Seat No. ______

M.Sc Sem-2 April 2025

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 2 APRIL 2025 (Regular) APRIL - 2025

COMPLEX ANALYSIS

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-02-00002

Time : 2.30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) **14**

1. Define Chordal metric on $\mathbb{C}_\infty$.
2. Define: Fixed point. If $\lambda \in \mathbb{C}, \lambda \neq 0, \lambda \neq 1$ then find the fixed points of the bilinear transformation S defined by $S_z = \lambda z$.
3. Define the right side and left side of the circle Γ in $\mathbb{C}_\infty$ with respect to an orientation of Γ.
4. State: Riemann-Stieltje's theorem.
5. Define the following terms: (a) Function of bounded variation. (b) Total variation of a function.
6. If $\gamma: [0,1] \to \mathbb{C}$ is defined by $\gamma(t) = a + re^{2\pi int}, \forall t \in [0,1]$, for some $a \in \mathbb{C}$, $r > 0$ and $n \in \mathbb{Z}$ then find $V(\gamma)$.
7. Give the statement: Minimum modulus theorem.
8. Define: Rectifiable path and length of rectifiable path.
9. If $f: \mathbb{C} \to \mathbb{C}$ is an entire function and $f\left(\frac{1}{n}\right) = 0$, $\forall n \in \mathbb{N}$ then prove that $f \equiv 0$ on $\mathbb{C}$.
10. Give the statement: Cauchy's integral formula of second version.

Q.2 Answer the following (Any Two) **14**

1. Find the bilinear transformations taking
   (i) $1 \to i, 0 \to -i, \ -1 \to 0$
   (ii) $i \to 1, 0 \to \infty, \ -i \to 0$.
2. Given two circles Γ_1 and Γ_2 in $\mathbb{C}_\infty$. Given two circles Γ_1 and Γ_2 in $\mathbb{C}_\infty$ and distinct $z_2, z_3, z_4 \in \Gamma_1$, distinct $w_2, w_3, w_4 \in \Gamma_2$ then prove that $\exists$ a unique bilinear transformation S such that $S(\Gamma_1) = \Gamma_2$ and $S(z_j) = w_j$; $\forall j = 2,3,4$.

Prove that for an analytic function $f: G \to \mathbb{C}$; where G be an open connected subset of $\mathbb{C}$ and

$G^* = \{\bar{z} / z \in G\}$ then $f^*: G^* \to \mathbb{C}$ defined by $f^*(z) = \overline{f(\bar{z})}$; $\forall z \in G^*$ is analytic.

Q.3 Answer the following 14

Let $a \in \mathbb{C}, R > 0$, $f: B(a,R) \to \mathbb{C}$ be analytic and $|f(z)| \le M$, $\forall z \in B(a,R)$ for some $M > 0$ .Then prove that $|f^n(a)| \le \frac{n! M}{R^n}, \forall n = 0,1,2,\ldots$

If $\gamma: [a,b] \to \mathbb{C}$ is a rectifiable path and $f: \{\gamma\} \to \mathbb{C}$ is continuous then prove that

$$\left| \int_\gamma f \right| \le \int_\gamma |f|\,|dz| \le V(\gamma) \cdot \sup_{z \in \{\gamma\}} |f(z)|.$$

OR

Answer the following 14

State without proof Cauchy's theorem for an open disc and find $\int_\sigma \frac{dz}{z^2 - 1}$; where $\sigma(t) = 1 + e^{it}$,

$\forall t \in [0, 2\pi]$.

Prove that, $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$, $\forall z \in \mathbb{C}$, $|z| < 1$.

Q.4 Answer the following questions (Any Two) 14

1 By using Cauchy's theorem 1st version prove Cauchy's integral formula 2nd version.

2 State and prove, Maximum modulus Theorem.

Q.5 Answer the following (Any Two) 14

Find $\int_\gamma \frac{1}{z} dz$, where $\gamma(t) = [1 - i, 1 + i, -1 + i, 1 - i]$.

Prove that every piecewise smooth path $\gamma: [a,b] \to \mathbb{C}$ is a function of bounded variation and $V(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

State and prove, Identity Theorem.

Evaluate: $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$; where $n \in \mathbb{N}$ and $\gamma(t) = e^{it}, \forall t \in [0, 2\pi]$.