SHREE H. N. SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics) Semester-4

IMP Questions of Integration Theory

1. State and prove Hahn decomposition theorem.

2. Define Jordan decomposition of a signed measure on a measurable space and prove that it is unique.

   If μ_1, μ_2 are two measures on a measurable space $(X, \mathscr{A})$ and at least one of them is finite then prove that $\mu_1 - \mu_2$ is a signed measure on $(X, \mathscr{A})$.

3. Prove that if (X, A) is a measurable space and $f : X \to [0, \infty]$ be measurable then there exists a sequence $\{S_n\}_{n=1}^{\infty}$ of simple measurable function such that

   (i) $0 \le S_1 \le S_2 \le \cdots .... \le S_n ...... \le f$; on X.

   (ii) $\lim_{n\to\infty} S_n = f(x); \forall x \in X.$

   If X is a locally compact separable metric space then prove that $Bo(X) = Ba(X)$.

4. Define Baire measure on the real line. Prove that the cumulative distribution function "F" of a finite signed measure on the real line is bdd, monotonically increasing and $\lim_{x\to-\infty} f(x) = 0$.

5. Prove that if X be a countable set and μ be the counting measure then $L^P(\mu) \cong l^P; \forall\, 1 \le P < \infty.$

6. Define σ-compact set in a locally compact Hausdorff space. Prove that every σ-compact open set in a locally compact Hausdorff space is a Baire set.

7. Give an example of baire measure on a locally compact Hausdorff space which is not regular. Justify.

8. State, without proof, Caratheodary extension theorem. Give an example to show that σ–finite assumption in the theorem cannot be dropped.

9. State, without proof, Fubim's theorem and Tonelli's theorem.

10. If μ, γ are measures on a measurable space then with usual notation prove that $\mu << \gamma$ and $\mu \perp \gamma \Rightarrow \mu = 0$. Does $\mu << \gamma \Rightarrow \gamma << \mu$? Justify.

11. If μ is a measure on an algebra $\mathscr{A}$ of subsets of a set X and μ^* is the outer measure on X induced by μ then prove that every $E \in \mathscr{A}$ is μ^*-measurable.

12. Give an example of a compact Hausdorff space X

$st\ Ba(X) \underset{\mp}{\subset} Bo(X)$.

13. State and prove Hahn Decomposition theorem.

14. State and prove Jordan Composition theorem.

15. State and prove Radon-Nikodyan theorem for signed measure.

16. Prove that Hahn Decomposition is unique except null sets.

17. Prove that countable union of positive sets is positive and countable union of negative sets is negative.

18. Define L^p – space. Prove Riesz-Fisher theorem for $L^p(\mu), 1 \le p < \infty$.