every complete regular space is regular.**

3 **State Urysohn's lemma.**

4 **Define open cover.**

5 **Define finite intersection property (F.I.P.).**

6 **Define: Limit point compact space.**

7 **Prove that every compact space is locally compact space.**

8 **Define: Compactification.**

9 Let (X,d) be a metric space and $(x_n) \to x$ then show that $(x_n)_{n\in\mathbb{N}}$ is Cauchy sequence in (X,d).

10 **Prove that Q is not complete metric space.**

Q.2 Answer the following (Any Two) 14

1 If d is a metric on X then show that $(X, \tau(d))$ is Hausdroff space.

2 **Prove that A space X is T_1 iff every singleton subset of X are closed.**

3 Prove that a space X is compact iff whenever $\mu = \{c_\alpha | \alpha \in I\}$ is a family of closed subsets of X having finite intersection property then $\cap\{c_\alpha\} \neq \phi, \forall \alpha \in I$.

Q.3 Answer the following 14

1 **State and prove Lebesgue covering lemma.**

2 **Prove that closed subspace of normal space is normal.**

OR

Answer the following 14

1 **State and prove tube lemma.**

2 **Prove that continuous image of compact space is compact.**

Q.4 Answer the following questions (Any Two) 14

1 **Prove that compact subspace of Hausdroff space is closed.**

2 **State and prove Heine-Borel theorem.**

Q.5 Answer the following (Any Two)

1 **Prove that X and Y are regular spaces iff $X \times Y$ is regular space.**

2 **Prove that if X is compact then X is limit point compact. Is converse true? Explain.**

Define one point compactification (X^*,τ^*). Prove that (X^*,τ^*) is compact and T_2 space.

Prove that $(\mathbb{R}, d)$ is complete metric space.