M – Sem-4 March, 2025.

Seat No.

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 4 MARCH 2025 (Regular) MARCH - 2025

GRAPH THEORY

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-04-00004

Time : 2:30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Draw a graph G, with |V(G)| = 5 and E(G) = empty set .

2 Define terms: Self loop, Parallel edges (Multiple edges).

3 Define terms: Simple graph, Null graph.

4 State and Prove, Bondy and Chvatal's theorem about Hamiltonian graph.

5 Define terms: Tree, Acyclic graph.

6 Write down terms: Fundamental cycle and Fundamental cut-set of a connected graph G with respect to a spanning tree T.

7 Define term: Weighted graph and minimal spanning tree.

8 Define term: Edge connectivity and Vertex connectivity.

9 Write down atleast three facts about dual of a planner graph.

10 Write down atleast three properties of an incidence matrix of a graph G.

Q.2 Answer the following (Any Two) 14

1 Let G = (V, E) be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) V = $V_1 \cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

2 Let G be a graph and it contains exactly two odd vertices, say x, y $\in V(G)$. Prove that, there is a path in G between x and y.

3 Let G be a simple graph with n vertices, q edges and k number of components in G. Prove that, q ≤ $\frac{1}{2}(n-k)(n-k+1)$.

Q.3 Answer the following 14

1 Let G be a connected graph. Prove that, G is an open Euler graph if and only if G has precisely two odd vertices and remaining are even vertices.

2 Let G be a simple graph, $|V(G)| > 2$ and $d_G(v) \geq \frac{n}{2}$, $\forall v \in V(G)$. Prove that, G is a Hamiltonian graph.

OR

Answer the following 14

Let T be a tree and it has atleast two vertices. Let $P = u_0 - u_1 - u_2 - \ldots\ldots - u_n$ be a longest path in T. Prove that, u_0 and u_n both are pendent vertices in T.

For a tree T, with $|V(T)| = n$, prove that, T has $n-1$ edges.

Q.4 Answer the following questions (Any Two) 14

1 For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$ and also prove that, (i) $e \geq \frac{3f}{2}$ (ii) $e \leq 3n - 6$. Using these, prove that, K_5 and $K_{3,3}$ both are non-planner graphs, where e = number of edges in G, n = number of vertices in G and f = number of faces in the planner graph G.

Let X(G) = $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ be an adjacency matrix of a graph G.

Find $Y = X + X^2 + X^3 + X^4$, where X = X(G). Determine G is a connected graph or it is not a connected graph.

Q.5 Answer the following (Any Two) 14

Let G be a connected graph with n vertices and $S \subseteq E(G)$. Prove that, S is a cut-set for G if and only if rank of $G - S$ is $n-2$ and rank of $G - S_1$ is $n-1$, for every S_1 proper subset of S.

2 Let G be a connected graph and S is a cut-set for G. Let T be a spanning tree for G. Prove that, $S \cap E(T) \neq \emptyset$.

Let $S \subseteq E(G)$ be a subset in a graph G and $S \cap E(T) \neq \phi$, for all the spanning trees T of G and no proper subset of S has above property. Prove that, S is a cut-set for the graph G.

Let G be a connected graph and S be a cut-set for G. Let F be any cycle in G. Prove that, $|E(F) \cap S|$ is even.