Seat No. ______________

F8X-003-1161001

M. Sc. (Sem. I) Examination

December - 2022

Mathematics - 1001

(Algebra - I)

Faculty Code : 003

Subject Code : 1161001

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer any **seven** short questions : 7×2=14

(1) Define with an example : Simple Group.

(2) Define a subgroup of a group G. Write down at least two subgroups of $(\mathbb{Z},+)$.

(3) Prove or disprove, that S_3 is a simple group.

(4) Let G be a group and H be a subgroup of G. Prove that, $ab^{-1}c^{-1} \in H, \forall\, a, b, c \in H$.

(5) Define terms : Cycle and Transposition in a symmetric group S_n.

(6) Define maximal normal subgroup of a group G.

(7) Define a complete group and give an example of a complete group.

(8) When group G act on a non-empty set X ? Define an action of a group G on the non-empty set X.

(9) Let G be the group with an internal direct product of its normal subgroups $N_1, N_2, \ldots\ldots, N_k$. Let $x \in N_i$ and $y \in N_j$, for some $i \neq j$ and $i, j \in \{1, 2, \ldots, k\}$. Prove that $xy = yx$.

(10) Define term : Integral Domain. Also prove that, every field is an integral domain.

2 Attempt any **two** : $2 \times 7 = 14$

(1) State and prove, Second Isomorphism Theorem of Groups.

(2) State and prove, Second Sylow's Theorem.

(3) Let G be a group and H be a subgroup of G. Suppose $O(H) = \frac{1}{2}O(G)$. Prove that, H is a maximal normal subgroup of G.

3 Attempt any **one** : $1 \times 14 = 14$

(1) Let R be a ring. Prove that, for any positive integer n, any ideal of $M_n(R)$, [the ring of all the $n \times n$ matrices over R] is given by $M_n(I)$, where I ranges through all the ideals of R.

(2) State and prove, Third Sylow's Theorem.

4 Attempt following **two** : $2 \times 7 = 14$

(1) Let G_1, G_2 be two groups, N_1 be a normal subgroup of G_1 and N_2 be a normal subgroup of G_2. In standard notation prove that,

(i) $N_1 \times N_2$ is a normal subgroup of $G_1 \times G_2$ and

(ii) $\frac{G_1 \times G_2}{N_1 \times N_2} \simeq \left[\frac{G_1}{N_1}\right] \times \left[\frac{G_2}{N_2}\right]$.

(2) Prove or disprove, the center of a group G is a normal subgroup of G. Also prove that, G is an abelian group if and only if its center is itself.

5 Attempt any **two** : $2 \times 7 = 14$

(1) Let G be a finite group and $O(G) = 48$. Prove that, G can't be a simple group.

(2) Let R be a ring and I be an ideal of R. Let $\phi : R \to \frac{R}{I}$ defined by $\phi(r) : r + I, \forall\, r \in R$, where $\frac{R}{I}$ is the quotient ring of R by the ideal I. Prove that, ϕ is a surjective ring homomorphism and $\ker\phi = I$.

(3) Let $\phi : G \to G'$ be a group homomorphism. In standard notation prove that, $\ker\phi$ is normal subgroup of G and $\phi(G)$ is subgroup of G'.

(4) State and prove, Cayley's Theorem.

SBS-003-1161001 Seat No. ________

M. Sc. (Sem. I) Examination

February – 2022

Mathematics : CMT-1001

(Algebra-I)

Faculty Code : 003

Subject Code : 1161001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Answer following seven questions : 7×2=14

(i) Define terms : Cycle and Transposition in a symmetric group S_n.

(ii) Define maximal normal subgroup of a group G.

(iii) Define a complete group and give an example of a complete group.

(iv) Let G_1, G_2 be two groups and $a,b \in G_1, c,d \in G_2$. Write down the identity element of $G_1 \times G_2$ and $(ab,cd)^{-1}$.

(v) Define a prime ideal of a ring R. Give an example of a prime ideal of $(\mathbb{Z},+,\cdot)$.

(vi) Prove or disprove, A_3 is a simple group.

(vii) In standard notation define $Z(G)$, the center of a group G. Is it a normal subgroup of G ? (Y/N).

2 Answer following seven questions : 7×2=14

(i) Define maximal normal subgroup of a group G. Write down a maximal normal subgroup of S_n.

(ii) In standard notation, define an inner automorphism T_g of a group G by an element $g \in G$. Also define $In(G)$.

(iii) Prove or disprove, $Z(S_n) = \{e\}$.

(iv) Write down four subgroups of S_3, where

$S_3 = \{e, \sigma, \sigma^2, \psi, \sigma\psi, \sigma^2\psi\}$.

(v) Let G be a finite group and a prime p divide to O(G). Define a p-Sylow subgroup of G.

(vi) Write down $\sigma \in S_9$ as a finite product of disjoint cycles,

where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 3 & 9 & 5 & 8 & 1 & 7 & 6 & 2 \end{pmatrix}$.

(vii) Let G be a group and N be a normal subgroup of G. In standard notation what is G/N ? Write down the identity element of G/N.

3 Answer following two questions : **2×7=14**

(a) Let X, Y be two non-empty sets and $f : X \to Y$ be a bijection. Prove that, S_x and S_y both are isomorphic groups.

(b) Let G_1, G_2 be two groups, N_1 be a normal subgroup of G_1 and N_2 be a normal subgroup of G_2. In standard notation prove that,

(i) $N_1 \times N_2$ is a normal subgroup of $G_1 \times G_2$ and

(ii) $G_1 \times G_2 / N_1 \times N_2 \simeq \left[G_1 / N_1 \right] \times \left[G_2 / N_2 \right]$

4 Answer following two questions : **2×7=14**

(a) State and prove, First isomorphism theorem of groups.

(b) State and prove, Second Sylow's theorem.

5 Answer following two questions : **2×7=14**

(a) State and prove, Third isomorphism theorem of groups.

(b) State and prove, Second isomorphism theorem of rings.

6 Answer following two questions : $2\times7=14$

(1) Let G be a group and H be a normal subgroup of G. Prove that, H is a maximal normal subgroup of G if any only if G/H is a simple group.

(2) Let G be a group. In standard notation prove that, In (G) is a subset of Aut (G) and it is also a subgroup of Aut (G).

7 Answer following two questions : $2\times7=14$

(a) Let G be a non-abelian group of order six. Prove that, $G \simeq S_3$.

(b) For a group G, in standard notation prove that,

(i) G' is normal subgroup of G.

(ii) G/G' is an abelian group.

(iii) For any normal subgroup H of G, if G/H is abelian, prove that G' is a subset of H.

8 Answer following two questions : $2\times7=14$

(1) Let $f: R \to S$ be a ring homomorphism. Prove that, $\{r \in R / f(r) = 0\}$ is an ideal of R.

(2) Let R be a ring and $1 \in R$. Let M be an ideal of R with $M \neq R$. Prove that, M is a maximal ideal of R if any only if R/M is a field.

9 Answer following two questions : $2\times7=14$

(a) Let F be a field. Prove that, F has precisely two ideals.

(b) Let R be a ring and A, B be two ideals of R. Prove that, $\left\{\sum_{i=1}^{t} a_i b_i / t \geq 1, a_i \in A, b_i \in B, \text{ for all } i = 1, 2, 3, \ldots, \ldots, t\right\}$ and $A \cap B$ both are ideals of R.

10 Answer following question : **1×14=14**

Let $\phi : G \to G'$ be a surjective group homomorphism. Prove that,

(i) $H < G \Rightarrow \phi(H) < G'$,

(ii) $H' < G' \Rightarrow \phi^{-1}(H') < G$,

(iii) $H \lhd G \Rightarrow \phi(H) \lhd G'$,

(iv) $H' \lhd G' \Rightarrow \phi^{-1}(H') \lhd G$,

(v) $H < G$ with $Ker\ \phi \subseteq H \Rightarrow H = \phi^{-1}(\phi(H))$ and

(vi) $\phi\left(\phi^{-1}(K)\right) = K$, for any subgroup K of G'.

MBN-003-1161001 Seat No. ________

M. Sc. (Sem. I) Examination

February - 2021

Mathematics : Paper - CMT-1001

(Algebra-I)

Faculty Code : 003
Subject Code : 1161001

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer any five questions.
(2) Each question carries 14 marks.

1 Answer following seven questions : 7×2=14

(i) Define a normal subgroup of a group G and write down a normal subgroups of S_3, where

$$S_3 = \{e, \sigma, \sigma^2, \psi, \sigma\psi, \sigma^2\psi\}$$

(ii) In standard notation, prove or disprove that S_3 is an abelian group.

(iii) Let $S_3 = \{e, \sigma, \sigma^2, \phi, \sigma\phi, \sigma^2\phi\}$. Take $K = \{e, \phi\}$.

Write down all the left cosets of K in S_3.

(iv) Let G_1, G_2 be two groups and $a, b \in G_1, c, d \in G_2$. Write down the identity element of $G_1 \times G_2$ and $(ab, cd)^{-1}$.

(v) Define a prime ideal of a ring R. Give an example of a prime ideal of $(\mathbb{Z}, +, \cdot)$.

(vi) Prove or disprove A_3 is a simple group.

(vii) In standard notation define $Z(G)$, the center of a group G. Is it a normal subgroup of G ? (Y / N).

2 Answer following seven questions : 7×2=14

(i) Define maximal normal subgroup of a group G. Write down a maximal normal subgroup of S_n.

(ii) In standard notation, define an inner automorphism T_g of a group G by an element $g \in G$. Also define $I_n(G)$.

(iii) Prove or disprove $Z(S_n) = \{e\}$.

(iv) Prove or disprove A_4 has no subgroup of order six.

(v) Let G be a group and $a \in G$. Prove that $N(a) = \{g \in G \mid ga = ag\}$ is a subgroup of G.

(vi) Define term: External direct product of groups.

(vii) Define ring homomorphism and give two ring homomorphisms on a ring Z into Z.

3 Answer following two questions : 2×7=14

(a) Let X, Y be two non-empty sets and $f : X \to Y$ be a bijection. Prove that, S_X and S_Y both are isomorphic groups.

(b) Let G_1, G_2 be two groups, N_1 be a normal subgroup of G_1 and N_2 be a normal subgroup of G_2. In standard notation prove that :

(i) $N_1 \times N_2$ is a normal subgroup of $G_1 \times G_2$ and

(ii) $G_1 \times G_2 / N_1 \times N_2 \simeq [G_1/N_1] \times [G_2/N_2]$.

4 Answer following two questions 2 × 7 = 14

(a) State and Prove First Isomorphism Theorem of Rings.

(b) State and Prove Second Sylow's Theorem.

5 Answer following two questions : 2 × 7 = 14

(a) State and Prove Third Isomorphism Theorem of Rings.

(b) State and Prove Second Isomorphism Theorem of Groups.

6 Answer following two questions : 2 × 7 = 14

(a) Let G be a group and H be a subgroup of G. Suppose $O(H) = \frac{1}{2} O(G)$. Prove that, H is a maximal normal subgroup of G.

(b) Prove or disprove the center of a group G is a normal subgroup of G. Also prove that, G is an ablelian group if and only if its center is itself.

7 Answer following two questions : $2 \times 7 = 14$

(a) Let G be a non-abelian group of order six. Prove that, $G \simeq S_3$.

(b) For a group G, in standard notation prove that,

(i) G' is normal subgroup of G.

(ii) G/G' is an abelian group.

(iii) For any normal subgroup H of G, if G/H is abelian, then prove that G' is a subset of H.

8 Answer following two questions : $2 \times 7 = 14$

(a) Let $G = \langle g \rangle$ be a cyclic group and $O(G) = mn$, where m and n are relatively primes. Let $H = \langle g^m \rangle$ and $K = \langle g^n \rangle$. Prove that G is the internal direct product of its subgroups H and K.

(b) Let G be a finite group and p is divisor of $O(G)$, for some prime p. Let P be a sylow p-subgroup of G. Prove that, P is only Sylow p-subgroup of G if and only if P is the normal subgroup of G.

9 Answer following two questions : $2 \times 7 = 14$

(a) Let F be a field. Prove that, F has precisely two ideals.

(b) Let R be a ring and A, B be two ideals of R. Prove that

$\{\sum_i^t a_i b_i \big/ t \geq 1,\ a_i \in A, b_i \in B$, for all i = 1, 2, 3,......,$t\}$ and

$A \cap B$ both are ideals of R.

10 Answer following one question : $1 \times 14 = 14$

(1) Let R be a ring and $1 \in R$. Let M be an ideal of R with $M \# R$. Prove that, following statement are equivalent.

(a) M is a maximal ideal of R.

(b) R/M has no non-trivial ideal.

(c) $M + (x) = R$, for every $x \in R - M$

JBD-003-1161001

JBD-003-1161001

Seat No. ______

M. Sc. (Sem. I) (CBCS) Examination

December - 2019

Mathematics : CMT - 1001

(Algebra - I)

Faculty Code : 003
Subject Code : 1161001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each question carries 14 marks.

1 Answer any **seven** questions : **7×2=14**

(i) Write down two subgroups of S_3 which are not normal, where $S_3 = \{e, \sigma, \sigma^2, \psi, \sigma\psi, \sigma^2\psi\}$.

(ii) Define a simple group and give an example of a simple group. Is A_4 a simple group ? (Y/N).

(iii) Prove or disprove that S_3 is a simple group.

(iv) Define an ideal I of a ring R. Let $a, b, c \in I$. Deduce that $a-b-2c \in I$.

(v) Let G be a finite group and a prime p divide to o(G). Define a p-Sylow subgroup of G.

(vi) Let A, B, C ideals of a ring R. Prove that $A \cap B \cap C$ is also an ideal of R.

(vii) Let G be a finite group with o(G) = 147. Write down order of 3-Sylow and 7-Sylow subgroups of G.

(viii) Define a prime ideal of a ring R. Is all prime ideals of $(\mathbb{Z}, +, \cdot)$ are maximal ideals ? Justify.

2 Answer any **two** questions : **2×7=14**

(a) State and prove Third Fundamental Theorem of Groups.

(b) Let G be a group and

$G' = \left\{\prod_{i-1}^{t} a_i b_i a_i^{-1} b_i^{-1} / a_i, b_i \in G, \forall i = 1, 2, \ldots, t\right\}$ be the commutator subgroup G. In standard notation prove that G' is a normal subgroup of G and G/G' is an abelian group.

(c) Let G be a non-abelian group of order 6. Prove that G is isomorphic to S_3 .

3 Answer any **one** question : **1×14=14**

(a) (i) State and Prove Sylow's Third Theorem.

(ii) Let G be a finite abelian group and a prime p divide to $o(G)$. Let P be a Sylow p-subgroup of G. Prove that P is only Sylow p-subgroup of $G \Leftrightarrow P$ is normal subgroup of G.

(b) Let R be a ring. Prove that for any positive integer n, any ideal of $M_n(R)$, the ring of all the nxn matrices over R is given by $M_n(I)$, where I ranges through all the ideals of R.

(c) Prove that $A_n\,(n \geq 5)$ is a simple group. For $n \geq 5$, prove that the collection of all normal subgroups of S_n is $\{\{e\}, A_n, S_n\}$.

4 Answer any **two** questions : **2×7=14**

(a) State and Prove First Isomorphism Theorem of Rings.

(b) Let A, B be two ideals of a ring R. Define product AB and sum $A + B$ of two ideals in R. Prove that AB, $A + B$ and $AB \cap (A+B)$ all are ideals of R.

(c) Let $f : R \rightarrow T$ be an onto ring homomorphism. Let $\mathcal{C}$ the collection of ideals of R which contains ker f and $\mathcal{D}$ be the collection of all ideals of T. Prove that there is a bijective map from $\mathcal{C}$ into $\mathcal{D}$.

5 Answer any **two** questions : **2×7=14**

(a) Let G be a finite group, with $O(G) = p \cdot q$, where p and q both are primes ($p < q$). If $p + q - 1$, then prove that G must be a cyclic group.

(b) Let R be a commutative ring and M be an ideal of R. Prove that M is a maximal ideal of R if and only if R/M is a field.

(c) Let G be a group and N_i be normal subgroups of G, $\forall\, i = 1, 2, ..., n$. Prove that G is the internal direct product of $N_1, N_2, ..., N_n$ iff $G = N_1\, N_2 \,..\, N_n$ and $N_i \cap N_1 \,..\, N_{i-1}\, N_{i+1} .. N_n = \{e\}$, for every $i \in \{1, 2, ..., n\}$.

(d) Prove that :

(i) Every irreducible element of a Principle Ideal Domain R is always a prime element of R and

(ii) Every Euclidean Domain is also Principle Ideal Domain.

Seat No. ______________

F8Y-003-1161002

M. Sc. (Sem. I) Examination

December - 2022

Mathematics : Paper - CMT-1002

(Real Analysis)

Faculty Code : 003
Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) There are total five questions.

(2) All questions are mandatory.

(3) Each question carries equal marks.

1 Answer any seven questions : **7×2=14**

(1) Define Boolean algebra on a non-empty set X.

(2) Define Lebesgue outer measure of a subset E of $\mathbb{R}$.

(3) Write down $m^*(\mathbb{N})$ and $m^*([2, 4] \cup (5, 8))$.

(4) Describe that countable union of F_σ sets is F_σ.

(5) Define Lebesgue measurable set.

(6) Show that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$ if E_1 and E_2 are measurable.

(7) Show that $|f|$ is integrable over a measurable set E then f is integrable over E.

(8) Define the term Convergence in measure.

(9) If f is integrable over a measurable set E and A, B are disjoint measurable subset of E then show that

$$\int_{A \cup B} f = \int_{A} f + \int_{B} f.$$

(10) Why is the condition $m^* A \geq m^*(A \cap E) + m^*\left(A \cap E^C\right)$ sufficient to become the set E is measurable ?

2 Answer any two of the following : 2×7=14

(1) Show that any Borel set is measurable.

(2) If $E_1 \supseteq E_2 \supseteq \cdots$ be a decreasing sequence of measurable sets with $mE_1 < \infty$ then show that

$$m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} mE_n.$$

(3) If for given $\varepsilon > 0$, $\exists$ a subset U of $\mathbb{R}$ such that U is the union of finite number of open interval in $\mathbb{R}$ with $m^*(U \Delta E) < \varepsilon$ then show that E is measurable.

3 Answer the following : 2×7=14

(1) State and prove Fatou's Lemma.

(2) State and prove Egoroff's theorem.

OR

3 Answer the following : 2×7=14

(1) If f, g are bounded measurable functions define on a measurable set E with $mE < \infty$ then prove that

$$\int_E f+g = \int_E f + \int_E g.$$

(2) If $f : [a, b] \to \mathbb{R}$ is bounded function and Riemann integrable over $[a, b]$ then prove that f is measurable and moreover $R\int_a^b f(x)\,dx = \int_{[a,\, b]} f(x)\,dx.$

4 Answer the following : 2×7=14

(1) If $\langle f_n \rangle$ is a sequence of measurable function defined on E and f is a real valued function defined on E such that $f_n \to f$ in measure on E then prove that there exists a subsequence $\langle f_{nk} \rangle$ of $\langle f_n \rangle$ such that $f_{nk} \to f$ almost everywhere on E.

(2) Prove that Lebesgue convergence theorem holds good if convergence a.e. is replaced by convergence in measure.

5 Answer any two of the following : 2×7=14

(1) If f, g are measurable functions define on a measurable set E then show that the following hold :

(i) If f is integrable over a measurable set E then for any $c \in \mathbb{R}$, cf is integrable over E and moreover

$$\int_E cf = c\int_E f.$$

(ii) If f, g are integrable over E and $f \leq g$ almost everywhere on E then $\int_E f \leq \int_E g$.

(2) State and prove Holder's inequality.

(3) If $f : [a, b] \to \mathbb{R}$ is a function of bounded variation then prove that $P - N = f(b) - f(a)$ and $P + N = T$. Where P, N, T are positive, negative and total variation of f over $[a, b]$ respectively.

(4) If $f : [a, b] \to \mathbb{R}$ is a bounded function and Riemann integrable over $[a, b]$ then prove that f is measurable and moreover $R\int_a^b f(x)\,dx = \int_{[a, b]} f(x)\,dx$.

SBT-003-1161002 Seat No. ________

M. Sc. (Sem. I) Examination

February – 2022

Mathematics : CMT-1002

(Real Analysis)

Faculty Code : 003

Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer any five questions.

(2) Each question carries 14 marks.

(3) There are 10 questions in total.

1 Answer the following seven questions : **14**

(1) Define : Algebra of sets of a non empty set X.

(2) Let $F_1, F_2, \dots$ be F_σ – sets. Then prove that, $\bigcup_{i=1}^{\infty} F_i$ is also an F_σ – set.

(3) Define : G_δ – set. Justify that, a closed interval in $\mathbb{R}$ is a G_δ – set.

(4) Give an example of a G_δ – set, which is not an F_σ – set. Also give an example of F_σ – set which is not a G_δ – set.

(5) Let $A \subseteq \mathbb{R}$. Then prove that, A is a G_δ – set if and only if A^c is an F_σ – set.

(6) Define : Borel field and Borel set.

(7) Using outer measure, prove that, $[1, 2021]$ is not a countable subset of $\mathbb{R}$.

2 Answer the following seven questions : **14**

(1) Define : Lebesgue outer measure of a subset A of $\mathbb{R}$.

(2) Write down $m^*(\mathbb{Q}\times\mathbb{N})$ and $m^*([1,3]\cap\mathbb{R})$.

(3) Let $E\subseteq\mathbb{R}$ and $m^*E=0$. Then prove that, E is a Legesgue measurable set.

(4) Let $E\in m$ and $\phi:E\to\mathbb{R}$ be a simple map with $\phi(E)=\{a_1,a_2,......,a_n\}$. Write down canonical representation of ϕ.

(5) Let $A,B,C\subseteq\mathbb{R}$. Let $m^*A=0$. Verify that, $m^*(A\cup B\cup C)=m^*(B\cup C)$.

(6) Prove or disprove, the continuous function $f:\mathbb{R}\to\mathbb{R}$ is a measurable function.

(7) Define : Measurable function. Also give an example of a measurable function on □ .

3 Answer the following two questions : **14**

(1) Let $X\neq\phi$ and $C\subseteq P(X)$. Let A be the algebra on X, generated by C. Let R_1= the σ – algebra on X, generated by C and R_1= the σ – algebra on X, geneared by A. Then prove that, $R_1=R_2$.

(2) Prove that, Lebesgue outer measure of any interval is its length.

4 Answer the following two questions : **14**

(1) Let $X\neq\phi$ and R be an algebra of sets on X. Let $<A_i>\subseteq R$ be a sequence. Then prove that, $\exists<B_i>\subseteq R$ such that B_i's are mutually disjoint, $B_i\subseteq A_i$, $\forall_i=1,2,....$ and for any $n\in\mathbb{N}$,

$$\bigcup_{i=1}^{n}B_i=\bigcup_{i=1}^{n}A_i.$$

(2) Prove that, m is an algebra on $\mathbb{R}$, where m is the family of all measurable sets on $\mathbb{R}$.

5 Answer the following two questions : **14**

(1) Let $E_1, E_2 \in m$ then prove that,
$m(E_1 \cap E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$, where m is the family of all measurable sets on $\mathbb{R}$.

(2) Let $f, b : E \to \mathbb{R}$ be two extended real valued measurable functions on a measurable set E. Let $c \in R$. Then prove that, $f+g, f+c, cf, g-f$ and $f\,g$ all ar measurable functions on E.

6 Answer the following two questions : **14**

(1) Let $<E_n> \subseteq M$ and $E_{n+1} \subseteq E_n, \forall n \in \mathbb{N}$. Let $m(E_1) < \infty$. Then prove that, $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$.

(2) (a) Prove that, $m^*(A+y) = m^*A, \forall A \subseteq \mathbb{R}$, where $A+y = \{x+y / x \in A\}$.

(b) Construct the Cantor set and show that, it is an uncountable, measurable set with required justification.

7 Answer the following two questions : **14**

(1) Let $<f_n>$ be a sequence of non-negative measurable functions such that $f_n \le f_{n+1}, \forall n \in \mathbb{N}$. Let $f_n(x) \to f(x), \forall x \in E$. Then prove that, $\int_E f = \lim_n \int_E f_n$.

(2) Let g be an integrable function over E and $<f_n>$ be a sequence of measurable functions on E such that $|f_n| \le g \; \forall n \in \mathbb{N}$ on E. Let $f(x) = \lim_n f_n(x)$ a.e. on E. Then prove that, f, f_n's are integrable over $E, \forall n \in \mathbb{N}$ and $\int_E f = \lim_n \int_E f_n$.

8 Answer the following two questions : **14**

(1) Let $f:[a,b]\to R$ be a bounded and measurable function. Let $F:[a,b]\to R$ be given by $F(x)=F(a)+\int_a^x f(t)\,dt$ then prove that, $F'(x)=f(x)$ a.e. on $[a,b]$.

(2) State and prove, Holder's inequality.

9 Answer the following one questions : **14**

(1) Let f,g be bounded measurable functions on E and $mE<\infty$. Then prove that,

(a) $\int_E (af+bg)=a\int_E f+b\int_E g,\ \forall a,b\in\mathbb{R}$

(b) $f\le g$ a.e. on E then $\int_E f\le\int_E g$.

(c) $f=g$ a.e. on E then $\int_E f=\int_E g$.

(d) If $a\le f(x)\le b,\ \forall x\in E$, then $a\le\frac{1}{mE}\int_E f\le b$.

(e) For any disjoint subset A and B of E,

$\int_{A\cup B} f=\int_A f+\int_B f$.

10 Answer the following one question : **14**

(1) Let f be a bounded function on a measurable set E and mE is finite. Then prove that,

$$\inf_{\substack{\psi\ge f\\ \psi \text{ is simple}}}\int_E \psi=\sup_{\substack{\phi\le f\\ \phi \text{ is simple}}}\int_E \phi$$

if and only if f is a measurable function.

2 Answer following **seven** questions : 7×2=14

(1) Let $E \subseteq \mathbb{R}$ and $m^*(E) = 0$. Prove that E is a Lebesgue measurable set.

(2) Let $A \subseteq \mathbb{R}$ be any subset. Prove that, A is a G_δ – set if and only if A^c is an F_σ – set.

(3) Define term: Measurable function. Also give an example of a measurable function on $\mathbb{R}$.

(4) Write down any two from Littlewood's three principles without proof.

(5) Write down Lebesgue integral of a non-negative measurable function on a measurable set – E.

(6) Define a characteristic function on a measurable set – D.

(7) Define the property Almost Everywhere.

3 Answer following **two** questions : 2×7=14

(a) Let X be the set of all natural numbers. Let $R = \{A \subseteq X /$ either A is finite or its complement is finite$\}$. Prove that, R is a Boolean algebra on X.

(b) Let X be a non-empty set and R is an algebra of sets on X. Let $\langle A_i \rangle \subseteq R$. Prove that, there is $\langle B_i \rangle \subseteq R$ such that, B_i's are mutually disjoint, $B_i \subseteq A_i$, for every $i = 1, 2, \ldots \ldots$ and for any positive integer n, $\bigcup_{i=1}^{n} B_i = \bigcup_{i=1}^{n} A_i$.

4 Answer following **two** questions : 2×7=14

(a) Let $\langle A_n \rangle \subseteq P(\mathbb{R})$. In standard notation, prove that

$$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^* A_n.$$

(b) Construct the Cantor Set and prove that, it is an uncountable, measurable set.

5 Answer following **two** questions : **2×7=14**

(a) Let X be a non-empty set and $C \subseteq P(X)$. Prove that, there is a smallest σ-algebra on X, which contains the given collection C.

(b) Let β_1 be the $\sigma-$algebra on R, generated by the collection of all closed sets on R and β_2 be the $\sigma-$algebra on R, generated by the collection of all open sets on R. Prove that $\beta_1 = \beta_2 = B_0$, where B_0 = the Borel field on R.

6 Answer following **two** questions : **2×7=14**

(a) Prove that, the Borel field on R is the subcollection of $\mathcal{M}$, where $\mathcal{M}$ is the set of all measurable sets.

(b) Let E be a measurable set and f be an extended real valued function on E. Prove that, for any real number α following statements are equivalent:

(1) $\{x \in E / f(x) \geq \alpha\}$ is a measurable set.

(2) $\{x \in E / f(x) < \alpha\}$ is a measurable set.

(3) $\{x \in E / f(x) \leq \alpha\}$ is a measurable set.

(4) $\{x \in E / f(x) > \alpha\}$ is a measurable set.

7 Answer following **two** questions : **2×7=14**

(1) Let $f, g : E \to R$ be two real valued simple functions on a measurable set E. Let $c \in \mathbb{R}$ be any real. Prove that, $f + g$, cf, $f - g$ and fg all are simple functions on E.

(2) Let ϕ and ψ be simple functions and they vanish outside of a set E, with $m(E) < \infty$. Let a, b be any real numbers. Prove that,

(i) $\int_E (a\phi + b\psi) = a \int_E \phi + b \int_E \psi$ and

(ii) If $\phi \geq \psi$ a.e. on E, then $\int_E \phi \geq \int_E \psi$.

8 Answer following **two** questions : 2×7=14

(a) State and Prove Bounded Convergence Theorem.

(b) State and Prove the Lebesgue Dominate Convergence Theorem.

9 Answer following **one** question : 1×14=14

Let f be a bounded function on a measurable set E and measure of E is finite. Prove that,

$\text{Inf} \quad \psi \geq f \int_E \psi = \sup \phi \leq f \int_E \phi$ for all simple functions ϕ and ψ if and only if f is a measurable function on E.

10 Answer following **one** question : 1×14=14

Construct a non-measurable subset of $\mathbb{R}$ with required justification.

JBE-003-1161002 Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

December – 2019

Mathematics : Paper - CMT-1002

(Real Analysis)

Faculty Code : 003
Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer any seven questions : **7 × 2 = 14**

(i) Define Countable set and give an example of a countable set.

(ii) Define Boolean algebra on a non-empty set X.

(iii) Define Borel field and Borel Set.

(iv) Define Outer measure and give an example of an infinite subset of $\mathbb{R}$ whose outer measure is zero.

(v) Give an example of a subset of nowhere dense set.

(vi) Prove or disprove, $\mathbb{R}$ is a measurable set.

(vii) Write down outer measure of following. sets: Q, [2,5] and (–3,5).

(viii) Is Cantor set measurable? Justify.

(ix) Define almost everywhere property.

(x) Define convergence in sense of measure.

2 Answer any two questions : **2 × 7 = 14**

(a) Let X be a non-empty set and a be a Boolean algebra on X. Let $< A_i > \subseteq a$ be any sequence in a. Prove that there is a sequence $< B_i >$ in a such that each B_i's are mutually disjoint, $Bi \subseteq Ai$, $\forall\ i \in N$ and

$\bigcup_{i=1}^{n} B_1 = \bigcup_{i=1}^{n} A_i$, for each $n \in N$.

(b) Give an example of a Boolean algebra on N, which is not a σ-algebra on N. Justify your answer.

(c) Let F, E $\in$ m, where m is the collection of all measurable sets. Prove that $F \cup E \in m$.

(d) Prove that the outer measure is translate invariant (i.e. $m^*(A) = m^*(A + y),\ \forall\ y \in \mathbb{R}$).

3 Answer any one question : **1 × 14 = 14**

(a) Construct a non-measurable subset of [0, 1].

(b) Let f be a bounded function on a measurable set E and $m\,E < \infty$. Prove that $\inf_{\psi \geq f} \int_E \psi \ \sup_{\phi \leq f} \int_E \phi$, for all simple functions ϕ and ψ on E if and only if f is a measurable function.

(c) State and Prove Vitali's Lemma.

4 Answer any two questions **2 × 7 = 14**

(a) Let $1 \leq p < \infty$. If $f,g \in L^p[0\ ,\ 1]$, the prove that $f + g \in L^p[0, 1]$ and $\| f+g \|_p \leq \| f \|_p + \| g \|_p$, where

$$\| f \|_p = \left[\int_0^1 \ |f|^p \right]^{1/p}.$$

(b) Let f be a bounded measurable function on [a, b] and $F(x) = \int_a^x f(t)\,dt + F(a), \forall\ x \in [a, b]$. Prove that $F'(X) = f(X)$ almost everywhere on [a, b].

(c) Let f : [0, 1] $\rightarrow$ $\mathbb{R}$ and f (0) = 0, f(x) = x^2 sin ($1/x^2$), $\forall$ $\times$ E (0 , 1]. Prove that f is not a function of bounded variation on [0, 1].

5 Answer any two questions : **2 × 7 = 14**

(a) State and prove Bounded Convergence Theorem.

(b) State and prove Fatou's Lemma.

(c) Let $\{f_n\}$ be a sequence of non-negative measurable functions such that $f_n \leq f_{n+1}$, $\forall$ $n \in \mathbb{N}$. Suppose

$f_n(X) \rightarrow f(X)$, $\forall x \in E$. Prove that $\int_E f = \overset{\lim}{n} \int_E f_n$.

(d) Let $\{f_n\}$ be a sequence of non-negative measurable functions such that fn $f_n \leq f$, $\forall n \in \mathbb{N}$, where f is also a non-negative measurable function. Suppose

$f_n(x) \rightarrow f(x), \forall x \in E$. Prove that $\int_E f = \overset{\lim}{n} \int_E f_n$.

PCD-003-1161002

Seat No. 015044

M. Sc. (Sem. I) Examination

December - 2018

Mathematics : CMT-1002

(Real Analysis)

Faculty Code : 003
Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Each questions carries 14 marks.

1 Answer any seven questions : 7×2=14

(i) Define terms : Sequence and Countable set.

(ii) Define Boolean algebra on a non-empty set X.

(iii) Give an example of a σ-algebra on a non-empty set.

(iv) Define Borel field and Borel Set.

(v) Define Nowhere Dense Set.

(vi) Give an example of a subset of $\mathbb{R}$ which is a no where dense set.

(vii) Give an example of G_δ-set, but is not a F_σ- set.

(viii) Write down ,outer measure of following sets: Q, [2, 5] and (–3, 5).

(ix) Is Cantor set a measurable set? Justify your answer.

(x) Define almost everywhere property.

2 Answer any two questions : 2×7=14

(a) Let X be a non-empty set and $\mathfrak{a}$ be a Boolean algebra on X. Let $\langle A_i \rangle \subseteq \mathfrak{a}$ be any sequence in $\mathfrak{a}$. Prove that there is a sequence $\langle B_i \rangle$ in $\mathfrak{a}$ such that each B_i's are mutually disjoint, $B_i \subseteq A_i, \forall i = 1, 2, \ldots\ldots$ and $\bigcup_{i=1}^{n} B_i = \bigcup_{i=1}^{n} A_i$, for each $n \in N$.

(b) Give an example of a Boolean algebra on N, which is not a σ-algebra on N. Justify your answer.

(c) Prove that the collection of all measurable sets $\mathfrak{m}$ is a Boolean algebra.

(d) Prove that the outer measure is translate invariant (i.e.. $m^*(A) = m^*(A+y), \forall\, y \in \mathbb{R}$).

3 Answer any **one** question : 1×14=14

(a) Construct a non-measurable subset of $[0,1]$.

(b) State and Prove Holder's Inequality.

(c) State and Prove Vitali's Lemma.

4 Answer any **two** questions : 2×7=14

(a) Let $1 \le p < \infty$. If $f, g \in L^p[0, 1]$, the prove that $f+g \in L^p[0,1]$ and $\|f+g\|_p \le \|f\|_p + \|g\|_p$, where $\|f\|_p = \left[\int_0^1 |f|^p\right]^{1/p}$.

(b) Prove that $L^p[0, 1]$ is a normed linear space over $\mathbb{R}$.

(c) Let $f:[0, 1] \to \mathbb{R}$ and $f(0)=0$, $f(x) = x^2 \sin(1/x^2)$, $\forall x \in (0,1]$. Prove that f is not a function of bounded variation on $[0,1]$.

(d) Let f be a real valued function on $[a,b]$. Prove that f is a function of bounded variation on $[a,b]$ if and only if f can be express as difference of two monotone real valued functions on $[a,b]$.

5 Answer any two questions : 2×7=14

(a) State and prove Bounded Convergence Theorem.

(b) State and prove Fatou's Lemma.

(c) Let $\{f_n\}$ be a sequence of non-negative measurable functions on a measurable set E and $f_n \le f_{n+1}, \forall n$. If $f_n(x) \to f(x)$ for some function f on E then prove that $\int_E f = \lim_n \int_E f_n$.

(d) Let f, g both are integrable functions on a measurable set E. Prove that cf and $f + g$ are also integrable functions on E, for any $c \in \mathbb{R}$.

M. Sc. (Mathematics) (Sem. I) (CBCS) Examination

November / December – 2017

Real Analysis : MATH CMT-1002

(New Course)

Faculty Code : 003
Subject Code : 1161002

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Answer all questions.

(2) Each question carries 14 marks.

(3) The figures to the right indicate marks allotted to the question.

1 All are compulsory (Each question carries **two** marks) 14

(a) Define algebra of sets.

(b) Give an example of a set that is σ algebra of sets.

(c) Give an example of a F_σ – set.

(d) True or false : Q, the set of rationals, is a G_δ – set.

(e) Define measurable function.

(f) State Littelwood's third principle.

(g) Define function of bounded variation.

2 Answer any **two** : 14

(a) Prove that every closed and open set are measurable. 7

(b) Define Lebesgue outer measure of a set and show that Lebesgue outer measure of a finite interval is its length. 7

(c) Show that countable union of measurable sets is again measurable. 7

3 All are compulsory : 14

(a) Prove that if f and g are measurable functions then fg is also measurable. 7

(b) Show that f is a function of bounded variation on [a, b] if and only if there exists monotonically increasing functions g, h: [a, b] $\to$ $\mathbb{R}$ such that $f = g - h$. 7

OR

3 All are compulsory : 14

(a) State and prove Egoroff's theorem. 7

(b) State and prove Fatou's Lemma. 7

4 Answer any **two** : 14

(a) State and prove Lebesgue dominated convergence theorem. 7

(b) Define lebesgue integral of a bounded measurable function. If f and g are bounded measurable functions defined on measurable set E then show that $\int_E af + bg = a\int_E f + b\int_E g$. 7

(c) State and prove Bounded convergence theorem. 7

5 All are compulsory (each question carries **two** marks) 14

(a) Show that if E is measurable set then its complement is also measurable.

(b) Show that [a, b] is uncountable.

(c) Give the Lebesgue outer measure of a countable subset of $\mathbb{R}$.

(d) Let $<f_n>$ be a sequence of measurable functions defined on E. If f: E $\to$ $\mathbb{R}$ then when do we say that $<f_n>$ converges to f in measure.

(e) Show that every step function is measurable.

(f) State monotone convergence theorem.

(g) True false : Fatou's Lemma and Lebesgue dominated convergence theorem holds good if almost every where is replaced by convergence in measure ?

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MBY-003-1161002 Seat No. ____________

M. Sc. (Sem. I) (CBCS) Examination

December – 2016

Mathematics : MATH.CMT-1002

[Real Analysis]

(New Course)

Faculty Code : 003

Subject Code : 1161002

Time : 2.30 Hours] [Total Marks : 70

Instructions : (1) Answer **all** the questions.

(2) **Each** questions carries 14 marks.

1 Answer any seven : (7x2=14) 14

(a) Let $A \subseteq \mathbf{R}$. When is A said to be of type G_δ ?

(b) Let $f:[a,b] \to \mathbf{R}$ be a step function. Prove that f is measurable.

(c) Let $E \subseteq \mathbf{R}$ be such that $m^*(E) = 0$. Such that E is measurable.

(d) Let $f:[0,1] \to \mathbf{R}$ be bounded and measurable. Define Lebesgue integral of f over [0,1].

(e) State Monotone convergence theorem.

(f) Let $R = \{A \subseteq \mathbf{N}$: either A is finite or $\mathbf{N} \setminus A$ is finite$\}$. Show that R is not closed under countable union.

(g) Find $m([0,5]\cup[2,7])$.

(h) Let $f:\mathbb{R}\to\mathbb{R}$ be a simple function which vanishes outside a set of finite measure. Define Lebesgue integral of f over $\mathbb{R}$.

(i) Let $f:[0,2]\to\mathbb{R}$ be the characteristic function of $\mathbb{Q}\cap[0,2]$. Show that $f(x)=0$ for almost all $x\in[0,2]$.

(j) When is $f:[a,b]\to\mathbb{R}$ said to be of bounded variation on $[a,b]$?

2 Answer any two : (2x7=14) 14

(a) Let X be a nonempty set. Let C be a subcollection of the collection of all subsets of X. Prove that there exists a smallest σ-algebra of sets R on X such that $C\subseteq R$.

(b) If E_1 and E_2 are measurable, then show that $E_1\cup E_2$ is measurable.

(c) Let $A\subseteq\mathbb{R}$. Prove that $m^*A=m^*(A+y)$ for any $y\in\mathbb{R}$.

3 (a) If f and g are real-valued measurable functions defined on $\mathbb{R}$, then show that fg is measurable.

(b) $D\subseteq\mathbb{R}$ be measurable. Let $f:D\to\mathbb{R}$. If $\{d\in D: f(d)<\alpha\}$ is measurable for each $\alpha\in\mathbb{R}$, then prove that $\{d\in D: f(d)\leq\alpha\}$ is measurable for any $\alpha\in\mathbb{R}$. 5

(c) Let $f:[0,1]\to\mathbb{R}$ be defined by $f(x)=1$ if x is rational and $f(x)=0$ if x is not rational. Show that f is not Riemann integrable over [0,1]. 4

OR

3 (a) Let $E \subseteq \mathbb{R}$ be measurable with $mE < \infty$. Let $f : E \to \mathbb{R}$ be bounded and measurable. Prove that $\int_E cf = c\int_E f$ for any $c \in \mathbb{R}$ with $c > 0$. 5

(b) Mention with details an example of a sequence $\langle f_n \rangle$ of measurable functions defined on $[0,1]$ such that f_n converges in measure to the zero function on $[0,1]$. 5

(c) Let $E \subseteq \mathbb{R}$ be measurable. Let $f : E \to \mathbb{R}$ be measurable. If f is integrable over E, then show that $-f$ and $|f|$ are integrable over E. 4

4 Answer any two : (2x7=14) 14

(a) Let $E \subseteq \mathbb{R}$ be measurable with $mE < \infty$. Let $\langle f_n \rangle$ be a sequence of measurable functions defined on E and let $f : E \to \mathbb{R}$ be measurable such that f_n converges to f pointwise on E. Then given $\epsilon > 0$ and $\delta > 0$, prove that there is a measurable set $A \subseteq E$ with $mA < \delta$ and $N \in \mathbb{N}$ such that $|f_n(x) - f(x)| < \epsilon$ for every $n \geq N$ and for all $x \in N \setminus A$.

(b) Prove Monotone convergence theorem.

(c) Let $f : [a,b] \to \mathbb{R}$ be such that f is integrable on $[a,b]$. Prove that the function $F : [a,b] \to \mathbb{R}$ defined by $F(x) = \int_a^x f(t)\,dt$ is of bounded variation on $[a,b]$.

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9

5 Answer any two (2x7=14) 14

(a) Let $a,b \in \mathbb{R}$ with $a<b$. Show that $m^*([a,b]) = b-a$.

(b) Let $A \subseteq \mathbb{R}$ and $E_1, \ldots, E_n$ be a finite sequence of disjoint measurable sets. Prove that

$$m^*\left(A \cap \left(\cup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$$

(c) Let $f:[a,b] \to \mathbb{R}$ be of bounded variation on $[a,b]$. Prove that there exists $g,h:[a,b] \to \mathbb{R}$ such that g and h are monotonically increasing and $f(x) = g(x) - h(x)$ for all $x \in [a,b]$.

(d) Let $E \subseteq \mathbb{R}$ be measurable with $mE < \infty$. Let $f: E \to \mathbb{R}$ be bounded and measurable. Let C be the collection of all simple functions ψ defined on E such that $\psi \geq f$ on E and D be the collection of all simple functions ϕ defined on E such that $\phi \leq f$ on E. Show that $\inf\left\{\int_E \psi : \psi \in C\right\} = \sup\left\{\int_E \phi : \phi \in D\right\}$.

SARVAIYA JENTILAL B.

$\inf_{\psi \in C} \int_E \psi = \sup_{\phi \in D} \int_E \phi$

BBM-003-016102 Seat No. ________

M. Sc. (Mathematics) (Sem. I) (CBCS) Examination

December - 2015

Mathematics : CMT - 1002

(Real Analysis)

Faculty Code : 003

Subject Code : 016102

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) Answer all the questions.

(2) Each question carries 14 marks.

1. Answer any Seven $7 \times 2 = 14$

(a) If $m^*(A) = 0$, then prove that $m^*(A \cup B) = m^*(B)$.

(b) When is a subset A of $\mathbb{R}$ said to be of type F_σ? If $E \subseteq \mathbb{R}$ is of type G_δ, then show that the complement of E in $\mathbb{R}$ is of type F_σ.

(c) Define σ-algebra of sets on a nonempty set X. Illustrate it with a non-trivial example.

(d) Let $A \subseteq \mathbb{R}$. Show that for any $r \in \mathbb{R}$, $m^*(A) = m^*(A + r)$.

(e) Define a measurable function. Let E be a nonmeasurable subset of $\mathbb{R}$. Verify that χ_E is nonmeasurable.

(f) State bounded convergence theorem.

(g) When is a function $f : [0,1] \to \mathbb{R}$ said to be essentially bounded? Illustrate with an example.

(h) When is a function $f : [a,b] \to \mathbb{R}$ said to be a function of bounded variation? If $f, g \in BV[a,b]$, then show that $f + g \in BV[a,b]$.

(i) If a measurable function f is integrable over a measurable set E, then show that $|f|$ is integrable over E.

(j) State Fatou's Lemma. Mention an example to illustrate that we may have strict inequality in Fatou's Lemma.

2. Answer any Two $2 \times 7 = 14$

(a) Prove that the collection of all measurable sets is a σ-algebra of sets on $\mathbb{R}$.

(b) Let $E \subseteq [0,1)$ be measurable. Let $y \in [0,1)$. Prove that $E \dot{+} y$(the translate modulo 1 of E by y) is measurable and moreover, $m(E) = m(E \dot{+} y)$.

(c) Let $D \subseteq \mathbb{R}$ be measurable. Let f be an extended real-valued function defined on D. Prove that the following statements are equivalent:

(i) For any $\alpha \in \mathbb{R}$, $\{x \in D | f(x) > \alpha\}$ is measurable.

(ii) For any $\alpha \in \mathbb{R}$, $\{x \in D | f(x) < \alpha\}$ is measurable.

3. (a) Prove that every Borel set is measurable. 5

(b) Let f be a nonnegative measurable function which is integrable over a set E. Then prove that the following holds: Given $\epsilon > 0$, there exists a $\delta > 0$ such that for any measurable set $A \subseteq E$ with $mA < \delta$, we have $\int_A f < \epsilon$. 5

(c) Let $\phi = \sum_{i=1}^{n} a_i \chi_{E_i}$ with $E_i \cap E_j = \emptyset$ for $i \neq j$. Suppose that each E_i is a measurable set of finite measure. Show that $\int \phi = \sum_{i=1}^{n} a_i m(E_i)$. 4

OR

3. (a) If f is a measurable function and $f = g$ a.e., then prove that g is measurable. 5

(b) Let f be integrable over E and $c \in \mathbb{R}$. Prove that cf is integrable over E, and moreover, $\int_E cf = c\int_E f$. 5

(c) If $f \in BV[a,b]$, then show that $P_a^b f - N_a^b f = f(b) - f(a)$. 4

4. Answer any Two $2 \times 7 = 14$

(a) Prove Fatou's Lemma.

(b) Let $< f_n >$ be a sequence of measurable functions defined on E and f be a measurable real-valued function defined on E such that $f_n \to f$ in measure on E. Show that there is a subsequence $< f_{n_k} >$ which converges to f almost everywhere on E.

(c) If f is integrable on $[a,b]$ and $\int_a^x f(t)dt = 0$ for all $x \in [a,b]$, then prove that $f(t) = 0$ a.e. on $[a,b]$.

5. Answer any Two $2 \times 7 = 14$

(a) State and prove Holder's inequality.

(b) Let $E \subseteq \mathbb{R}$. Prove that the following statements are equivalent:

(i) E is measurable.

(ii) Given $\epsilon > 0$, there is a closed set $F \subseteq E$ such that $m^*(E \setminus F) < \epsilon$.

(c) Let E be a measurable set with $m(E) < \infty$. Let $f : E \to \mathbb{R}$ be bounded. Let $A = \{\psi : E \to \mathbb{R} | \psi$ is simple and $\psi \geq f\}$ and $B = \{\phi : E \to \mathbb{R} | \phi$ is simple and $\phi \leq f\}$. If $\inf\{\int_E \psi | \psi \in A\} = \sup\{\int_E \phi | \phi \in B\}$, then show that f is measurable.

(d) Let E be as in (c). Let $< f_n >$ be a sequence of measurable functions defined on E. Let f be a measurable real-valued function such that $f_n(x) \to f(x)$ for each $x \in E$. Prove that given $\epsilon > 0$ and $\delta > 0$, there is a measurable set $A \subseteq E$ with $m(A) < \delta$ and N such that for all $x \notin A$ and for all $n \geq N$, $|f_n(x) - f(x)| < \epsilon$. □

XA-147

003-016102

M. Sc. Mathematics (CBCS) Sem - I Examination

December 2014

MATH. CMT-1002 : REAL ANALYSIS

Faculty Code : 003

Subject Code : 016102

Time : 2½ Hours] [Total Marks : 70

1. Answer any Seven : $7 \times 2 = 14$

(a) Define a measurable set. Let E be a subset of R such that $m^*(E) = 0$. Show that any subset of E is measurable.

(b) Define a measurable function. Show that any continuous function $F : R \to R$ is measurable.

(c) Define a simple function and illustrate it with an example.

(d) State Monotone convergence theorem.

(e) Define the collection of Borel sets. Can a nonmeasurable set a Borel set ? If not, then why ?

(f) Let f be measurable function. When do we say that f is integrable over a measurable set E ? Verify that the characteristic function of any countable subset of R is integrable over R.

(g) Let $f : [2, 3] \to R$. When is f said to be a function of bounded variation over [2, 3] ?

(h) Let $f : [0,1] \to R$. What is the meaning of saying that f is continuous almost everywhere on [0,1] ?

(i) Define m^*A for any subset A of R. Determine $m^*([4, 8] \cup [10, 12])$.

(j) Let $f : R \to R$ be defined by $f(0) = 0$ and $f(x) = x \sin(1/x^2)$ for $x \neq 0$. Find all the four derivatives of f at $x = 0$.

2. Answer any two : $2 \times 7 = 14$

(a) Let $\{A_n\}$ [illegible] collection of subsets of R. Prove that $m^*(\cup A_n) \leq \Sigma m^* A_n$.

(b) Prove that for any a ∈ R, the interval (a, ∞) is measurable.

(c) Let c be any real [illegible] be measurable functions. Prove that the functions f + c, [illegible] also measurable.

(a) Let f be a bounded [illegible] f is Riemann [illegible] [a, b], then prove that f is measurable.

(b) Let f be a bounded measurable function defined on a measurable set E of finite measure. Prove that for any $a \in \mathbb{R}$, $\int_E af = a\int_E f$.

(c) State Fatou's lemma. Show that we may have strict inequality in Fatou's lemma.

OR

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be nonnegative and integrable. Prove that the function F defined by $\int_{-\infty}^{x} f$ is continuous. 5

(b) Let f, g be integrable over a measurable set E. If $f \leq g$ almost everywhere on E, then prove that $\int_E f \leq \int_E g$. 5

(c) Show that there exists a sequence $< f_n >$ of measurable functions which converges to the zero function in measure on [0,1] but $< f_n(x) >$ does not converge for any $x \in [0, 1]$. 4

4. Answer any two : $2 \times 7 = 14$

(a) Prove Lebesgue convergence theorem.

(b) Let $E \subseteq \mathbb{R}$. Prove that E is measurable if and only if given $\epsilon > 0$, there exists an open set O in $\mathbb{R}$ such that $O \supseteq E$ and $m^*(O \setminus E) < \epsilon$.

(c) Prove that a function f is of bounded variation on [a, b] if and only if f is the difference of two monotonically increasing functions on [a, b].

5. Answer any two : $2 \times 7 = 14$

(a) If E is measurable, then prove that for any $y \in \mathbb{R}$, E + y is measurable and moreover, $mE = m(E + y)$.

(b) State and prove Egoroff's theorem.

(c) Let f be integrable on [a, b]. If $\int_a^x f = 0$ for all $x \in [a, b]$, then prove that $f(t) = 0$ a.e. in [a, b].

(d) If f and g are bounded measurable functions defined on a measurable set E of finite measure, then prove that $\int_E f + g = \int_E f + \int_E g$.

003-016102

M.Sc. (MATHS) (CBCS) - (Sem.-I) Examination

November-2013

CMT-1002 : Mathematics (Real Analysis)

Faculty Code : 003

Subject Code : 016102

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer any seven : 7 × 2 = 14

(a) Define a σ-algebra of sets on a nonempty set X. Verify that there exists a subset A of N such that both A and A^c are not finite.

(b) Define a measurable function. If the characteristic function of a subset E of R is measurable, then show that E is a measurable set.

(c) State bounded convergence theorem.

(d) Let A ⊆ R. Define m^*A. If for some subset A of R, $m^*A = 0$, then verify that $m^*(A \cup B \cup C) = m^*(B \cup C)$ for any subsets B, C of R.

(e) Let f be a nonnegative measurable function defined on a measurable set E. Define $\int_E f$.

(f) Let f be a real-valued function defined on R. Define f^+ and f^-. For any real valued functions f, g defined on R, verify that $(f+g)^+ \le f^+ + g^+$.

(g) Let $f : [a, b] \to R$ be such that $f(x) \le f(y)$ for any $x, y \in [a, b]$ with $x \le y$. Prove that $f \in BV[a, b]$.

(h) Let X be a normed linear space over R. Define a Cauchy sequence in X.

(i) State Holder inequality.

(j) Let $f : [0, 1] \to \{0, 1\}$ be the characteristic function of $Q \cap [0, 1]$. Determine the upper Riemann integral of f over [0, 1].

2. Answer any two : $2 \times 7 = 14$

(a) Let $a, b \in R$ be such that $a < b$. Prove that $m^*[a, b] = b - a$.

(b) Let $A \subseteq R$ and $E_1, \ldots, E_n$ be a finite sequence of disjoint measurable sets. Prove that $m^*(A \cap (\cup_{i=1}^{n} E_i)) = \sum_{i=1}^{n} m^*(A \cap E_i)$.

(c) Let f be an extended real-valued function defined on a measurable set D. Prove that the following statements are equivalent :

(i) $\{x \in D : f(x) > \alpha\}$ is measurable for any $\alpha \in R$.

(ii) $\{x \in D : f(x) < \alpha\}$ is measurable for any $\alpha \in R$.

(a) If f and g are bounded measurable functions defined on a measurable set E of finite measure, then prove that $\int_E f + g = \int_E f + \int_E g$. 5

(b) Let f be a measurable function which is integrable over a measurable set E. Prove that for any real number c, cf is integrable over E. 5

(c) Let $A \subseteq R$. When is A called an F_σ? Prove that any open set in R is an F_σ. 4

OR

(a) Let $E \subseteq R$ be measurable. Prove that given $\epsilon > 0$, there is an open set O in R such that $O \supseteq E$ and $m(O \setminus E) < \epsilon$. 5

(b) Prove Fatou's lemma. 5

(c) Let f, g be functions defined on a measurable set D. If f is measurable and f = g a.e on D, then prove that g is measurable. 4

4. Answer any three. Part (d) is compulsory.

(a) Let X be a nonempty set and C be a nonempty collection of subsets of X. Prove that there is a smallest algebra R of subsets of X which contains C.

(b) Let f be a nonnegative integrable function. Show that the function F defined by $F(x) = \int_{-\infty}^{x} f$ is continuous. 3

(c) If f is a function of bounded variation on [a, b], then prove that $T_a^b f = P_a^b f + N_a^b f$. 5

(d) Let $f : R \to R$ be the function defined by $f(x) = |x|$. Find $D^+ f(0)$, $D_+ f(0)$, $D^- f(0)$ and $D_- f(0)$. 4

5. Answer any two : $2 \times 7 = 14$

(a) Let $< E_n >$ be an infinite decreasing sequence of measurable sets and let mE_1 be finite. Prove that $m(\cap_{i=1}^{\infty} E_i) = \lim_{n \to \infty} m E_n$.

(b) If every absolutely summable series in a normed linear space X is summable, then prove that X is complete.

(c) Prove Holder inequality.

(d) Prove that the collection of measurable sets is a σ-algebra.

SC-003016102

SC-003016102 SeatNo.__________

M.Sc. Mathematics (CBCS) Semester-01

October / November - 2012

CMT-1002: Real Aanlysic

Time : $2\frac{1}{2}$ Hours Total Marks : 70

Instructions:

(i) All question are compulsory.

(ii) Each question carries 14 marks.

Q:1 Answer Any Seven 14

a) Define a σ-algebra of sets on a nonempty set X.

b) Define (i) a F_σ-set (ii) the collection of Borel sets.

c) Define Lebesgue outer measure. $m^*((0,1)\cup(1,3))$

d) Let $E \subseteq R$. Verify that for nay subset A of R. $m^* A \le m^*(A \cap E) + m^*(A \cap E^c)$

e) Let f, g be real-valued functions defined on R. When de we say that $f = g$ almost everwhere?

f) Give an example of a setp function defined on [0,2]. Define the concept of a simple function.

g) State Fatou's lemma.

h) If f is integrable over E, then show that $|f|$ is integrable over E.

i) Let $f : R \to R$ be given by $f(0)$ 0 and $f(x) = x\sin(1/x)$ if $x \neq 0$. Find $D^+ f(0)$

j) Define a normed linear space over the field of real numbers.

19

14

Q:2 Answer Any Two

a. Let $X = R$. Let R be the collection of subsets A of X such that either A is countable or A^c is countable. Show that R is a σ-algebra of sets.

b. Let A be any subset of R. Prove the following statements:

(i) Given any $\epsilon > 0$, there exists an open set $O \subseteq R$ such that $A \subseteq O$ and $m^* O \leq m^* A + \epsilon$.

(ii) There is a subset G of R which is of type G_δ such that $A \subseteq G$ and $m^* A = m^* G$.

c. Prove the following:

(i) If $m^* E = 0$, then E is measurable.

(ii) If E_1 and E_2 are measurable then so is $E_1 \cup E_2$

Q:3 a. Prove that every Borel set is measurable. 05

b. Let E be measurable, Let $< f_n >$ be a sequance of measurable functions define on E. Prove that the functions $\sup\{f_1, \ldots, f_n\}$, $\inf\{f_1, \ldots, f_n\}$, $\sup\{f_n | n$ $\inf\{f_1, \ldots, f_n\}, \in N\}$ are all measurable 05

c. Let $f : [2,3] \to R$ be given by $f(x) = 1$ if x is rational and $f(x) = 0$ if x is irrational. Prove that f is not Riemann integrable on [2,3]. 04

OR

Q:3 a. If f and g are bonded measurable functions defined on a set E of finite measure, then prove that $\int_E f + g = \int_E f + \int_E g$. 14

b. Let $<f_n>$ be an increasing sequence of nonnegative measurable functions and let $f = \lim f_n$, prove that $\int f = \lim f_n$.

c. Let f of integrable over E. Let $\alpha \in R$. Prove that αf is integrable over E.

Q:4 Answer Any Two: 14

a. Let g be intebrable over E and let $< f_n >$ be a sequence of measurable functions such that $|f_n| \leq g$ on E and $f(x) = \lim f_n(x)$ for almost all $x \in E$. Prove that $\int_E f = \lim \int_E f_n$

b. Prove that a function f is of bounded variation on [a, b] if and only if f is the difference of two increasing real-valued functions on [a,b].

HF
I. Sc. (C

c. Let $f : [0,1] \to R$ be an essentially bounded measurable function. Prove that $|f| \leq \|f\|_\infty$ almost everywhere on $[0,1]$. If $f, g : [0,1] \to R$ are essentially bounded measurable functions, then show that $\|f + g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$ 14

Q:5 Answer Any Two)

a. State and prove Minkowski inequality.

b. Prove the following statements

(i) m^* is translation invariant.

(ii) If E_1 and E_2 are measureable, then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

c. Let and ϕ and ψ be simple functions which vanish outside a set of finite measure. Prove that for any real numbers a and b, $\int (a\phi + b\psi) = a\int\phi + b\int\psi$.

d. If f is integrable on [a, b] and $\int_a^x f = 0$ for all $x \in [a, b]$, then prove that $f(t) = 0$ a.e. on [a,b].

(1) L^p space

(2) vector space

(3) n.l.s

(4) essentially Bounded

(5) Holder inequality

~~(6)~~ minkowski's in

(7) convergence & complete

(8) convergent sequence

(9) cauchy

(10) complete, Banach space

Jay Mataje (11) Riesz-Fischer thm

(12) series

(13) summable

(14) Absolutely summable

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4 unit (1) ~~Vitali thm~~

(2) $(D^+F)(x)$ $(D^-F)(x)$

(3) Fn of bdd variation

(4) lipschitz condition

003-016102

M. Sc. (Sem. I) (CBCS) (Mathematics) Examination

December - 2011

MATH.CMT - 1002 : Real Analysis

Faculty Code : 003

Subject Code : 016102

Time : Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.

(2) Each question carries 14 marks.

1. Answer any Seven $7 \times 2 = 14$

(a) Define the collection of Borel sets. Let $F \subseteq \mathbb{R}$ be closed. Verify that F is a Borel set.

(b) For any subset A of $\mathbb{R}$, define $m^*(A)$. Show that $m^*(\{r\}) = 0$ for any $r \in \mathbb{R}$.

(c) Define a measurable function. If $P \subset [0, 1)$ is a nonmeasurable set, then verify that the characteristic function of P is not a measurable function.

(d) State bounded convergence theorem.

(e) Let $E \subseteq \mathbb{R}$ and let f be an extended real valued function defined on E. Define f^+, f^-, and show that $f(x) = f^+(x) - f^-(x)$ for each $x \in E$.

(f) Let $< f_n >$ be a sequence of real-valued measurable functions defined on E. Let f be a real-valued measurable function defined on E. When do we say that $< f_n >$ converges to f in measure?

(g) Let $f : [0, 1] \to \mathbb{R}$ be increasing. Verify that f is of bounded variation over $[0, 1]$.

(h) Let $(X, \| \ \|)$ be a normed linear space. Let $d : X \times X \to \mathbb{R}$ be given by $d(x, y) = \|x - y\|$. Show that d is a metric.

(i) Define a measurable set. If $E \subset \mathbb{R}$ is such that $m^*(E) = 0$, then verify that $\mathbb{R} \setminus E$ is measurable.

(j) Define an algebra of sets on a set X. Let $\mathcal{R}$ be an algebra of sets on a set X. If $A_1, A_2 \in \mathcal{R}$, then prove that there exist $B_1, B_2 \in \mathcal{R}$ such that $B_1 \cap B_2 = \emptyset$ and $A_1 \cup A_2 = B_1 \cup B_2$.

2. Answer any Two $2 \times 7 = 14$

[illegible] Illustrate by means of an example [illegible] need not be a σ-algebra of sets. Let C be a [illegible] that there is a smallest σ-algebra of sets on [illegible]

[illegible] ation of sets with $A_n \in \mathbb{R}$ for each n, then [illegible]

4 Answer any three, among first two have marks five, five each and third has four marks.

(1) Let $E \in m$ and $f : E \to \mathbb{R}$ be a map. Then prove that followings are equivalent.

(i) $\{x \in E \mid f(x) > \alpha\} \in m, \forall x \in \mathbb{R}$

(ii) $\{x \in E \mid f(x) \geq \alpha\} \in m, \forall x \in \mathbb{R}$

(iii) $\{x \in E \mid f(x) < \alpha\} \in m, \forall x \in \mathbb{R}$

(iv) $\{x \in E \mid f(x) \leq \alpha\} \in m, \forall x \in \mathbb{R}$

(2) Fatous' lemma state and prove.

(3) Define following terms :

(i) Convergence sequence in a nls

(ii) Cauchy sequence in a nls

(iii) Complete normed linear space

(iv) Summable sequence

(v) Absolute summable sequence.

(4) A nls $(X, \|\cdot\|)$ is complete iff every absolute summable sequence $\langle x_n \rangle$ in X is also a summable sequence in X, prove it.

5 Answer any two :

(1) State and prove Vitali's lemma. 7

(2) State and prove Holder's inequality. 7

(3) Let $X = [0,1)$ and the relation "~" on X defined by $x \sim y$ iff $x - y \in Q$, for any $x, y \in X$, which makes X into equivalence classes $E_\lambda, \lambda \in \wedge$ with $E_{\lambda_0} = [0,1) \cap Q$, for some $\lambda_0 \in \wedge$. If we choose P which contains exactly one element from each E_λ and $P_i = P + r_i$, for each $r_i \in E_{\lambda_0}$, then prove that $\cup P_i = X$ and each P_i's are non-measurable sets. 7

(4) Let f be a bounded function on a measurable set E with $mE < \infty$. Then prove that the necessary and sufficient condition for $\inf_{\psi \geq f} \int_E \psi = \sup_{\phi \leq f} \int_E \phi$, where ϕ [illegible] on E. 7

003-016102 / A-32 Seat No. ________

M. Sc. (Sem. I) (Maths) Examination

November / December - 2019

CMT-1002 : Real Analysis

Faculty Code : 003

Subject Code : 016102

Time : 3 Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.

(2) Each question carries 14 marks.

1 Answer any seven objective type questions : 14

(1) Which of followings may not hold for a boolean algebra on a set X ? .

(A) If $A, B \in G$ then $A^c \cap B^c \in G$

(B) If $A_1, A_2, ..., A_n \in G$ then $A_1 \cap A_2 \cap ... A_n \in G$

(C) If $\langle A_i \rangle \subseteq G$, then $\bigcup_{i=1}^{\infty} A_i \in G$

(D) If $A, B \in G$, then $A \cup B^c \in G$

(2) Every Borel set is always ________ set.

(A) finite (B) measurable

(C) uncountable (D) empty

(3) The set of all irrationals $\mathbb{R} \setminus \mathbb{Q}$ is not ________ set.

(A) measurable (B) G_δ -set

(C) F_σ – set (D) infinite

(4) For any $A \subseteq \mathbb{R}$, $m^*(A+x) =$ ________ , where $x \in \mathbb{R}$

(A) $m^* A$ (B) $m^* A + x$

(C) $m^* A + x + y$ (D) None of these

(5) The outer measure for the Cantor set C is ________.

(A) 1 (B) $\frac{1}{3}$

(C) $1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - ......$ (D) 0

(6) The relation "~" on $X = [0,1)$ defined by for any $x, y \in X$, $x \sim y$ if $x - y \in Q$ is ______

(A) not reflexive (B) not symmetric

(C) not transitive (D) an equivalence relation

(7) Range of a simple function is ______ set.

(A) a finite (B) an infinite

(C) a countable (D) an uncountable

(8) The characteristic function $\chi_{Q \cap [a,b]}$ on interval $[a,b]$ is ______ map.

(A) a Reimann integrable

(B) a Lesbegue integrable

(C) Reimann and Lebegue integrable

(D) a continuous

(9) If $f(x) = |x|$, then the value of $D_+ f(0)$ is ______.

(A) -1 (B) 0

(C) 1 (D) ∞

(10) In standard notation value of $|f| - f$ is ______.

(A) $2f^+$ (B) $2f^-$

(C) 0 (D) $2f$

2 Answer any two :

(1) Find a σ-algebra generated by the collection $\{\phi, \{a,b\}, \{c,d\}\}$ on X, where $X = \{a, b, c, d\}$. 7

(2) Show that energy Borel set is a measurable set. 7

(3) State and prove Bounded convergence theorem. 7

3 (1) Prove that the collection of all measurable set m is a σ-algebra. 7

(2) State and prove generalized Fatou's Lemma. 7

OR

3 (1) Let $X \neq \phi$ and $\mathcal{C} \subseteq P(X)$. Then prove that there is a smallest σ-algebra on X which contains the given collection $\mathcal{C}$. 4

(2) State and prove monotone convergence theorem. 5

(3) State and prove Minhoweski inequality. 5

4 Answer any three, among first two have marks five each and third has four marks. 14

(1) Let $E \in m$ and $f : E \to \mathbb{R}$ be a map. Then prove that followings are equivalent.

(i) $\{x \in E \mid f(x) > \alpha\} \in m, \forall\ x \in \mathbb{R}$

(ii) $\{x \in E \mid f(x) \geq \alpha\} \in m, \forall\ x \in \mathbb{R}$

(iii) $\{x \in E \mid f(x) < \alpha\} \in m, \forall\ x \in \mathbb{R}$

(iv) $\{x \in E \mid f(x) \leq \alpha\} \in m, \forall\ x \in \mathbb{R}$

(2) Fatous' lemma state and prove.

(3) Define following terms :

(i) Convergence sequence in a nls

(ii) Cauchy sequence in a nls

(iii) Complete normed linear space

(iv) Summable sequence

(v) Absolute summable sequence.

(4) A nls $(X, \|\cdot\|)$ is complete iff every absolute summable sequence $\langle x_n \rangle$ in X is also a summable sequence in X, prove it.

5 Answer any two :

(1) State and prove Vitali's lemma. 7

(2) State and prove Holder's inequality. 7

(3) Let $X = [0,1)$ and the relation "$\sim$" on X defined by $x \sim y$ if $x - y \in \mathbb{Q}$, for any $x, y \in X$, which makes X into equivalence classes $E_\lambda, x \in \wedge$ with $E_{\lambda_o} = [0,1) \cap \mathbb{Q}$, for some $\lambda_o \in \wedge$. If we choose P which contains exactly one element from each E_λ and $P_i = P + r_i$, for each $r_i \in E_{\lambda_o}$, then prove that $\cup P_i = X$ and each P_i's are non-measurable sets. 7

(4) Let f be a bounded function on a measurable set E with $mE < \infty$. Then prove that the necessary and sufficient condition for $\inf_{\psi \geq f} \int_E \psi = \sup_{\phi \leq f} \int_E \phi$ where ϕ and ψ are simple function is f be a measurable function on E. 7

Saurashtra University

Department of Mathematics

Semester-1 Real Analysis Test No.2 12.10.11

Answer any 10 questions. Each question carries 2 marks.

1. Let $\phi : \mathbb{R} \to \mathbb{R}$ be a simple function such that $m\{x \in \mathbb{R} : \phi(x) \neq 0\} < \infty$. Define $\int_{\mathbb{R}} \phi$.

2. Let E be measurable. When is a function from E into $\mathbb{R}$ said to be bounded? If $m(E) < \infty$ and $f : E \to \mathbb{R}$ is bounded and measurable, then define $\int_E f$.

3. State bounded convergence theorem.

4. State Fatou's lemma. Illustrate with the help of an example that we may have strict inequality in Fatou's lemma.

5. Let f be a non-negative measurable function defined on a measurable set E. Define $\int_E f$. State Monotone convergence theorem.

6. Let f be a nonnegative measurable function defined on $\mathbb{R}$. Let $\mathcal{M}$ denote the collection of all Lebesgue measurable sets.
Let $\mu : \mathcal{M} \to \{r \in \mathbb{R} : r \geq 0\} \cup \{\infty\}$ defined by $\mu(E) = \int_E f$. Verify that μ is a measure. 7. Let f be as in 6. If f is integrable over $\mathbb{R}$, then prove that the function F given by $F(x) = \int_{(-\infty,x)} f$ is continuous.

8. Let f be a measurable function. When is f said to be integrable over a measurable set E? Verify that f is integrable over E if and only if $|f|$ is integrable over E.

9. State Lebesgue convergence theorem.

10. Let $< f_n >$ be a sequence of real-valued functions defined on a measurable set E and let f be a real-valued measurable function defined on E. If $< f_n >$ converges to f in measure on E, then verify that any subsequence of $< f_n >$ also has the same property.

11. Let $< f_n >$ be a sequence of measurable functions defined on a measurable set E. Let f be an extended real-valued function defined on E. When do we say that $f_n \to f$ a.e. on E?

12. State Riemann-Lebesgue theorem.

13. Verify that $\chi_{\mathbb{Q}\cap[0,1]}$ is not Riemann integrable over $[0,1]$.

14. Illustrate that the Monotone convergence theorem need not hold for decreasing sequence of functions.

15. Let f be a non-negative measurable function defined on E. If $\int_E f = 0$, then verify that $f = 0$ a.e on E.

Best of luck

Test No. 2

(c) Let $< E_n >$ be an infinite decreasing sequence of measurable sets. Let $m(E_1) < \infty$. Prove that $m(\bigcap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} m(E_n)$. 99

3. (a) Let f and g be measurable real-valued functions defined on E. Let $c \in \mathbb{R}$. Prove that the functions $f + c, cf,$ and $f + g$ are measurable. 5
(b) Let E be a measurable set of finite measure. Let $< f_n >$ be a sequence of measurable functions defined on E. Let f be a measurable real-valued function such that $f_n(x) \to f(x)$ for each $x \in E$. Then, given $\epsilon > 0$ and $\delta > 0$, prove that there is a measurable set $A \subseteq E$ with $m(A) < \delta$ and an integer N such that for all $x \notin A$ and all $n \geq N$, $|f_n(x) - f(x)| < \epsilon$. 5
(c) Let f, g be bounded measurable functions defined on a set E of finite measure. If $f \leq g$ almost everywhere on E, then prove that $\int_E f \leq \int_E g$. 4

250

Or

3. (a) State and prove Fatou's lemma. 232 5
(b) Let f be a nonnegative measurable function which is integrable over E. Let $\epsilon > 0$ be given. Prove that there is a $\delta > 0$ such that for any measurable set A with $A \subseteq E$ and $m(A) < \delta$, $\int_A f < \epsilon$. 287 5
(c) Let f and g be integrable over E. Show that $f + g$ is integrable over E. 4 288

4. Answer any Three. Part(a) is compulsory.
(a) Let $< f_n >$ be a sequence of measurable real-valued functions defined on E. Let f be a measurable real-valued function defined on E such that $< f_n >$ converges in measure to f on E. Prove that there exists a subsequence $< f_{n_k} >$ which converges to f almost everywhere on E. 320 5
(b) Let $f : [a, b] \to \mathbb{R}$ be of bounded variation over $[a, b]$. Prove that $T_a^b(f) = P_a^b(f) + N_a^b(f)$ and moreover, prove that $f(b) - f(a) = P_a^b(f) - N_a^b(f)$. 5 326
(c) If f is integrable on $[a, b]$ and if $\int_{[a,x]} f(t)dt = 0$ for all $x \in [a, b]$, then prove that $f = 0$ a.e. on $[a, b]$. 44 (b-3) 5
(d) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x\sin(1/x)$ if $x \neq 0$ and $f(0) = 0$. Find $(D^+ f)(0)$, $(D_+ f)(0)$, $(D^- f)(0)$, and $(D_- f)(0)$. 4 b-3 15

Answer any Two 2 × 7 = 14

5. State and prove Hölder's inequality. 87
6. (i) If E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable. 77
(ii) Let $A \subseteq \mathbb{R}$. Let $E_1, E_2, \ldots, E_n$ be a finite disjoint sequence of measurable sets. Show that $m^*(A \cap (\bigcup_{i=1}^{n} E_i)) = \sum_{i=1}^{n} m^*(A \cap E_i)$. 79
7. State and prove Lebesgue convergence theorem. 293
8. Let f be integrable function on $[a, b]$. If $F : [a, b] \to \mathbb{R}$ is given by $F(x) = \int_a^x f(t)dt$, then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$. 47 (b-3)

Saurashtra University
Department of Mathematics
M.Sc. Semester-1, Test No.1, CMT-1002, Real Analysis,
9-9-16

Answer any 10 questions, Each question carries 2 marks.

1. Define an *algebra of sets* on a nonempty set X. Describe (with no proof) the smallest algebra of sets on X containing $\mathcal{C} = \{\{x\} | x \in X\}$.
2. Let $\mathcal{R}$ be an algebra of sets on a nonempty set X. For any $A, B \in \mathcal{R}$, show that $A \Delta B \in \mathcal{R}$.
3. Define the concept of σ*-algebra of sets* on a nonempty set X. Let $\mathcal{S}$ be a σ-algebra of sets on a set X. If $A_n \in \mathcal{S}$ for each $n \in \mathbb{N}$, then prove that $\cap_{n=1}^{\infty} A_n \in \mathcal{S}$.
4. Let A be a nonempty set and $\mathcal{C}$ be a subcollection of the collection of all subsets of A. Define the notion of σ*-algebra of sets generated* by $\mathcal{C}$. Define the concept of *Borel sets*.
5. Let $F \subseteq \mathbb{R}$. When is F said to be of *type* F_σ?
6. Mention an example (with brief details and no proof) of $F \subseteq \mathbb{R}$ such that F is of type F_σ, but F^c is not of type F_σ.
7. Let $G \subseteq \mathbb{R}$ be of type G_δ. Show that G^c is of type F_σ.
8. Verify that m^* is monotone.
9. Let $I = [0, 2)$ and $J = [2, 3]$. Find $m^*(I \cup J)$ and $m^*(I \cap J)$.
10. Define the concept of a measurable set. Verify that [illegible]
11. If $E \subseteq \mathbb{R}$ is measurable, then show that E^c is measurable. Mention the reason that the statement *the set of all irrationals is not measurable* is false.
12. Let $A, E \subseteq \mathbb{R}$. For any $y \in \mathbb{R}$, verify that $A \cap (E + y)^c = (A - y) \cap E^c$.
13. For each $n \in \mathbb{N}$, let $E_n \subseteq \mathbb{R}$ be such that $m^*(E_n) = 0$. Show that $m^*(\cup_{n=1}^{\infty} E_n) = 0$.
14. Is any algebra of sets on $\mathbb{N}$, a σ algebra of sets? If not, then why?
15. Let $A \subseteq \mathbb{R}$ be such that A admits a subset B which is not measurable. Verify the assertion that $m^*(A) > 0$.

Seat No. ______________

F8Z-003-1161003

M. Sc. (Sem. I) (CBCS)

Examination

December - 2022

Mathematics : 1003

(Topology - 1)

Faculty Code : 003

Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) There are five questions.
(2) Answer all the questions.
(3) Each question carries 14 marks.

1 Answer any seven of the following : $7 \times 2 = 14$

(1) Define (i) Discrete topology (ii) Indiscrete topology.

(2) Give an example of a set X and a sub collection of P(X), which is not topology.

(3) Define first and second projection map on $X \times Y$.

(4) Define interior and closure of a topological space.

(5) Define with example : Homeomorphism.

(6) Define with example : Separation of a topological space.

(7) Define with example : Locally connected space.

(8) Define : Linear Continuum.

(9) When a sequence is said to be converges uniformly?

(10) Define with example : Strictly finer topology.

2 Answer any two from the following questions. 2×7=14

(a) Let $'\mathcal{B} = \{(a, b) \mid a < b,\ a, b \in \mathbb{R}\}$ prove that, $'\mathcal{B}$ is a basis for some topology on $\mathbb{R}$.

(b) Let Y be a subspace of X. Prove that, $F \subset Y$ is a closed subset of Y if and only if $F = K \cap Y$ for some closed subset K of X.

(c) Let A be a subset of a topological space X. Prove that, $x \in \overline{A}$ if and only if every open set U containing x intersect A.

3 Answer the following both questions. 2×7=14

(a) Consider $\mathbb{R}$ with standard topology, Let A = (0, 1). Find A'.

(b) If $f : X \to Y$ is continuous and $g : Y \to Z$ is continuous then prove that, $g \circ f : X \to Z$ is also a continuous map.

OR

3 Answer the following both questions. 2×7=14

(a) Let (X, d) is a metric space. Prove that, $'\mathcal{B} = \{B_d(x, \in) / x \in X, \in > 0\}$ is a basis for the metric space (X, d).

(b) State and prove, sequence lemma.

4 Answer the following both questions. 2×7=14

(a) Prove that, a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

(b) State and prove, intermediate value theorem.

5 Answer any two from the following questions. **2×7=14**

(a) Prove that, lower limit topology on $\mathbb{R}$ is strictly finer than the standard topology on $\mathbb{R}$.

(b) Let X be an infinite set.

Let $\tau = \{G \subset X \mid G = \phi \text{ or } X - G \text{ is a finite set}\}$ prove that, τ is a topology on X.

(c) Prove that, if a space X is locally path connected and connected then it is path connected.

(d) State and prove, pasting lemma.

M. Sc. (Sem. I) Examination

February - 2022

Mathematics : CMT-1003

(Topology - I)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal (14) marks.

1 Answer the following :

(1) Define with example : Co-finite Topology on a Set.

(2) Define with example : Homeomorphism.

(3) Define with example : Lower limit Topology on $\mathbb{R}$.

(4) Define with example : Convex set.

(5) Is boundedness a topological property ? Justify your answer.

(6) Define with example : Product Topology

(7) Define with example : Convergence of a sequence.

2 Answer the followings :

(1) Define with example : Path between two elements in topological space.

(2) Define with example : Square Metric.

(3) Define with example : Limit point of a set.

(4) Let X and Y be topological space. Consider the function $\pi_1 : X \to Y$ defined by $\pi_1(x, y) = x$, for all $(x, y) \in X \times Y$. Is π_1 is continuous ? Justify your answer.

(5) Define with example : Connected Topological space.

(6) Let X be a discrete topological space and Y be a metric space. Let $f : X \to Y$ be any function. Is f continuous ? Justify your answer.

(7) Define with example : Metric space.

3 Answer the following :

(a) On the set of real numbers $\mathbb{R}$, Define $\tau_c \{U \subseteq \mathbb{R} / \mathbb{R} - U$ is either countable or all of $\mathbb{R}$? Prove that, τ_c is topology on $\mathbb{R}$.

(b) Let X be any set of B be a basis of X. Define $\tau = \{U \subseteq X :$ if $x \in U$ then there exists $B \in B$ such that $x \in B \subseteq U\}$. Prove that, τ is a topology on X.

4 Answer the followings :

(a) Let $(X, <)$ be a simply ordered set and

$$B = \{(a,b)/a,b \in X\} \cup \{[a_0, b)/a_0, b \in X\} \cup \{(a, b_0]/a, b_0 \in X\}$$

where a_0, b_0 are the smallest and largest elements in X. Prove that B is basis for X.

(b) Let B be a basis for the topology of X and $\mathcal{C}$ be a basis for the topology of Y. Prove that the collection $D = \{B \times C / B \in B \text{ and } C \in \mathcal{C}\}$ is a basis for the product topology of $X \times Y$.

5 Answer the following :

(a) State and prove, Pasting lemma.

(b) Prove that, the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology of $\mathbb{R}$, but are not comparable with each other.

6 Answer the followings :

(a) Let X and Y be two topological spaces. Let $f:X\to Y$ be a function. If f is continuous then prove that, for every subset A of $X, f(\overline{A})\subseteq\overline{f(A)}$.

(b) Let X, Y and Z be a topological spaces. Let A be a subset of X. Let $f:X\to Y$ and $g:Y\to Z$ be continuous functions. Prove that

(i) The inclusion function $j:A\to X$ is continuous

(ii) The composite function $g\circ f:X\to Z$ is continuous.

7 Answer the following :

(a) Let X and Y be topological spaces and π_1, π_2 be the projection maps. Prove that

$\mathcal{S}=\{\pi_1^{-1}(U)/U$ is open in $X\}\cup\{\pi_2^{-1}(V)/V$ is open in $Y\}$ is a sub basis for the product topology on $X\times Y$.

(b) Let X be a topological space and Y be a subspace of X. Prove that, a set A is closed in Y if and only if it equals the intersection of a closed set in X with Y.

8 Answer the following :

(a) Let A be subset of a topological space X; Let A' denote the set of all limit points of A. Prove that $\overline{A}=A\cup A'$.

(b) Let (X,d) be metric space.

Prove that, $B_d=\{B_d(x,r)/x\in X, r>0\}$ is a basis for the metric space (X,d).

9 Answer the followings :

(a) State and prove, Uniform limit theorem.

(b) Let d and d' be two metrics on the set X. Let τ and τ' be the topologies induced by d and d' respectively. Prove that, τ' is finer than τ if and only if for each x in X and each $\varepsilon > 0$, there exists a $\delta > 0$ such that $B_{d'}(x, \delta) \subset B_d(x, \varepsilon)$.

10 Answer the following :

(a) Let X be a topological space with a basis B for topology on X. Prove that, $x_n \to x$ if and only if for every basis element B containing x, there exists $n_0 \in \mathbb{N}$ such that $x_n \in B, \ \forall n \geq n_0$.

(b) Let X and Y be two metrizable spaces with metrics d_x and d_y, respectively. Let $f: X \to Y$ be a function. Prove that, f is continuous if and only if for given $x_0 \in X$ and $\in > 0$, there exists $\delta > 0$ such that

$$d_x(x, x_0) < \delta \Rightarrow d_y(f(x), f(x_0)) < \in.$$

MBP-003-1161003 Seat No. ________

M. Sc. (Sem. I) Examination

February - 2021

Mathematics : CMT-1003

(Topology-I)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Answer the following : **14**

(1) Define with example : Topological space.

(2) Let X be any set and $\{\tau_\beta\}$ be a collection of topologies on X. If $\tau = \bigcap_\beta \tau_\beta$ then prove that τ is a topology on X.

(3) Define with example : Basis of a set.

(4) Define with example :
 (i) Finer topology
 (ii) Coarser topology

(5) Define with example : Simple order relation.

(6) Let X and Y be topological spaces. Consider the function $\pi_1 : X \to Y$ defined by :

$$\pi_1(x, y) = x, \textit{for all}\,(x, y) \in X \times Y.$$

Is π_1 continuous ? Justify your answer.

(7) Define with example : Homeomorphism between two topological spaces.

2 Answer the following : **14**

(1) Is boundedness a topological property ? Justify your answer.

(2) Define with example :

(i) Sub basis

(ii) Subspace Topology

(3) Define with example : Convex subset of an ordered set.

(4) Define with example :

(i) Closure of a set

(ii) Limit point of a set

(5) Prove that the sequence $\left(\frac{1}{n}\right)_{n=1}^{\infty}$ converges to any $x \in \mathbb{R}$ in co-finite topology.

(6) Define with example : Metric topology.

(7) Define with example : Linear continuum.

3 Answer the following : **14**

(a) Let X be any set and $\mathbb{B}$ be a basis of X. Define $\tau = \{U \subset X \mid if\ x \in U \text{ then there exists } B \in \mathbb{B} \text{ such that } x \in B \subset U\}$ prove that τ is a topology on X.

(b) Let $\mathbb{B}_1$ and $\mathbb{B}_2$ are basis of the topologies τ_1 and τ_2 respectively, then the following are equivalent :

(i) τ_2 is finer than τ_1.

(ii) For each basis element $B_1 \in \mathbb{B}_1$ and $x \in B_1$, there is a $B_2 \in \mathbb{B}_2$ such that $x \in B_1 \subset B_2$.

4 Answer the following : **14**

(a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology of $\mathbb{R}$, but they are not comparable with each other.

(b) Prove that the topology generated by a sub basis δ is defined to be the collection τ of all unions of finite intersections of elements of δ.

5 Answer the following : **14**

(a) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

(b) Let X be an ordered set with ordered topology; let Y be a subset of X that is convex in X. Prove that the order topology on Y is same as the topology Y inherits as a subspace of X.

6 Answer the following : **14**

(a) Let X, Y be topological spaces and let $f : X \rightarrow Y$, then the following are equivalent :

(i) f is continuous.

(ii) For every subset A of X one has $f(\overline{A}) \subset \overline{f(A)}$.

(iii) For every closed subset B of Y, the set $f^{-1}(B)$ is closed in X.

(b) Let $f : X \rightarrow Y$ be continuous. If Z is a subspace of Y containing the image set $f(X)$, then prove that the function $g : X \rightarrow Y$ obtained by restricting the range of f is continuous. If Z is a space having Y as a subspace, then prove that the function $h : X \rightarrow Y$ obtained by expanding the range of f is continuous.

7 Answer the following : **14**

(a) State and prove the Pasting lemma.

(b) Let $f : A \rightarrow X \times Y$ be given by the equation

$$f(a) = (f_1(a), f_2(a)).$$

Prove that f is continuous if and only if the functions $f_1 : A \rightarrow X \times Y$ and $f_2 : A \rightarrow X \times Y$ are continuous.

8 Answer the following : **14**

(a) Let A be subset of a topological space X. Prove that $x \in \overline{A}$ if and only if every open set containing U intersects A.

(b) Let A be subset of a topological space X ; Let A' denote the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

9 Answer the following : **14**

(a) Let X be a metric space with metric d. Define $\overline{d} : X \times X \to \mathbb{R}$ by the equation

$$\overline{d} : (x, y) = \min\{d(x, y), 1\}.$$

Prove that $\overline{d}$ is a metrics that induces the same topology as d.

(b) State and prove the sequence lemma.

10 Answer the following : **14**

(a) Prove that a finite Cartesian product of connected spaces is connected.

(b) (i) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open is X.

(ii) If X is a topological space, each path component of X lies in a component of X. If X is locally path connected, then prove that the components and path components of X are same.

JBF-003-1161003

Seat No. ________

M. Sc. (Sem. I) Examination

December – 2019

Mathematics : CMT-1003

(Topology - I)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Answer any seven questions. $7 \times 2 = 14$

a) Define: Closed set. Give an example to show that arbitrary union of closed set need not be closed.

b) Prove that a space (X, τ) is a discrete space if and only if $\forall x \epsilon X, \{x\} \epsilon \tau$.

c) Define: Convergence sequence in a metric space.

d) State Hausdorff's Criterian.

e) Define: Interior of a set. If $A \subset B$ then prove that $A^\circ \subset B^\circ$.

f) Define: Continuity of a function at a point.

g) Prove that locally connectedness is topological property.

h) Define: Co-finite topology.

i) Define: Homeomorphism with an example.

j) Define: Locally path connected space.

2 Answer any two. $2 \times 7 = 14$

a) Prove that lower limit topology on $\mathbb{R}$ is finer than the standard topology on $\mathbb{R}$.

b) Prove that $\tau = \{U \subseteq \mathbb{R};$ for each $x \in U,$ there is an open interval $(a, b) \ni (a, b) \subset U\}$

c) Let (X, τ) be topological space. Then prove that

1) $X, \emptyset$ are closed set.
2) Arbitrary intersection of closed set is closed.
3) Finite union of closed set is closed.

3 Answer the following. **$2 \times 7 = 14$**

a) Let (X, τ) be topological space and Y be non-empty subset of X.
Let $\tau_Y = \{G \cap Y;\ G \in \tau\}$.

b) Let X and Y be topological spaces. Then prove that
$\mathcal{B}_{X \times Y} = \{U \times V; U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$ is a basis for some topology on $X \times Y$.

OR

a) If (X, d) be a metric space and $\mathcal{B} = \{Bd(x, \varepsilon)/x \epsilon X, \varepsilon > 0\}$ then prove that $\mathcal{B}$ is a basis for some topology on X.

b) Let X and Y be spaces. $A \subset X$ and $B \subset Y$. Then prove that $\overline{(A \times B)} = \bar{A} \times \bar{B}$

4 Answer any two. **$2 \times 7 = 14$**

a) Suppose X and Y are topological space and $f: X \to Y$ be any function. Prove that f is continuous iff f is continuous at every point of X.

b) State and prove Pasting Lemma.

c) Prove that

1) If $A \subset X$ then $\bar{A} = \{x \in X, \text{for any open set } U \text{ containing } x, U \cap A \neq \emptyset\}$.
2) If $A \subset X$ then $\bar{A} = A' \cup A$.

5 Answer any two. **$2 \times 7 = 14$**

a) Prove that $X \times Y$ is a locally path connected if and only if X and Y are locally path connected.

b) If X is connected and locally path connected then prove that X is path connected

c) Suppose X and Y are topological space. If $f: X \to Y$ is continuous and onto. If X is connected then prove that Y is also connected

d) Prove that

1) If C is a component and A is a connected set then either $A \cap C = \emptyset$ or $A \subset C$.
2) If C is a component then C is a maximal connected subset of X.
3) If C is a component then C is a closed subset of X.

HEI-003-1161003 Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

November / December - 2017

Mathematics : 1003

(Topology-I) (New Course)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions :

(1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries two marks) 14

(1) If every subset of X is closed set of X then the topology on X is .....topology.

(2) In a topological space X ______ and ______ are both open and closed set.

(3) In $\mathbb{R}$ the closure of the set of rational numbers is ______.

(4) If A contains all its limit points then A is ______ set.

(5) The set of natural numbers is a closed set in $\mathbb{R}$ when $\mathbb{R}$ has ______ topology.

(6) The number of components of a disconnected space is at least ______.

(7) A subset G of X is open if and only if G° = ______.

2 Attempt any two :

(a) Give an example to show that denumerable intersection of open set need not be open. 7

(b) Let A be a subset of X. Prove that $X \setminus \bar{A} = (X \setminus A)^0$ 7

(c) Let A subset of X and B subset of Y. Prove that : 7

(1) $\overline{(A \times B)} = \bar{A} \times \bar{B}$.

(2) Prove that for any subset A of X $(A^0)^0 = A^0$.

3 All are compulsory :

(a) Give the definition of separation of a space X. Find one separation for the subspace of natural numbers and deduce that this space is disconnected. 6

(b) Prove that every component is a maximal connected set and it is a closed set. 4

(c) Prove that every path connected space is connected. 4

OR

3 All are compulsory :

(a) Prove the subspace (0, 1) is homeomorphic to (a, b) of $\mathbb{R}$. 5

(b) Suppose f : X → Y is continuous and Z is a subspace of X. Then prove that the function f/Z : Z to Y is continuous. 4

(c) Prove that the set of all natural numbers has no limit point in $\mathbb{R}$ when $\mathbb{R}$ has the standard topology. 5

4 Attempt any two :

(a) Prove that X × Y is a locally connected if and only if X and Y are locally connected. 7

(b) Give an example of a connected space which is not locally connected and give an example of a locally connected space which is not connected. 7

(c) Suppose (X, τ) is a topological space where $\tau = \tau$ (d), for some metric d on X, let $E \subset X$ and $x \in X$. Prove that $x \in \overline{E}$ if and only if there is a sequence in E which converges to x. 7

5 Do as directed (Each question carries two marks) 14

(1) Give the definition of a convex subset of a simply ordered set.

(2) Give an example of a closed subset of $\mathbb{R}$ with discrete topology which is not a closed subset of $\mathbb{R}$ with standard topology.

(3) Give an example of a subset of $\mathbb{R}$ which is closed when $\mathbb{R}$ has standard topology but it is not closed when $\mathbb{R}$ has co - finite topology.

(4) Find all interior points of the set of all rational numbers when $\mathbb{R}$ has the standard topology.

(5) Give an infinite subset of $\mathbb{R}_l$, which is both open and closed.

(6) Give the definition of the dictionary order on $\mathbb{R} \times \mathbb{R}$.

(7) Give the definitions of closure and the interior of any subset A of a topological space X.

4

MBZ-003-1161003 Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

December – 2016

Mathematics : Course No. - 1003

[Topology - I]

(New Course)

Faculty Code : 003

Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries two marks)

(1) If every subset if X is open set of X then the topology on X is ________ topology.

(2) In a topological space X ________ is always a closed set.

(3) In $\mathbb{R}$ the closure of the set of rational numbers is ________.

(4) If A contains all its limit points then the closer of A is equal to ________.

(5) The set of irrational numbers is an open set in $\mathbb{R}$ when $\mathbb{R}$ has ________ topology.

(6) ________ is not a connected subset of the space of rationals.

(7) ________ is the intersection of all closed sets containing A.

2 Attempt any two :

(a) Give an example to show that denumerable union of closed set need not be closed. 7

(b) Prove that a subset F of X is closed if and only if F contains all its limit points. 7

(c) Let A subset of X and B subset of Y. Prove that : 7

(1) $(A \times B)^0 = A^0 \times B^0$

(2) Prove that for any subset A of X $\left(A^0\right)^0 = A^0$.

3 All compulsory :

(a) Give the definition of separation of a space X. Give one separation of a discrete space with atleast two points. Prove that such a space is always disconnected. 6

(b) Prove that every component is a maximal connected set and it is a closed set. 4

(c) Give an example of a connected space which is not path connected. Give an example of a connected space which is not locally connected. 4

OR

3 All compulsory :

(a) Prove the subspace (0,1) is homeomorphic to (a, b) of **R**. 5

(b) Suppose $f : X \to Y$ is continuous and $g : Y \to Z$ in continuous then prove that $gof : X \to Z$ is continuous. 4

(c) Prove that the interior of the set of all rational numbers in **R** (with standard topology) is empty. 5

4 Attempt any two :

(a) Prove that X×Y is a locally connected if and only if X and Y are locally connected. 7

(b) Prove that $X \times Y$ is path connected if and only if X and Y are path connected. 7

(c) Suppose (X, τ) is a topological space where $\tau = \tau(d)$, for some metric d on X, let $E \subset X$ and $x \in X$. Prove that $x \in \bar{E}$ if and only if there is a sequence in E which converges to x. 7

5 Do as directed : (Each question carries two marks)

(1) Give the definition of a limit point of a set.

(2) Give an example of a closed subset of $\mathbb{R}$ with discrete topology which is not a closed subset of $\mathbb{R}$ with standard topology.

(3) Give a separation of the space of all irrational numbers.

(4) Find all interior points of the set of all irrational numbers when $\mathbb{R}$ has the standard topology.

(5) Give an infinite subset of $\mathbb{R}$ which is neither open nor closed.

(6) Give the definition of a component of a space.

(7) Give the definitions of

(i) a continuous function and

(ii) the subspace topology.

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BBN-003-016103 Seat No. ______

M. Sc. Mathematics (Sem. I) (CBCS) Examination

December - 2015

1003 : Topology - I

[New Course]

Faculty Code : 003

Subject Code : 016103

Time : 2.30 Hours] [Total Marks : 70

Instructions :
(1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries two marks) 14

(1) If every subset of X is closed set of X then the topology on X is ______ topology.

(2) In a topological space X, ______ and ______ are both open and closed set.

(3) In R the closure of the set of irrationals is ______.

(4) If A is a closed set then A contains all its ______.

(5) [0, 1) is an open set in ______ topology on $\mathbb{R}$.

(6) If every finite subset of X is closed then the topology on X is ______ topology.

(7) Urysohn lemma is equivalent to ______ separation axioms.

2 Attempt any two :

(a) Prove that the finite union of closed sets is a closed set. 7

(b) Prove that a subset G of X is open if and only if $G^o = G$. Give an example of a nonempty subset of R whose interior is empty. 7

[Contd...

8

(c) Let A subset of X and B subset of Y. Prove that : 7

(1) $cl(A \times B) = cl(A) \times cl(B)$

(2) $A \times B$ is a closed subset of $X \times Y$ if and only if A is closed in X and B is closed in Y. (cl denotes the closure of a set)

3 All compulsory :

(a) Give the definition of closure of a subset of a topological space X. 6

Find the closure of $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots \frac{1}{n} \ldots\ldots\right\}$.

(b) Prove that every T_2 space is a T_1 space. 4

(c) Suppose Y is a subspace of a regular space X. Prove that Y is a regular space. 4

OR

3 All compulsory :

(a) If X is a topological space and f is defined as f(x)=x for all x then prove that f is a continuous function. 5

(b) Suppose $f : X \to Y$ and $g : Y \to Z$ are continuous. Prove that $gof : X \to Z$ is continuous. 4

(c) Prove that the set of all natural numbers has no limit point in R. 5

4 Attempt any two :

(a) Prove that X×Y is a Hausdorff space if and only if X and Y are Hausdorff spaces. 7

(b) Write the statement of the Uryhohn lemma and using it prove that every normal space is completely regular space. 7

(c) Suppose X,Y,Z are topological spaces and $f : Z \to X \times Y$ is a function. Prove that f is continuous if and only if the functions Π_1 of and Π_2 of are continuous functions.

14

5 Do as directed (Each question carries two marks)

(1) Give the definition of a simply ordered set

(2) Give an example of an open subset of R with lower limit topology which is not an open subset of R with standard topology.

(3) Give an example of a subset of $\mathbb{R}$ which is closed in the discrete topology of R but not closed in the standard topology.

(4) Give an example of a normal space X for which $X \times X$ is not normal.

(5) Give an infinite subset of $\mathbb{R}$ which is neither open nor closed.

(6) Give the definition of dictionary order on $R \times R$.

(7) Give the definitions of

(i) Homomorphism and

(ii) Topological property.

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XB-135

003-016103

M. SC. (Maths) Sem.-I (CBCS) Examination

December-2014

Mathematics

1003 : Topology – I

(New)

Faculty Code : 003

Subject Code : 016103

Time : 2½ Hours] [Total Marks : 70

Instructions : (1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries **14** marks.

1\. Fill in the blanks : (Each question carries two marks) 14

(a) If every subset of X is an open set of X then the topology on X is........ topology.

(Usual, Lower Limit, Discrete, Indiscrete)

(b) In every topological space there are.........open sets.

(Four , Two, Three, One)

(c) {0} is a......... set of $\mathbb{R}$

(open, infinite, empty, closed)

(d) If A contains all its limit points then A is ....

(open, closed, finite, infinite)

(e) [0, 1) is an open set in ....topology on $\mathbb{R}$.

(Lower limit, Standard, Cofinite, Indiscrete)

(g) If every infinite subset of X is closed then X must be a .... space.

(T_1, T_2, Regular, Normal).

2\. Attempt any two :

(a) Suppose $F_1, F_2, \ldots\ldots F_n$ are closed sets. Prove that their union is a closed set. 7

(b) Let X be an infinte set. Prove that the family $T = (G \subset X : X \backslash G \text{ is finite}) \cup \{\phi\}$ is topology on X. 7

(c) If A and B are subsets of X then prove that (i) If $A \subset B$ then $A^0 \subset B^0$ and (ii) $(A \cap B)^0 = A^0 \cap B^0$ 7

3. All compulsory :

(a) Give the definition of limit point of a set A of a topological space X. 6

Prove that 0 is the limit point of $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots \frac{1}{n} \ldots\ldots\right\}$ 4

(b) Prove that an infinite set with co finite topology is a T_1 space. 4

(c) Suppose Y is a subspace of a T_1 space X. Prove that Y is a T_1 space.

OR

All compulsory :

(a) If X is a topological space and f is defined as $f(x) = x$ for all x then prove that f is a continuous function. 5

(b) Suppose $f : X \to Y$. Prove that f is continuous if and only if the inverse image of every closed subset of Y is a closed subset of X. 6

(c) Establish that every real number is a limit point of Q- the set of rationals. 3

4. Attempt any two :

(a) Prove that $X \times Y$ is a T_1 space if and only if X and Y are T_1 spaces. 7

(b) Suppose $f : X \to Y$ and $g : Y \to Z$ are continuous. Prove that $gof : X \to Z$ is continuous. 7

(c) Suppose X and Y are topological spaces. Prove that the family $\{U \times V : U$ is open in X, V is open in Y$\}$ is a basis for some topology on $X \times Y$. 7

5. Do as directed (Each carries two marks) : 14

(a) Give the definition of a simply ordered set.

(b) Find the closure of Q- the set of rationals.

(c) Give an example of a subset of $\mathbb{R}$ which is open in lower limit topology but not open in the standard topology.

(d) Give the definition of a completely regular space.

(e) Give an infinite subset of $\mathbb{R}$ which is not open.

(f) Give the definition of the closure of any subset A of X.

(g) Give the definition of a convergent sequence.

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152/C

003-016103

M.Sc. (Maths) (CBCS) Sem.-I Examination
November-2013
Mathematics
Paper No. : 1003 (Topology – I)

Faculty Code : 003
Subject Code : 016103

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each questions carries 14 marks.
(4) Figures to the right indicates full marks.

1. Fill in the blanks : (Each carries two marks) $7 \times 2 = 14$
   (i) If X is a T_2 space then every finite subset of X is ________.
   (ii) If X is a Housdorff space then every subspace of X is ________.
   (iii) If A is closed then A contains all ________ point of A.
   (iv) If X discrete space then every subset of X is a ________ set.
   (v) If U is an open subset of X and Y is topological space then $U \times Y$ is a ________ subset of $X \times Y$.
   (vi) Arbitrary intersection of closed sets is a ________ set.
   (vii) Urysohn's lemma is equivalent to ________ space.

2. Attempt any two :
   (a) Let $p \in X$ and let $T = \{G \subset X : p \in G\} \cup \{\phi\}$, then prove that T is a topology on X. 7
   (b) Prove that the collection of all open intervals of R is basis for some topology on R. (R = the set of real numbers) 7
   (c) Prove that a finite union of closed sets is a closed set. Give an example to show that arbitrary union of closed sets may not be closed. 7

3. Attempt all :
   (a) Let Y be a subspace of X. Prove that a subset K of Y is closed in Y if and only if $K = F \cap Y$ for some closed subset F of X. 6

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(b) Let Y be a closed subspace of X. Prove that a subset F of Y is closed in Y if and only if it is closed in X.

(c) Prove that (0,1) is homeomorphic to (a, b) for any a and b in R with a < b.

OR

Attempt all .

(a) Define the interior A^0 of any subset A of a space X. Prove that ... if and only if A is an open set.

(b) If cl(A) denotes the closure of A for any subset A of X then prove that

(1) If $A \subset B$ then $cl(A) \subset cl(B)$.

(2) $cl(A \cup B) = cl(A) \cup cl(B)$.

(c) Let d be a metric on X and T(d) be the metric topology induced by d on X. Let $E \subset X$ and $x \in X$. Prove that $x \in \overline{E}$ if and only if there ... a sequence (xn) in E such that (xn) converges to x.

4. Attempt any two :

(a) Prove that every regular space is T_1 space. Give an example of a space which is not regular.

(b) Prove that every closed subset of a normal space is normal.

(c) Prove that :

(i) Regularity is a topological property.

(ii) X × Y is regular if X and Y are regular.

5. Do as directed: Each carries 2 marks :

(i) Define the closure $\bar{A}$ of any subset A of X

(ii) Find all limit points of (0,1).

(iii) Give the definition of a closure of a subset A of a space of X

(iv) Give an example of a non-empty set A such that A' is non-empty ... $A^1 \subset A$.

(v) Find two disjoint open subsets of R containing points 2 and ...

(vi) Give an example of a non-empty set T such that T^0 ...

(vii) Give the definition of a convergence sequence in a topological space ...

"SC-003016103"

SC-003016103 Seat No.________

M.Sc. Mathematics Semester-I (CBCS) Examination

October / November - 2012

Paper No. 1003 (Topology-I)

Time : 2 ½ Hours Total Marks : 70

Instructions: 1) There are give 5 questions
2) All questions are compulsory
3) Each question carries 14 marks
4) Figures to the right indicates marks

Q:1 Fill in the blanks : (Each carries two marks)

a) If X is a T_1 space then every singleton subset of X is............

b) If X is a Housdorff space then the set $\{(x,x) : x \in X\}$ is a .... set.

c) If A contains all its limit points the A must be..........

d) If every subset if X is an open subset of X then the topology on X must be ....

e) If U is open in X, V is open in Y then U x V is open in X x Y has.....topology.

f) Arbitrary intersection of open sets......open.

g) A completely regular space is also called a .... space or a.....space.

Q:2 Attempt any Two

a. Let X be infinite set and $\tau = \{G \subset U X : X\text{-}G \text{ is finite}\} \cup \{\varphi\}$. Prove that τ is a topology on X. (7)

b. Prove that the family $\beta = \{(a,b) : a < b\}$ is a basis for some topology on the set of all real numbers (7)

c. Prove that arbitrary intersection of closed sets is a closed set. Give an example to show that arbitrary union of closed sets need not be closed. (7)

Q:3 Answer the following

a) Let Y be a non-empty subset of a topological space (X, τ) (7)
Prove that the family $T_Y = \{G \cap Y : G \in T\}$ is a topology on Y.

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b) Prove that the subspace topology on the set of all natural number induced from the standard topology on ℝ is the discrete topology.

c) Let Y be a closed subspace of X. Prove that a subset F of Y is closed in Y iff it is closed in X.

OR

a) Define the interior A^o of any subset A of a space X. Prove that

(i) $A \subseteq B \rightarrow A^o \subseteq B^o$ and

(ii) $(A \cap B)^o = A^o \cap B^o$

b) Let d be a metric on X and T(d) be the metric topology induced by d on X. Let $E \subseteq X$ and $x \in X$. Prove that $x \in \bar{E}$ iff there is a sequence (Xn) in E such that (Xn) converges to x.

c) Let $f: X \rightarrow Y$ be a continuous onto function. Prove that f is a quotient map if and only if f(A) is open in Y whenever A is a saturated open subset of X.

Q:4 Attempt any two :

a) Prove that every T_2 space is a T_1 space. Give an example of a space X which is T_1 but not T_2.

b) Suppose for any closed subspace A of X and for any continuous function $f: A \rightarrow R$, there is a continuous function $g: X \rightarrow R$ such that $g/A = f$. Prove that X is a normal space.

c) Prove that :

(i) Complete regularity is a topological property.

(ii) X×Y is completely regular if X and Y are completely regular.

Q:5 Do as directed : Each carries 2 marks.

(i) Define the closure $\bar{A}$ of any subject A of X.

(ii) Find a limit point of the subset $A = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots\}$ of R.

(iii) Give the definition of a neighborhood of a point x of X.

(iv) Give an example of a non-empty set A such that A' is non-empty and $A \cap A' = \varphi$.

(v) Find two disjoint open subsets of R containing points 1 and 4.5.

(vi) Give an example of a non-empty set T such that $T^o = \varphi$.

(vii) Give the definition of a metric on a set X.

003-016103/B-51 Seat No. ________

M. Sc. (Sem. I) Examination

November / December - 2019

Mathematics

Faculty Code : 003

Subject Code : 016103

Time : 3 Hours] [Total Marks : 70

Instructions : (1) There are five questions.

(2) Each questions carry 14 marks.

1 Answer any seven :

(1) Write down all open sub sets of discrete topology on $\{a, b, c, d\}$.

(2) Write three subsets of $\mathbb{R}$, one is closed, second is open and third is neither open nor closed.

(3) Write down interior and non-interior points of $[0, 1]$.

(4) Suppose X has cofinite topology. Let $a, b \in X$ what is the closure of $\{a, b\}$.

(5) Write down a subset of $\mathbb{R}$ which contains all its limits points and its interior is empty.

(6) Explain why the set $A = \left\{1, \frac{1}{2}, \ldots \frac{1}{n}\right\}$ is not a closed subset of $\mathbb{R}$.

(7) Suppose X has cofinite topology and $a \in X$. Give a closed subset of X which does not contain a.

(8) Give an example to show that $(A \cup B)^{\circ} \neq A^{\circ} \cup B^{\circ}$.

(9) Give the definition of interior point.

(10) Give the definition of closure.

2 Answer any two :

(a) Prove that lower limit topology on $\mathbb{R}$ is strictly finer than the standard topology.

(b) Give an example to show that countable intersection of open sets need not be open.

(c) Let X be an infinite set and let $\tau = \{G \subset X : X - G \text{ is finite}\} \cup \{\phi\}$ prove that τ is a topology on X.

3 (a) Let X be a space and Y be a non-empty subset of X. Prove that $P_Y = \{G \cap Y : G \text{ is open in } X\}$ is a topology on Y

(b) Prove that $f : X \to Y$ is continuous iff $f(\bar{E}) \subset \overline{f(E)}$ for each subset E of X.

OR

3 (a) Prove that $A \cup A' = \bar{A}$. 5

(b) Prove that $\overline{(A \times B)} = \bar{A} \times \bar{B}$ 5

(c) Prove that a set A is closed iff $A' \subset A$. 4

4 Attempt any two :

(a) Prove that the collection of all open discs in a metric space (X, d) is a basis for some topology on X. 7

(b) Let d be a metric on X and $\tau = \tau(d)$. Let $A \subset X$. Prove that $x \in \bar{A}$ iff these is a sequence (x_n) in A such that $(x_n) \to x$. 7

(c) Give a topology on $\mathbb{R}^{\mathbb{R}}$, a subset E of $\mathbb{R}^{\mathbb{R}}$ and $x \in \bar{E}$ such that there is no sequence (x_n) in E such that $(x_n) \to x$ 7

2 Answer any two :

(a) Prove that lower limit topology on $\mathbb{R}$ is strictly finer than the standard topology. 7

(b) Give an example to show that countable intersection of open sets need not be open. 7

(c) Let X be an infinite set and let $\tau = \{G \subset X : X - G \text{ is finite}\} \cup \{\phi\}$ prove that τ is a topology on X. 7

3 (a) Let X be a space and $\underline{Y}$ be a non-empty subset of X. Prove that $P_{\underline{Y}} = \{G \cap \underline{Y} : G \text{ is open in } X\}$ is a topology on $\underline{Y}$ 7

(b) Prove that $f : X \to \underline{Y}$ is continuous iff $f(\bar{E}) \subset \overline{f(E)}$ for each subset E of X. 7

OR

3 (a) Prove that $A \cup A' = \bar{A}$. 5

(b) Prove that $\left(\overline{A \times B}\right) = \bar{A} \times \bar{B}$ 5

(c) Prove that a set A is closed iff $A' \subset A$. 4

4 Attempt any two :

(a) Prove that the collection of all open discs in a metric space (X, d) is a basis for some topology on X. 7

(b) Let d be a metric on X and $\tau = \tau(d)$. Let $A \subset X$. Prove that $x \in \bar{A}$ iff these is a sequence (x_n) in A such that $(x_n) \to x$. 7

(c) Give a topology on $\mathbb{R}^{\mathbb{R}}$, a subset E of $\mathbb{R}^{\mathbb{R}}$ and $x \in \bar{E}$ such that there is no sequence (x_n) in E such that $(x_n) \to x$ 7

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5 Attempt any two :

(a) Define a connected space. Let A be a connected subset of X and $A \subset B \subset \bar{A}$. Prove that B is connected. 7

(b) Suppose X and Y are connected prove that $X \times Y$ is connected. 7

(c) Prove that every path connected space is connected. Give an example of a connected space which is not connected. 7

(d) Prove that a space X is locally connected if and only if each component of each open set is open in X. 7

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003-016103 Seat No. ______

M. Sc. (CBCS) (Sem. I) Examination

December – 2011

Mathematics : 1003

(Topology - I)

(New Course)

Faculty Code : 003

Subject Code : 016103

Time : 2.30 Hours] [Total Marks : 7

Instructions : (1) There are 5 questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

(4) Figures on right indicate marks.

1 Fill in the blanks : (any seven)

(1) If every subset of X is closed then the topology of X is ______ topology.

(2) If T is a topology on X then ______ and ______ are always members of T.

(3) If A is a closed then the closure $\bar{A}$ = ______.

(4) The standard topology on R is ______ then the ______ lower limit topology on IR.

(5) Arbitrary intersection of ______ sets is closed in any topological space.

(6) If A contains all its limit points then the closure $\bar{A}$ = ______

(7) If $d(x, y) = |x - y|$ for all x, y in IR then the topology induced by d on IR is ______ topology.

(8) Interior of the set of all rational numbers in IR is ______

(9) The subspace topology on the set of natural numbers induced from $\mathbb{R}$ is ______

(10) A space X is a T_1 space iff every singleton set of X is ______

2 Answer any two :

(a) Give the definition : Basis for some topology on a set X. Let $B = \{(a, b). a < b, a, b \in \mathbb{R}\}$ prove that B is a basis for some topology on $\mathbb{R}$ -the set of red numbers. 7

(b) Let (X, τ) be a topological space and Y be a non-empty subset of X. Prove that $\tau_Y = \{G \cap Y : G \in \tau\}$ is a topology on Y. 7

(c) Define the closure $\bar{A}$ of a subset A of X. Prove that 7

(i) $\bar{\bar{A}} = \bar{A}$ for any $A \subset X$.

(ii) $\bar{A} = A \cup A'$.

3 (a) Define the interior A° of $A \subset X$. Prove that 4

(i) $(A \cap B)^\circ = A^\circ \cap B^\circ$

(ii) $(A^\circ)^\circ = A^\circ$.

(b) Prove that 5

(i) $\overline{X - A} = X - A^\circ$

(ii) $(X - A^\circ) = X - \bar{A}$ for $A \subseteq X$.

(c) Give an exam. to show that 3

(i) $(A \cup B)^\circ \neq A^\circ \cup B^\circ$

(ii) $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$

OR

3 (a) Let (X, d) be a metric space. Let $E \subseteq X$ and $x \in X$. Prove that $x \in \bar{E}$ iff there is a sequence (x_n) in E such that $(x_n) \to x$.

(b) Let d be the standard metric and ρ be the square metric on IR^2. Prove that $\rho(\bar{x}, \bar{y}) \le d(\bar{x}, \bar{y}) \le \sqrt{2}\, \rho(\bar{x}, \bar{y})$ for all $x, \bar{y}$ in IR^2. Also prove that the topology induced by d the topology induced by ρ.

4 Attempt any **two** :

(a) (i) Prove that a subspace of a regular space is regular. 7

(ii) Prove that $X \times Y$ is regular iff X and Y are both regular.

(b) State and prove Urysohn's lemma. 7

(c) State and prove Tietze's extension theorem. 7

5 (a) Give an example of a T_1 space which is not T_2 space. 4

(b) Give the definition of a metric on a non-empty set X. 4

(c) Give an example to show that infinite intersection of open sets may not be open. 3

(d) If $F : X \to Y$ is continuous then prove that $f^{-1}(K)$ is a closed subset of X whenever K is a closed subset of Y. 3

Topology
1003
External
– 2008, 2010 to 2016
23

M.Sc. Sem I Maths Examination March/April / October 200-8

Topology - I Subject [Total Marks 100]

(Old, New or Old & New to be mentioned where necessary)

Time: 3 hours

Instructions:

(1) There are nine questions in this paper.

(2) Answer any five questions.

(3) Each question carries twenty marks.

(4) Figures on right indicate marks.

N.B.: Gujarati Version of question paper is to be written first. English version should follow the Gujarati version of question paper.

Q.1 (a) Suppose $p \in X$ and $\tau = \{G \subseteq X / p \in G\} \cup \{\phi\}$. Show that τ is a topology on X. [7]

(b) Suppose X is an infinite set and τ be the co-finite topology on X. Prove that every single-ton set of element of X is closed in τ. [6]

(c) Prove that the Lower Limit topology on $\mathbb{R}$ is strictly finer than the standard topology on $\mathbb{R}$. [7]

---- 2

To be filled by the Press.
Note: Full Marks of each question to be indicated in a circle at the right end of the first line of each question e.g. (10)

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Q-2 (a) Suppose τ_1, τ_2 be two topologies on [illegible] and $\mathcal{B}_1$, $\mathcal{B}_2$ be basis for τ_1, τ_2 respec[tively]. Prove that τ_2 is finer than τ_1 iff f[or] each $C_1 \in \mathcal{B}_1$ and $x \in C_1$ $\exists$ $C_2 \in \mathcal{B}_2 \ni x \in C_2 \subseteq C_1$. [10]

(b) Suppose X be a topological space and A, B, C are subsets of X. Then prove that

(I) $\overline{A \cup B} = \bar{A} \cup \bar{B}$ and

(II) $\bar{A} = A$ iff A is closed in X. [1[illegible]

Q-3 (a) Prove that the family $\mathcal{B} = \{ U \times V \,/\, U$ is an open set in X and V is open in Y $\}$ is a basis for some topology on $X \times Y$. [5]

(b) Suppose (X, τ) be a topological space and $Y \subseteq X$. Then show that the family $\tau_Y = \{ G \cap Y \,/\, G \in \tau \}$ is a topology on Y. [8]

(c) Suppose X, Y be topological spaces. Then show that the projection map $\pi_1 : X \times Y \rightarrow X$ defined by $\pi_1(x, y) = x$, $\forall (x, y) \in X \times Y$ is a continuous map. [4]

Q-4 (a) Define a continuous map $f : X \rightarrow Y$. Prove that $f : X \longrightarrow Y$ is continuous iff $f^{-1}(K)$ is closed in X whenever K is closed in Y. [10]

---- 3

Q-4 (b) Prove that the co-finite topology on a finite set and the subspace topology on $\mathbb{N}$, as a subset of $\mathbb{R}$ with standard topology are discrete topologies [10]

Q-5 (a) Suppose (X, d) be a metric space and $\mathcal{B} = \{B_d(x, \epsilon) \,/\, x \in X \text{ and } \epsilon > 0\}$, where $B_d(x, \epsilon) = \{y \in X \,/\, d(x, y) < \epsilon\}$. Then prove that $\mathcal{B}$ is a basis for some topology on X. [10]

(b) Suppose X be a metrizable space and $E \subseteq X$. If $x \in \bar{E}$ then prove that $\exists$ a sequence $(x_n) \subseteq E \ni x_n \longrightarrow x$ in X. [10]

Q-6 (a) Suppose $f : X \longrightarrow Y$ be an onto map and X be a topological space. Show that the collection $\tau = \{G \subseteq Y \,/\, f^{-1}(G) \text{ is open in } X\}$ is a topology on Y. Also prove that $f : X \longrightarrow Y$ is a quotient map. [10]

(b) Suppose $f : X \longrightarrow Y$ is a quotient map and $g : Y \longrightarrow Z$ be a map, where X, Y, Z are topological spaces. Prove that g is continuous iff $g \circ f : X \rightarrow Z$ is a continuous map. [10]

------ 4

Q-7 (a) Suppose X and Y be connected spaces. Prove that X×Y is also connected space. [8]

(b) Suppose X be a space and $A, B \subseteq X$ be two connected subsets of X with $A \cap B \neq \phi$. Prove that $A \cup B$ is a connected subset of X. [6]

(c) Suppose X is a connected space and $f: X \to Y$ be an onto continuous map. Then prove that Y is connected space. [6]

Q-8 (a) Prove that a space X is locally connected iff each component of each open set is an open subset of X. [10]

(b) Prove that I × I with dictionary order topology is connected but not path connected, where I = [0,1]. [10]

Q-9 (a) Prove that a space X is locally path connected iff each component of each open set of X is an open subset of X. [12]

(b) Give an example to show that a path component need not be equal to a component. [8]

PCE-003-1161003

Seat No. 15044

M. Sc. (Sem. I) Examination

December - 2018

Mathematics : CMT-1003

(Toplogy - I)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.
(2) There are 5 questions.
(3) Figures to the right indicate full marks.

1 Attempt any seven : (Each question carries two marks) 14

(1) Give an example of a subset of $\mathbb{R}$ which is open in lower limit topology but not but not open in the standard topology.

(2) Give the definition of a convergence sequence in a topological space X.

(3) Give the definition of a closure of a subset A of a space of X.

(4) Give an example of a homeomorphism from $\mathbb{R}$ to $\mathbb{R}$.

(5) Give an example of an uncountable subset of $\mathbb{R}$ which is not an open set.

(6) Give an example such that $\overline{A}\cap\overline{B}\not\subset\overline{A\cap B}$, where A and B are subsets of X.

(7) Give an example of an infinite subset of $\mathbb{R}$ whose interior is empty.

(8) Give the definition of locally connected space and example of connected but not locally connected space.

(9) Give an example to show that arbitrary intersection of open sets need not be open.

(10) Show that the subspace $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is homeomorphic to $\mathbb{R}$.

2 Attempt any two :

(a) If X is an infinite set and $\tau = \{G \subset X : G \neq \phi$ and $X - G$ is finite$\} \cup \{\phi\}$ then prove that τ is topology on X.

(b) If X, Y, Z be spaces then prove that $f : Z \to X \times Y$ is continuous if and only if the functions $\pi_1 \circ f : Z \to X$ and $\pi_2 \circ f : Z \to Y$ are continuous.

(c) If X and be spaces and $A \subset X, B \subset X$ then prove that

(1) $\overline{A \times B} = \overline{A} \times \overline{B}$ and (2) $(A \times B)^* = A^* \times B^*$.

3 Attempt the following :

(a) If Y be a subspace of X then prove that 7

(1) A subset A of Y is closed $\Leftrightarrow A = C \cap Y$ for some closed subset C of X.

(2) For any subset A of Y, $Cl_y A = Cl_x A \cap Y$

(b) State and prove Hausdroff's Criterion. 7

OR

3 Attempt the following :

(a) If X is connected and locally path connected then prove that X is path connected. 7

(b) Prove that $X \times Y$ is a path connected if and only if X and Y are path connected. 7

4 Attempt the following :

(a) If (X, d) be a metric space and $B = \{Bd(x, \in) / x \in X, \in > 0\}$ then prove that B is a basis for some topology on X. 7

(b) Prove that $X \times Y$ is a connected if and only if X and Y are connected. 7

5 Attempt any two :

(a) Prove that 7

$\tau = \{U \subseteq \mathbb{R}$: for each $x \in U$, there is an open interval $(a, b) \ni x \in (a, b) \subseteq U\}$ is topology on $\mathbb{R}$.

(b) Prove the followings : 7

(1) Every path connected space is connected.

(2) Prove that continuous image of connected set is connected.

(c) Prove that a space X is locally path connected if and only if each path component of each open subspace of X is an open subset of X. 7

(d) Prove that B_1 and B_2 generate the same topology, 7

where $B_1 = \{(a, b) / a, b \in \mathbb{R}, a < b\}$ and

$B_2 = \{(a, b) / a, b \in \mathbb{Q}, a < b\}$.

Seat No. ____________

F8AA-003-1161004

M. Sc. (Sem. I) Examination

December - 2022

Mathematics : CMT - 1004

(Theory of Ordinary Differential Equation)

Faculty Code : 003

Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) Attempt are total five questions.

(2) All are compulsory.

(3) Each question carries equal marks.

1 Answer the following : (any seven) **7×2=14**

(a) Write linear differential equation $y_1' = y_1 + y_2 + f(t)$ and $y_2' = y_1 + y_2$ in the matrix form.

(b) Define with an example :

(i) Degree of a differential equation and

(ii) Linear differential equation.

(c) Show that : $\Gamma(z) = (z-1)\Gamma(z-1)$.

(d) State first and second fundamental theorem of calculus.

(e) Define :

(i) Wronskian and

(ii) Singular Point.

(f) State :

Shifting Property of Laplace transform for $z \in C$.

(g) If A and B be $n * n$ matrix and $AB = BA$ then prove that $\exp(A+B) = \exp(A) \cdot \exp(B)$.

(h) Show that e^{3t} and te^{3t} are linearly Independent solutions of $y'' - 6y' + 9y = 0$ on R.

(i) Determine the largest interval of existence of solution for the I.V.P. :

$$\left(t^2+4\right)y''+ty'+(\sin t)y=1; \text{ with } y(1)=2 \text{ and } y'(1)=0.$$

(j) State any two test for the test of convergence.

2 Answer any two of the following : **2×7=14**

(a) State and prove Gronwall's Inequality.

(b) Let $p_1, p_2, p_3, \ldots\ldots, p_n : I \to R$ be continuous then show that n solutions $\psi_1, \psi_2, \psi_3, \ldots\ldots, \psi_n$ of

$y^n + p_1(t)y^{n-1} + \ldots \ldots + p_n(t)y = 0; \forall t \in I$ are linearly independent if and only if

$w(\psi_1, \psi_2, \psi_3 \ldots\ldots, \psi_n)(t) \neq 0; \forall t \in I$.

(c) Prove that the solution the I.V.P.

$$y''-2ty'+2ny=0;\ y(0)=0 \text{ and } y'(0)=\frac{2(-1)^m(2m+1)!}{(m)!};$$

where $n = 2m+1; m \geq 0$ is an integer is a Hermite's Polynomial of degree $2m+1$.

3 Answer the following : **2×7=14**

(a) Find the Eigen values and Eigen vectors of

Matrix $\begin{bmatrix} 6 & 3 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) If $p, q : I \to R$ be continuous functions on I with $t_0 \in I$ and $y_0 \in R$ then prove that the initial value problem $y' + p(t)y = q(t)$ with $y(t_0) = y_0$ has a unique solution.

$$u(t) = y_0 e^{\{-p(t)\}} + e^{\{-p(t)\}} \int e^{\{p(t)\}} q(t) \cdot dt \quad \text{on } I.$$

OR

3 Answer the following : 2×7=14

(a) State and prove variation of constant formula for second order non-homogenous linear differential equation.

(b) Let A be a constant 2 * 2 complex matrix then prove that ∃ a constant 2 * 2 non-singular real matrix T such that $T^{-1}AT = \left[\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}\right]$.

4 Answer the following : 2×7=14

(a) Prove that if $a_0(t), a_1(t), a_2(t)$ which are analytic at t_0 and t_0 is a regular singular point of $a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$ then given equation can be written in the form

$(t - t_0)^2 y'' + (t - t_0)\alpha(t)y' + \beta(t)y = 0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at t_0 and not all $\alpha(t_0), \beta(t_0)$ and $\beta'(t_0)$ are zero.

(b) State and prove Abel's Formula.

5 Answer any two of the following : 2×7=14

(a) Define Legendre's polynomial and compute it for 1st, 2nd, 3rd, 4th and 5th degree.

(b) If $f(t) = \begin{cases} e^{-\frac{1}{t^2}} & ; \text{if } t \neq 0 \\ 0 & ; \text{if } t = 0 \end{cases}$. Then prove that the Initial value problem $y' = f(t), y(0) = 0$, has no analytic solution at 0.

(c) Solve $y''+25y=10\cos t$ with $y(0)=2, y'(0)=0$ using Laplace transform.

(d) Show that if $f(t), \frac{f(t)}{t} \in H$, then prove that :

$$L\left(\frac{f(t)}{t}\right)=\int_z^{\infty} L(f(w))\, dw$$

For which Im (w) is bounded and Re (w) tends to infinite.

SBV-003-1161004 Seat No. ________

M. Sc. (Sem. I) Examination

February – 2022

Mathematics : CMT-1004

(Theory of Ordinary Differential Equation)

Faculty Code : 003

Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal marks.

1 Answer the following : **7×2=14**

(1) Show that, $u(t)=\begin{bmatrix} e^t \\ e^t \end{bmatrix}$ is a solution of $y'=A(t)y$ on $(-\infty,\infty)$, where $A(t)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ for every $t\in(-\infty,\infty)$.

(2) Define : Exponential of an $n\times n$ matrix.

(3) Reduce $y''+2y'+7ty=e^{-t}$; $y(1)=7$ and $y'(1)=-2$ to an IVP of system of 1st order linear differential equation.

(4) State, second fundamental theorem of calculus and find γ_1.

(5) Find the general solution of $y''+16y=0$ on $\mathbb{R}$.

(6) Define : Inverse Laplace Transform and find $L[\cos at]$.

(7) State and prove, Linearity of Laplace transform.

2 Answer the following : **7×2=14**

(1) (a) Define : Gamma function.

(b) Define : Irregular singular point.

(2) Let A be a $n \times n$ matrix then show that, A has at most n distinct eigen values and A has atmost n Linearly independent eigen vectors.

(3) State, the Abel's formula.

(4) Locate and classify the singularity of $t^2 y'' + ty' + (t^2 - n^2) y = 0$.

(5) Show that, $\Gamma(z) = (z-1)\Gamma(z-1)$.

(6) Show that, $u(t) = \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$ is a solution of the initial value problem $y' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ on □.

(7) Construct the successive approximation $\phi_0, \phi_1, \phi_2, \phi_3$ to a solution of $y' = \cos y, y'(0) = 0$.

3 Answer the following : **2×7=14**

(1) Prove that, for a continuous matrix $A(t)$ of order $n \times n$ on I, the solution matrix $\phi(t)$ of $y'' = A(t)$ on I is a fundamental matrix if and only if $\deg(\phi(t)) \neq 0; \forall t \in I$. Further if $\det(\phi(t_0)) \neq 0$; for some $t_0 \in I$ then $\det(\phi(t)) \neq 0; \forall\, t \in I$.

(2) State and prove, variation of constant formula for scalar linear second order non-homogeneous differential equation.

4 Answer the following : **2×7=14**

(1) Prove that, the solution of IVP $y'' - 2ty' + nty = 0; y'(0) = 0$ and $y(0) = \frac{2(-1)^m (2m)!}{m!}$; where $n = 2m; m \geq 0$ is an integer is Hermite's polynomial of degree $2m$.

(2) Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$ and A^{-1}.

5 Answer the following : **2×7=14**

(1) Find the fundamental matrix of $y' = Ay$ on $\mathbb{R}$, where

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

(2) Find $\exp(tA)$; $\forall t \in (-\infty, \infty)$ for the matrix $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

using its solution matrix.

6 Answer the following : **2×7=14**

(1) If $f(t) = t;\ 0 \le t \le 1$ and $f(t+1) = f(t);\ \forall t \in [0, \infty)$ then find $L(f)(z)$.

(2) Solve $y'' + y = 2e^t$, $y(0) = 2 = y'(0)$, using Laplace transform.

7 Answer the following : **2×7=14**

(1) Define convolution. Further show that,

$$L\left(\int_0^t f(s)\,ds\right)(z) = \frac{1}{z} L(f(z)), \forall f \in \mathcal{H}.$$

(2) Find, $L^{-1}\left(\dfrac{1}{z\left(z^2+4\right)^2}\right)$.

8 Answer the followings : **2×7=14**

(1) Find the solution of the IVP $y' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} y + \begin{bmatrix} 0 \\ e^{-2t} \end{bmatrix}$ with

$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ on $\mathbb{R}$.

(2) Let A be a constant 2×2 complex matrix then prove that, there exists a constant 2×2 non-singular matrix T such that $T^{-1}AT$ has the following forms :

(a) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$

(b) $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

9 Answer the following : **2×7=14**

(1) Find the eigen values and the corresponding eigen vectors of the matrix $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -1 & -6 \end{bmatrix}$.

(2) Prove that, if $\alpha = 2m+1$ where m is a non-negative integer then the solution ϕ of the Legendre's equation with $y(0)=0$ and $y'(0)=1$ is polynomial of degree $2m+1$. Compute this polynomial for $m=0,1,2$.

10 Answer the following : **2×7=14**

(1) Find the particular solution of $y''+y=\sec t;\ \forall t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

(2) Let $A(t)$ and $g(t)$ be two continuous matrices of order $n*n$ and $n*1$ respectively on $(-\infty,\infty)$ and consider any $n_0\in R^n$ then prove that, the unique solution of the intial value problem $Y'=A(t)\cdot Y+g(t)$ with $Y(t_0)=n_0$ is :

$$u(t)=\exp(t-t_0)A\cdot n_0+\int_{t_0}^{t}(\exp(t-s)\cdot A)\,g(s)\,ds;\ \forall t\in(-\infty,\infty).$$

B-003-1161004 Seat No. ________

M. Sc. (Sem. I) Examination

March – 2021

CMT - 1004 : Mathematics

(Theory of Ordinary Differential Equation)

Faculty Code : 003
Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions:

(1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1) Answer the following: 14

1) Define Linear Differential Equation and Linear Homogenous Differential Equation with an example.
2) Prove that for every $n * n$ real matrix $\exp(A+B) = e^A \cdot e^B$ provided $AB = BA$.
3) State and prove change of scale property in Laplace Transform.
4) Find two linearly Independent solutions of $y'' + y = 0$ on R.
5) Show that $u(t) = \begin{pmatrix} cost \\ -sint \end{pmatrix}$ is a solution matrix of the initial value problem$y' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} y,\ y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
6) Find $L(Sin(ct));\ \forall c \in C$.
7) State Variation of Constant Formula for First Order Differential Equation.

2) Answer the following: 14

1) If y_1, y_2 are solutions of $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ with initial condition $y_1(0) = 0,\ y_1'(0) = -1,\ y_2(0) = 1$ and $y_2'(0) = 0$ then find $w(y_1, y_2)\left(\frac{1}{2}\right)$.
2) Define Power Series and Bessel's Function.
3) Determine the largest interval of Exisistance of the solution for the I.V.P for the equation:
$$y''' + (t^2-1)^{\frac{1}{2}}y = 0\ with\ y(-1) = 1; y'(-1) = 0; y''(-1) = -1$$
4) Prove that $v_1, v_2, \dots\dots v_n \in K^n$are linearly dependent if and only if $\det[v_1, v_2, \dots\dots v_n] \neq 0$.
5) Construct the successive approximation $\phi_0, \phi_1, \phi_2, \phi_3$ to a solution of $y' = cosy$, $y'(0) = 0$.
6) Check whether the Legendre's equation $(1-t^2)y'' - 2ty' + n(n+1)y = 0$ has a series solution near 0 or not?
7) Define Heavy Side Function and Show that Laplace Transform is linear.

3) **Answer the following:** 14

1) Prove that the solution of the I.V.P $y'' - 2ty' + 2ny = 0; y'(0) = 0$ and $y(0) = \frac{2(-1)^m(2m)!}{(m)!}$; where $n = 2m; m \geq 0$ is an integer is a Hermite's Polynomial of degree 2m.
2) State and prove variation of constant formula for scalar linear second order non-homogenous differential equation.

4) **Answer the following:** 14

1) i) Find $L^{-1}\left(\frac{3z+7}{z^2-2z-3}\right)$ and ii) Find $L(Cosct)$.
2) Solve $y'' + y = t$; $y = 1$ and $y' = -2$ at $t = 0$ using Laplace Transform.

5) **Answer the following:** 14

1) State and Prove Gronwall's Inequality.
2) Define Legendre's polynomial and compute the polynomial for 1st, 2nd, 3rd, 4th and 5th degree.

6) **Answer the following:** 14

1) Prove that if $\emptyset$ is a solution of the I.V.P: $y' = f(t, y); y(t_0) = y_0$ if and only if $\emptyset$ is a solution of the Voltera's equation $y(t) = y_0 \int_{t_0}^{t} f(s, y(s))ds$.
2) Define Convolution. Further show that if $f \in \mathcal{H}$ and $\frac{f(t)}{t} \in \mathcal{H}$ then $L\left(\frac{f(t)}{t}\right)(z) = \int_z^{\infty}(Lf(w))dw$ for which img(w) is bounded and Re(w) → ∞ .

7) **Answer the following:** 14

1) Find $exp(tA)$ for $y' = Ay$ on R where $A=\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ on R.
2) i) Classify and locate all the singularities of
$$t^2y'' + ty' + (n^2 - t^2)y = 0; n \neq 0.$$
ii) Prove that $\Gamma(z) = (z-1)\Gamma(z-1); \forall\, z \in \mathbb{C}$ and $Re(z) > 1$.

8) **Answer the following:** 14

1) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$ for every $t \in (-\infty, \infty)$
2) Find exp (tA) ; $\forall t \in (-\infty, \infty)$ for the above given matrix using its solution matrix.

9) **Answer the following:** 14

1) i) If $f(t) = t; 0 \leq t \leq 1$ and $f(t+1) = f(t); \forall t \in [0, \infty)$ then find L(f)(z).
ii) Find $L(e^t sin^2 t)(z)$.
2) Let A be a constant 2 x 2 complex matrix then prove that there exists a constant 2 x 2 non- singular matrix T such that $T^{-1}AT$ has the following forms:

$$a) \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} \text{ and b) } \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

10) **Answer the following:** **14**

1) State and Prove Abel's Formula.
2) Prove that if $a_0(t), a_1(t), a_2(t)$ which are analytic at t_0 and t_0 is a regular singular point of $a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$ then given equation can be written in the form $(t-t_0)^2y'' + (t-t_0)\alpha(t)y' + \beta(t)y = 0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at t_0 and not all $\alpha(t_0), \beta(t_0)$ and $\beta'(t_0)$ are zero.

JBG-003-1161004 Seat No. ______

M. Sc. (Sem. I) (CBCS) Examination

December - 2019

Mathematics : CMT - 1004

(Theory of Ordinary Differential Equation)

Faculty Code : 003
Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions.
(2) The figures on the right hand side indicate the marks allotted to the questions.

1 Answer any seven : **7×2=14**

(a) Define Degree of a differential equation and linear differential equation with examples.

(b) Show that $\Gamma z = (z-1)\Gamma(z-1)$.

(c) State Variation of constant formulae for scalar linear second order non-homogenous differential equation.

(d) Define Laplace Transform of a function in $\mathcal{K}$ and Show that it converges absolutely.

(e) Prove that $\exp\left(T^{-1}AT\right) = T^{-1}\exp(A)T$.

(f) State change of scale property and 1^{st} shifting property in Laplace Transform.

(g) Find general solution of $y^4 + 16y = 0$ on $\mathbb{R}$.

(h) State the First fundamental theorem of calculus.

(i) State the Abel's formula.

(j) Locate and classify the singularities of

$$t^2 y'' + ty' + \left(t^2 - n^2\right)y = 0.$$

2 Answer any **two** : **2×7=14**

(a) Let A be a constant 2×2 complex matrix then prove that there exists a constant 2×2 non-singular matrix T such that $T^{-1}AT$ has the following forms :

(a) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$ and (b) $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$.

(b) Let A be a constant $n \times n$ real matrix. Let $g(t)$ be a continuous $n \times 1$ matrix on $(-\infty, \infty)$ and $n_0 \in R^n$ then prove that the unique solution of IVP :

$y' = A(t)y + g(t); y(t_0) = n_0$ is

$$u(t) = \exp(t - t_0) A \cdot n_0 + \int_{t_0}^{t} (\exp(t - s) A) \cdot g(s) ds; \forall t \in (-\infty, \infty)$$

Further find the solution of the IVP :

$$y' = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}; y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

(c) Find the Eigen values and the corresponding Eigen vector of matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -1 & -6 \end{bmatrix}$.

3 All are compulsory : **2×7=14**

(1) State and prove Variation of constant formulae for scalar linear 1st order non-homogenous differential equation.

(2) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix} \forall t \in (-\infty, \infty)$ and Find $\exp(tA); \forall t \in (-\infty, \infty)$.

OR

3 All are compulsory : **2×7=14**

(1) Find the solution of the I.V.P :

$$y' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} y + \begin{pmatrix} 0 \\ e^{-2t} \end{pmatrix}; y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ on } R.$$

(2) Prove that Eigen vectors corresponding to the distinct Eigen values of $n * n$ matrix are linearly independent in $\mathbb{R}^n$ or $\mathbb{C}^n$.

4 Answer any two : **2×7=14**

(1) Justify weather the Legendre's equation

$\left(1-t^2\right)y''-2ty'+n(n+1)y=0$; (where n is constant)

has a solution or not.

(2) Prove that if $a_0(t)$, $a_1(t)$, $a_2(t)$ which are analytic at t_0 and t_0 is a regular singular point of $a_0(t)y''+a_1(t)y'+a_2(t)y=0$ then given equation can be written in the form $\left(t-t_0\right)^2 y''+\left(t-t_0\right)\alpha(t)y'+\beta(t)y=0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at t_0 and not all $\alpha\left(t_0\right), \beta\left(t_0\right)$ and $\beta'\left(t_0\right)$ are zero.

(3) Compute the first five terms of the series expansion at zero of the solution of the Legendre's equation $\left[1-(t)^2\right]y''-2ty'+\alpha(\alpha+1)y=0$, where α is a constant and can you guess the general term of the coefficient of the solution.

5 Answer any **two** : **2×7=14**

(1) (i) Find $L^{-1}\left(\dfrac{1}{z\left(z^2+4\right)^2}\right)$ and

(ii) Find $L(\cos ct)$.

(2) (i) Define second shifting theorem and

(ii) Find $L\left(e^{ct}\right)(z)$ using definition of Laplace Transform.

(3) Solve $y''-y'-2y=60e^t \sin 2t$ with $y=0$ and $y'=0$ when $t=0$ using Laplace Transform.

(4) State and prove Laplace Transform of Integral.

Janki I

2017
2016
2015
2014
2013
2012
2011
2010

HEJ-003-1161004

HEJ-003-1161004 Seat No. 15008

M. Sc. (Maths) (CBCS) (Sem. I) Examination

November/December – 2017

CMT-1004 : Maths

(Theory of Ordinary Differential Equation)

Faculty Code : 003
Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all questions.
(2) The figures on the right hand side indicate the marks allotted to the questions.

1 Answer all questions : 7×2=14

(1) Define Laplace transform of a function in H and show that it converges absolutely.

(2) Show that $u(t)=\begin{pmatrix} e^t \\ e^t \end{pmatrix}$ is a solution of $y'=A(t)y$ on $(-\infty,\infty)$ where $A(t)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ for every $t\in(-\infty,\infty)$.

(3) Define Eigen values and Eigen vectors.

(4) Prove that sint and cost are two linearly independent solution of $y''+y=0$ on $(-\infty,\infty)$.

(5) State and prove Cauchy inequality.

(6) Define :

(1) Power series

(2) Regular singular point.

(7) Reduce $y''+2y'+7ty=e^{-t}$; $y(1)=7$ and $y'(1)=-2$ to an IVP of system of 1st order linear differential equation.

$2 \times 7 = 14$

2 Answer any two :

(1) Let A be a constant 2×2 matrix with eigen values $\alpha \pm i\beta$ where $\alpha, \beta \in R$. Prove that there exists a constant 2×2 non-singular real matrix T such that $T^{-1}AT = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$.

(2) Let A be a constant $n \times n$ real matrix. Let $g(t)$ be a continuous $n \times 1$ matrix on $(-\infty, \infty)$ and $\eta_0 \in R^n$ then prove that the unique solution of IVP $y' = A(t)y + g(t)$; $y(t_0) = \eta_0$ is :

$$u(t) = \exp(t - t_0)A \cdot \eta_0$$

$$+ \int_{t_0}^{t} (\exp(t-s)A) \cdot g(s)\, ds; \ \forall t \in (-\infty, \infty).$$

Further find the solution of the IVP :

$$y' = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}; \ y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

(3) Find fundamental matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$ for every $t \in (-\infty, \infty)$ and find $\exp(tA)$; $\forall t \in (-\infty, \infty)$.

3 All are compulsory : 2×7=14

(1) Prove that for a continuous matrix $A(t)$ of order $n*n$ on I. The solution matrix $\varnothing(t)$ of $y'' = A(t)$ on I is a fundamental matrix if and only if $\det(\varnothing(t)) \neq 0;\ \forall t \in I$. Further if $\det(\varnothing(t_0)) \neq 0;$ for some $t_0 \in I$ then $\det(\varnothing(t)) \neq 0;\ \forall t \in I$.

(2) Find the general solution of $y'' - 6y' + 9y = e^t$ on R.

OR

3 All are compulsory : 2×7=14

(1) Find the solution of the I.V.P : $y' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} y + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix};$

$$y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ on } R.$$

(2) State and prove variation of constant formulae for scalar linear 2nd order non-homogenous differential equation.

4 Answer any two : 2×7=14

(1) Prove that the solution of the I.V.P. $y'' - 2ty' + 2ny = 0;$

$$y'(0) = 0 \text{ and } y(0) = \frac{2(-1)^m (2m)!}{(m)!}; \text{ where } n = 2m;$$

$m \geq 0$ is an integer is a Hermite's polynomial of degree 2 m.

(2) State and prove the Existence and Uniqueness theorem for the first order I.V.P. of the form $y' = f(t, y);\ y(0) = y_0$.

(3) (a) Classify and locate all the singularities of

$$t^4 \left(1 - t^2\right)^3 y''' + 5t^5 (1 + t) y'' - 2t^2 \left(1 - t^2\right) y' + y = 0.$$

(b) Prove that if $\varnothing$ is a solution of the I.V.P. :

$y' = f(t, y);\ y(t_0) = y_0$ if and only if $\varnothing$ is a solution of the Voltera's equation

$$y(t) = y_0 \int_{t_0}^{t} f(s, y(s))\, ds.$$

5 Answer any two : 2×7=14

(1) Solve $y'' + 9y = \cos t;\ y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform.

(2) Find $L\left(t^n e^{ct}\right)(z)$.

(3) Define convolution. Further show that if $f, g \in H$ then $L(f * g) \in H$.

(4) (a) $L^{-1}\left(\frac{3z+7}{z^2 - 2z - 3}\right)$.

(b) If $f(t) = t;\ 0 \le t \le 1$ and $f(t+1) = f(t);$ $\forall t \in [0, \infty)$ then find $L(f)(z)$.

Sarasara Vaishali J.

MCA-003-1161004 Seat No. ________

M. Sc. (CBCS) (Sem. I) Examination

December – 2016

Mathematics : CMT-1004

[Theory of Ordinary Differential Equations]

(New Course)

Faculty Code : 003

Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all questions.

(2) Each question carries 14 marks.

(3) The figures on the right indicate marks alloted to the question.

1 Answer any seven questions : 2×7=14

(i) Solve $y'+ay=0$, where "a" is a constant.

(ii) True or false ? Justify.

If $\phi(t)$ is a fundamental matrix of $Y'=A(t)Y$ on I and C is a non-singular $n\times n$ matrix then $C\phi(t)$ is a fundamental matrix of $Y'=A(t)Y$ on I.

(iii) If the columns of a continuous $n\times n$ matrix $A(t)$ on I are linearly independent then is it true that $\det A(t)\neq 0,\ \forall t\in I$? Justify.

$y'+p(t)y$

$y'+ay=0$

$y'=-ay$

(iv) If $a_0, a_1, a_2 : I \to \mathbb{R}$ are continuous, $a_0(t) \neq 0,\ \forall t \in I,\ t_0 \in I$ and ϕ_1, ϕ_2 are solutions of $a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$ then prove that $w(\phi_1, \phi_2)(t) = w(\phi_1, \phi_2)(t_0)\exp\left(-\int_{t_0}^{t} \frac{a_1(s)}{a_0(s)} ds\right),\ \forall t \in I$.

(v) Find two linearly independent solutions of $y'' - 8y' + 16y = 0$ on $\mathbb{R}$.

(vi) If A is a constant $n \times n$ matrix then prove that $\exp(tA)$ is a fundamental matrix of $Y' = AY$ on $\mathbb{R}$.

(vii) Define Legendre polynomial of degree n, where $n \in \{0, 1, 2, \ldots\}$.

Is $p(t) = -\frac{3}{2}\left(t + \frac{10t^3}{3!}\right)$ a Legendre polynomial of degree 3 ?

Justify.

(viii) Find the indicial equation of $2ty'' + y' + ty = 0$.

(ix) Define Gamma function and state, without proof, its recursion formula.

(x) Find $L(\sinh ct)$, where $c \in \mathbb{C}$.

2 Answer any two questions : **2×7=14**

(a) If $p, q : I \to \mathbb{R}$ are continuous, $t_0 \in I$, $y_0 \in \mathbb{R}$ then find the unique solution of the $I_{vp} : y' + p(t)y = q(t),\ y(t_0) = y_0$.

(b) If $A(t)$ is a continuous $n \times n$ matrix on I then prove that a solution matrix $\phi(t)$ of $Y' = A(t)Y$ on I is a fundamental matrix iff $\det \phi(t) \neq 0,\ \forall t \in I$.

(c) State and prove variation of constant formula for a non-homogeneous system of first order differential equations.

3 (a) Prove that $-\cos t \log|\sec t + \tan t|$ is a solution of $y'' + y = \tan t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. 7

(b) Find a fundamental matrix of $Y' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} Y$ on $(-\infty, \infty)$. 7

OR

(c) Define eigen-values and eigen vectors of an $n \times n$ matrix. If A is a constant real or complex $n \times n$ matrix and $v_1, v_2, \ldots\ldots\ldots, v_n$ are linearly independent eigen-vectors corresponding to the eigen-values $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ of A then prove that $\phi(t) = \left[e^{\lambda_1 t} v_1, e^{\lambda_2 t} v_2, \ldots\ldots\ldots, e^{\lambda_n t} v_n\right]$ is a fundamental matrix of $Y' = AY$ on $(-\infty, \infty)$. 7

(d) Find $\exp(tA)$, $\forall t \in \mathbb{R}$, where $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$. 7

4 Answer any two questions : 2×7=14

(a) If $\alpha = 2m+1$, where $m \geq 0$ is an integer then prove that the solution of the

$I_{vp} : \left(1-t^2\right) y'' - 2t\, y' + \alpha(\alpha+1) y = 0,\ y(0) = 0,\ y'(0) = 1$ is a polynomial of degree $2m+1$.

(b) Solve the $I_{vp} : y'' - 2t\, y' + 2ny = 0$, $n = 2m$, an even integer,

$$y(0) = \frac{(-1)^m (2m)!}{m!},\ y'(0) = 0.$$

(c) If $f(t) = \begin{cases} e^{-\frac{1}{t^2}} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$

then prove that the $I_{vp} : y' = f(t),\ y(0) = 0$ has no analytic solution at 0.

5 Answer any two questions : $2 \times 7 = 14$

(a) Define successive approximations to a solution of the integral equation $y(t) = y_0 + \int_{t_0}^{t} f(s, y(B))\, ds$ and construct the successive approximations $\phi_n, n = 0,1,2,3$ to a solution of $y' = \cos y,\ y(0) = 0$.

(b) State and prove Gronwall's inequality.

(c) Find $L(\cosh ct)$, where $c \in \mathbb{C}$.

(d) Solve $y'' + y = 2e^t,\ y(0) = 2 = y'(0)$ using Laplace transform.

BBO-003-016104 Seat No. _

M. Sc. (Sem. I) (CBCS) Examination

December - 2015

CMT-1004 : Mathematics

(Theory of Ordinary Differential Equations)

Faculty Code : 003

Subject Code : 016104

Time : 2.30 Hours] [Total Marks : 70

Instructions :

(1) Answer all questions. Each question carries 14 marks.

(2) The figures on the right indicate marks alloted to the question.

1 Choose the correct answer : 2x7=14

(1) The solution of the I_{vp} : $y' = -y$, $y(0) = 2$ is ______

(A) $2e^{-2t}$ (B) $2e^{-t}$

(C) e^{-2t} (D) $2e^{2t}$

(2) If ϕ_1, ϕ_2 are solutions of $e^t\ y'' + \cos t\ y' + \sin t\ y = 0$ on I the wronskian $w(\phi_1, \phi_2)$ satisfies the differential equation $w' = w$ on I

(A) $\frac{e^t}{\cos t}$ (B) $\frac{e^t}{\sin t}$

(C) $\frac{\cos t}{e^t}$ (D) $\frac{-\cos t}{e^t}$

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(3) A fundamental matrix of $Y' = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} Y$ on $\mathbb{R}$ is ______

(A) $\begin{pmatrix} 2e^{2t} & t \\ 0 & 3e^{3t} \end{pmatrix}$ (B) $\begin{pmatrix} 2e^{t} & 0 \\ 0 & 3e^{t} \end{pmatrix}$

(C) $\begin{pmatrix} 2e^{2t} & t \\ 0 & 3e^{3t} \end{pmatrix}$ (D) $\begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-3t} \end{pmatrix}$

(4) If A is a constant n×n matrix and $\phi(t)$ is a solution of $Y' = A(t)Y$ on $\mathbb{R}$ then $\exists$ a unique $c \in \mathbb{R}^n$ s.t. ______, $\forall t \in \mathbb{R}$

(A) $\phi(t) = C \exp A(t)$ (B) $\phi(t) = \exp A(t) \cdot C$

(C) $\exp A(t) = C\,\phi(t)$ (D) $\exp A(t) = \phi(t) \cdot C$

(5) ______ are two linearly independent solutions of $y'' + y = 0$

(A) e^{it}, e^{-it} (B) e^{it}, te^{it}

(C) e^{-it}, te^{-it} (D) e^{t}, te^{t}

(6) The indicial equation of $y'' + \alpha(t)y' + \beta(t)y = 0$, where $\alpha(t) = \sum_{k=0}^{\infty} \alpha_k t^k$, $\beta(t) = \sum_{k=0}^{\infty} \beta_k t^k$ in $|t| < r$, is ______

(A) $z^2 + \alpha_0 z + \beta_0$ (B) $z(z+1) + \alpha_0 z + \beta_0$

(C) $z^2 - z + \alpha_0 z + \beta_0$ (D) $z(z-1) - \alpha_0 z + \beta_0$

(7) 0 is a ______ point of $t^4(1-t^2)y^{(3)} + st^5 + (1+t)y'' - 2t^2(1-t^2)y' + y = 0$

(A) regular point (B) singular point

(C) regular singular point (D) irregular singular point

(8) $hf(at)(z) =$ _______, $\forall z \in dom\ hf$, $\forall f \in \mathcal{F}, a > 0$

(A) $hf\left(\frac{z}{a}\right)$ (B) $\frac{1}{a} hf\left(\frac{z}{a}\right)$

(C) $\frac{1}{z} hf(az)$ (D) $\frac{1}{z} hf\left(\frac{z}{a}\right)$

(9) The legendre polynomial of degree 2 is _______

(A) $t^2 + 1$ (B) $t^2 - t$

(C) $t^2 + 2t$ (D) None of (a), (b), (c)

(10) For $f, g \in \mathcal{F}$, $(f * g)(x) =$ ______, $\forall x \in [0, \infty)$

(A) $\int_0^\infty f(x-y)g(y)\,dy$ (B) $\int_0^x f(x-y)g(y)\,dy$

(C) $\int_0^x f(y)g(y)\,dy$ (D) $\int_0^\infty f(y)g(x-y)\,dy$

2 Answer any two : 2x7=14

(a) Solve the I_{vp} : $\begin{array}{l} y_1' = -y_1 \\ y_2' = y_1 + y_2 \end{array}$, $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(b) Find the solution of the I_{vp} :

$$Y' = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} Y + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix}, \; Y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(c) Find a particular solution of $y'' + y = \sec t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3 (a) Find a fundamental matrix of $Y' = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix} Y$ on $\mathbb{R}$.

(b) If $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, find $\exp(tA)$, $\forall t \in (-\infty, \infty)$. 7

OR

(c) Prove that the eigen vectors corresponding to distinct eigen values of an n×n matrix are linearly independent. 7

(d) For $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$, find $\exp(tA)$, $\forall t \in (-\infty, \infty)$. 7

4 Answer any two : 2x7=14

(a) Find the solution of the I_{vp} : $y'' - ty = 0$, $y(0) = 1$, $y^1(0) = 0$.

(b) If p is not zero or a positive integer then prove that

$$J_p(t) = \left|\frac{t}{2}\right|^p \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(p+k+1)} \left(\frac{t}{2}\right)^{2k}$$ is a solution of

$t^2 y'' + ty' + (t^2 - p^2) y = 0$ in any excluded nbhd of 0.

(c) State, without proof, Gronwall's inequality. Using Gronwall's inequality, prove that the I_{vp} : $y' = f(t, y)$, $y(t_0) = y_0$ has a unique solution.

5 Answer any two : 2x7=14

(a) If $f \in \ni$ and $hf = F$ then prove that

$$h^{-1}\left(F^n(z)\right)(t) = (-1)^n t^n f(t), \forall t \in [0, \infty), \forall n = 1, 2, \ldots\ldots$$

(b) Find $h^{-1}\left(\frac{3z+7}{z^2 - 2z - 3}\right)$.

(c) Solve $y'' + y = t$, $y = 1$, $y' = -2$ when $t = 0$ using Laplace transform.

(d) If $f(t) = e^{-\frac{1}{t^2}}$, $\forall_0 \neq t \in \mathbb{R}$ and $f(0) = 0$ then prove that the I_{vp} $y' = f(t)$, $y(0) = 0$ has no analytic solution.

XC-137

003-016104

M.Sc. (Maths) (CBCS) Sem.-I Examination

December-2014

CMT-1004 : Maths

(Theory of Ordinary Differential Equations) (Set-1)

Faculty Code : 003

Subject Code : 016104

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) Answer all questions.

(2) The figures on the right indicate the marks allotted to the question.

1. Answer any seven questions : $2 \times 7 = 14$

(1) A matrix $\phi(t)$ is a fundamental matrix of $y' = A(t)\,y$ on I if

(a) $\phi(t)$ is a solution matrix of $y' = A(t)y$ on I

(b) $\phi(t)$ is a solution matrix and $\det \phi(t_0) \neq 0$ for some $t_0 \in I$.

(c) the columns of $\phi(t)$ are linearly independent on I

(d) $\det \phi(t) \neq 0, \forall t \in I$

(2) $y'' - \cos t\, y' + e^t\, y = \sin t,\ y(0) = 1,\ y'(0) = 0$ is equivalent to

(a) $y' = \begin{pmatrix} 0 & 1 \\ -e^t & -\cos t \end{pmatrix} y,\ y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(b) $y' = \begin{pmatrix} 0 & 1 \\ -e^t & \cos t \end{pmatrix} y,\ y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(c) $y' = \begin{pmatrix} 0 & 1 \\ e^t & \cos t \end{pmatrix} y,\ y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(d) $y' = \begin{pmatrix} 0 & 1 \\ e^t & -\cos t \end{pmatrix} y,\ y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(3) If v_1, v_2 are eigen vectors corresponding to distinct eigen values λ_1, λ_2 of a constant 2×2 matrix A then a fundamental matrix of $y' = Ay$ on $(-\infty, \infty)$ is ____________

(a) $[v_1, v_2]$ (b) $[\lambda_1 v_1, \lambda_2 v_2]$

(c) $[\lambda_2 v_1, \lambda_2 v_2]$ (d) $[\lambda_1 v_2, \lambda_1 v_1]$

(4) The Wronskian of two differentiable functions ϕ_1, ϕ_2 is defined as ________.

(a) $\phi_1' \phi_2' - \phi_1 \phi_2$ (b) $\phi_1 \phi_1' - \phi_2 \phi_2'$

(c) $\phi_1 \phi_2 - \phi_1' \phi_2'$ (d) $\phi_1 \phi_2' - \phi_1' \phi_2$

(5) The indicial equation of $t^2y'' + ty' - ty = 0$ is ________.

(a) $z(z-1) = 0$ (b) $z(z+1) = 0$

(c) $(z+1)(z-1) = 0$ (d) $(z-1)^2 = 0$

(6) Two linearly independent solutions of $y'' + y' - 2y = 0$ on $(-\infty, \infty)$ are ________.

(a) $e^{2t}, t e^{2t}$ (b) e^{t}, e^{2t}

(c) e^{t}, e^{-2t} (d) $e^{t}, t e^{t}$

(7) $L(t^n e^{ct})(z) =$ ________

(a) $\dfrac{n!}{(z-c)^{n+1}}$ (b) $\dfrac{n!}{(z-c)^{n}}$

(c) $\dfrac{(n+1)!}{(z-c)^{n}}$ (d) $\dfrac{(n+1)!}{(z-c)^{n+1}}$

(8) $(L f)(z) =$ ________

(a) $\int_0^\infty e^{-Zt} f(t)\, dt$ (b) $\int_0^\infty e^{Zt} f(t)\, dt$

(c) $\int_{-\infty}^\infty e^{-Zt} f(t)\, dt$ (d) $\int_{-\infty}^\infty e^{Zt} f(t)\, dt$

(9) ________ has infinitely many solutions

(a) $y' = A(t)y,\ y(t_0) = \eta$

(b) $y_1' = y_2',\ y_1(0) = 1,\ y_2(0) = 0$

(c) $y'' + p(t)y' + q(t)y = 0,\ y(0) = 1,\ y'(0) = 0$

~~(d) $y' + p(t)y = q(t);\ y(t_0) = y_0$~~

(10) A fundamental matrix of $y' = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ in $(-\infty, \infty)$ is ________.

(a) $\begin{pmatrix} e^{\lambda t} & t \\ 0 & e^{\lambda t} \end{pmatrix}$ (b) $\begin{pmatrix} e^{\lambda t} & t e^{\lambda t} \\ 0 & e^{\lambda t} \end{pmatrix}$

(c) $\begin{pmatrix} e^{t} & t e^{\lambda t} \\ 0 & e^{t} \end{pmatrix}$ (d) $\begin{pmatrix} t e^{\lambda t} & e^{t} \\ 0 & t e^{\lambda t} \end{pmatrix}$

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$\exp At = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots + \frac{A^nt^n}{n!}$

2. Answer any two questions : $2 \times 7 = 14$

(a) If $p, q : I \longrightarrow \mathbb{R}$ are continuous, $t_0 \in I$ and $y_0 \in \mathbb{R}$, solve the Ivp : $y' + p(t)\,y = q(t)$, $y(t_0) = y_0$

(b) If $p_1, p_2, \dots, p_n$ are continuous on I then prove that $y^n + p_1(t)\,y^{n-1} + \dots + p_n(t)\,y = 0$ has n linearly independent solutions and for every solution ψ of this equation on I, $\exists$ constants $C_1, C_2, \dots, C_n$ s.t. $\psi(t) = C_1\psi_1(t) + C_2\psi_2(t) + \dots + C_n\psi_n(t)$, $\forall\, t \in I$.

(c) State and prove variation of constants formula for a linear system of first order equations.

3. (a) Define exponential of an $n \times n$ matrix. Find $\exp\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}^t$ 14

(b) Find a fundamental matrix of $y' = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{pmatrix} y$ on $(-\infty, \infty)$.

OR

(c) Find the eigen values and corresponding eigen vectors of $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$ and find a fundamental matrix of $y' = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} y$.

(d) Given a 2×2 real matrix A with complex conjugate eigen values $\alpha \pm i\beta$, prove that $\exists$ a real constant non-singular maotrix T s.t. $T^{-1} A T = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$

4. Answer any two questions : $2 \times 7 = 14$

(a) If $\alpha = 2m$ where m is a non-negative integer then prove that the solution of the Ivp $= (1 - t^2)y'' - 2ty' + \alpha(\alpha + 1)y = 0$, $y(0) = 1$, $y'(0) = 0$ is a polynomial of degree 2m.

(b) Define singular points, regular singular points and regular points of $a_0(t)\,y^n + a_1(t)\,y^{n-1} + \dots + a_{n-1}(t)y' + a_n(t)y = 0$. Locate and classify the singular points of $(t - 1)^3 y'' + 2(t - 1)^2 y' - 7ty = 0$

(c) Prove that $ty'' + ty' - y = 0$ has only one solution of the form $|t|^r \sum_{k=0}^{\infty} c_k t^k$, ... h is an excluded nbhd of 0.

$y(t) = y_0 e^{-P(t)} + e^{-P(t)} \int^t e^{P(s)} q(s)\, ds$

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$2 \times 7 = 14$

5. Answer any two questions :

(a) Solve the IVP = $y' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$, $y(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(b) State and prove first shifting theorem. Deduce that $L(t^n e^{ct})(z) = \frac{n!}{(z-c)^{n+1}}$, $\forall n = 0, 1, 2, \ldots\ldots, c \in \mathbb{R}$.

(c) Prove that $L(\sinh ct)(z) = \frac{c}{z^2 - c^2}$, $\forall z \in 4$, $\text{Re}\, z > |c|$, $\forall c \in \mathbb{R}$.

(d) Solve $y'' - y = 0$, $y(0) = 0$, $y'(0) = 1$ using Laplace transform.

$u(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$ $\begin{pmatrix} -\cos t \\ \sin t \end{pmatrix}$

$u'(t) = \begin{pmatrix} -\cos t \\ -\sin t \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$ $u(t) = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ $u'(t) = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$

$= \begin{pmatrix} -e^{-t} \\ e^{t} \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -e^{-t} \\ e^{-t} \end{pmatrix}$ $u'(t) = \begin{pmatrix} t \cdot e^{t} \\ e^{t} \end{pmatrix}$

$\begin{pmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{-2t} \end{pmatrix} + \begin{pmatrix} 1 & t & t^2/2! \\ 0 & 1 & t \\ 0 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} e^{-2t} & t & t^2/2! \\ 0 & e^{-2t} & t \\ 0 & 0 & e^{-2t} \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -e^{-t} \\ e^{t} \end{pmatrix}$

$= \begin{pmatrix} -e^{-t} + e^{t} \\ 0 + e^{t} \end{pmatrix}$

$-e^{-t}(-1)$ e^{-t}

=

146/D

003-016104

M.Sc. (CBCS) Sem.-I Examination
November-2013
Mathematics
CMT-1004 : Theory of Ordinary Differential Equations

Faculty Code : 003
Subject Code : 016104

Time : 2½ Hours] **[Total Marks : 70**

1. Answer any seven question : $7 \times 2 = 14$

(1) The order of $y' - t^4 y'' + 0 - y''' + 7t\,y - 6 = 0$ is ______.

(a) 3 (b) 1 (c) 2 (d) 4.

(2) The solution of the Ivp $y' = \frac{y}{2}$, $y'(0) = \frac{1}{2}$ is ______.

(a) $\frac{1}{2}t + 1$ (b) $\frac{1}{2}e^t$ (c) $2e^t$ (d) $\frac{1}{2}e^{2t}$

(3) The solution of the Ivp : $y' = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is

(a) $\begin{pmatrix} e^{2t} \\ e^{-t} \end{pmatrix}$ (b) $\begin{pmatrix} e^{-2t} \\ e^{t} \end{pmatrix}$ (c) $\begin{pmatrix} e^{t} \\ -e^{2t} \end{pmatrix}$ (d) $\begin{pmatrix} -e^{t} \\ e^{2t} \end{pmatrix}$

(4) ______ is a fundamental matrix of $y' = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} y$ on $(-\infty, \infty)$

(a) $\begin{pmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{pmatrix}$

(b) $e^{2t}\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}$

(c) $e^{t}\begin{pmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{pmatrix}$

(5) ______ are two linearly independent solutions of $y'' - 6y' + 9 = 0$ on $(-\infty, \infty)$

(a) $\cos t, \sin t$ (b) e^{3t}, e^{t} (c) e^{3t}, te^{t} (d) te^{3t}, e^{3t}

(6) If A is a constant $n \times n$ matrix and $f(x)$ is a fundamental matrix of $y' = A(t)\,y$ on $(-\infty, \infty)$ then $\exists$ a non-singular $n \times n$ matrix C s.t. $\forall t \in (-\infty, \infty)$

(a) $\exp At = \phi(t)\,C$

(b) $\exp At = C\,\phi(t)$

(c) $\phi(t) = C \exp At$

(d) $\exp At = \phi(t) + C$

(7) $\exp\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} t =$ ______

(a) $e^{t}\begin{pmatrix} 1 & 3t \\ 0 & 1 \end{pmatrix}$

(b) $e^{3t}\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} \cos t & \sin 3t \\ -\sin 3t & \cos t \end{pmatrix}$

(d) $\begin{pmatrix} \cos 3t & \sin t \\ -\sin t & \cos 3t \end{pmatrix}$

(8) ______ is the Legendre polynomial of degree 3

(a) $t^3 + 1$ (b) $1 - t^3$ (c) $\frac{3}{2}\left(t^2 - \frac{10t^3}{6}\right)$ (d) $-\frac{3}{2}\left(t - \frac{10t^3}{6}\right)$

(9) $L(\cosh ct)(z) =$ ______

(a) $\frac{z}{z^2 + c^2}$ (b) $\frac{z}{z^2 - c^2}$ (c) $\frac{c}{z^2 + c^2}$ (d) $\frac{c}{z^2 - c^2}$

(10) $L(f')(z) =$ ______

(a) $z\,L(f)(z)$

(b) $L(f)(3) - f(0)$

(c) $z^2 L f(z)$

(d) $z\,L(f)(z) - f(0)$

2. Answer any two questions : $2 \times 7 = 14$

(a) State and prove the necessary and sufficient condition for a solution matrix of $y' = A(t)y$ on I to be a fundamental matrix.

(b) If $p, q : I \longrightarrow \mathbb{R}$ are continuous, $t_0 \in I$ and $y_0 \in \mathbb{R}$ then solve the Ivp : $y' + p(t)y = q(t)$, $y(t_0) = y_0$.

(c) State and prove variation of constant formula for the solution of the Ivp : $Y' = A(t)y + g(t)$, $y(t_0) = 0$ on an interval containing t_0.

3. (a) If A is a constant $n \times n$ matrix and $v_1, v_2, \ldots, v_n$ are linearly independent eigen vectors corresponding to the eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ of A then prove that $\phi(t) = [\exp(\lambda_1 t)v_1, \exp(\lambda_2 t)v_2, \ldots, \exp(\lambda_n t)v_n]$ is a fundamental matrix of $y' = Ay$ on $(-\infty, \infty)$. 7

(b) Find a fundamental matrix of $y' = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix} y$ on $(-\infty, \infty)$. 7

OR

(c) Find a fundamental matrix of $y' = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$ on $(-\infty, \infty)$. 7

(d) Find the general solution of $y'' - 6y' + 9y = e^t$. 7

4. Answer any two questions : $2 \times 7 = 14$

(a) If $\alpha = 2m$ where m is a non-negative integer then prove that $(1 - t^2)$ $y'' - 2t\, y' + \alpha(\alpha + 1)y = 0$ has a power series solution in $|t| < 1$.

(b) Solve the Ivp : $y'' - 2ty' + 2n\, y = 0$, where $n = 2m + 1$, m is a non-negative integer, $y(0) = 0$, $y'(0) = 0$, $= \dfrac{2(-1)^m (2m+1)!}{m!}$.

(c) If $f \in \exists_1$ and $L\, f = F$ then prove that

$L^{-1}(F^n(z))\,(t) = (-1)^n t^n f(t)$, $\forall\, t \in (0, \infty)$, $\forall\, n = 1, 2, \ldots$

5. Answer any two questions : $2 \times 7 = 14$

(a) Define Wronskian $w(f_1, f_2)$ of two times different functions f_1, f_2 on I. If p, q are continuous functions on I then prove that two solutions ψ_1, ψ_2 of $y'' + p(t)y' + q(t)y = 0$ on I are linearly independent iff $w(\psi_1, \psi_2)(t) \neq 0, \forall t \in I$.

(b) State without proof. Gronwal's inequality. If $R = \{(t, y) \in \mathbb{R}^2 \mid |t - t_0| < a, |y - y_0| < b\}$ is a rectangle with center at (t_0, y_0), $f : R \longrightarrow \mathbb{R}$ is continuous s.t. f is bdd, $\frac{\partial f}{\partial y}$ exists, continuous and bdd on R then prove that the Ivp : $y' = f(t, y)$, $y(t_0) = y_0$ has a unique solution.

(c) Find $L(e^t \sin^2 t)$.

(d) Solve $y'' + 9y = \cos 2t$, $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform.

SC-003016104

SC-003016104 Seat No.________

M.Sc. Mathematics Semester-I (CBCS) Examination

October / November - 2012

CMT-1004: Theory of ordinary Differential Equations

Time : 2 ½ Hours Total Marks : 70

Q:1 Answer any Seven questions. 14

(i) The order of $y^{I} - y + t^{5}y^{II} + t^{4}y^{III} = 0$ is (a) 5 (b) 4 (c) 3 (d) 2

(ii) The solution of the IVP= $y^{I} = y$, $y(o) = 1$ is (a) e^{t} (b) $t + 1$ (c) e^{t+1} (d) cost

(iii) If A(t) is a continuous 2x2 matrix on I then $y^{I} = A(t)y$ has..........
(a) infinitely many solutions (b) unique solution (c) no solution
(d) finitely many solutions.

(iv) if A is a constant nxn Matrix and $\varphi(t)$ is a fundamental matrix of $y^{I} = Ay$ on $(-\infty, \infty)$ then $\exists$ a non-singular matrix C s-t $\forall$ $t \in (-\infty, \infty)$

(a) $\varphi(t) = C \exp At$ (b) $\varphi(t) = \exp At\ C$ (c) $\exp At = c\,\varphi(t)$
(d) $\varphi(t) = \exp At + c$

v) .... has no analytic solution of o (a) $y^{II} + t\,y^{I} + t^{2}y = o$

(b) $y^{I} = f(t)$, $y(o) = o$ where $f(t) = e^{-\frac{1}{t^{2}}}$ if $t \neq o$ and $f(t) = o$ if $t = o$

(c) $y^{II} + y^{I} + y = o$ (d) $y^{II} + t^{2}y + ty = o$.

vi) If $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$ then $\exp At =$ ____ (a) $e^{3t}\begin{pmatrix} \cos 5t & \sin 5t \\ -\sin 5t & \cos 5t \end{pmatrix}$

(b) $e^{3t}\begin{pmatrix} \cos 5t & -\sin 5t \\ \sin 5t & \cos 5t \end{pmatrix}$ (c) $e^{3t}\begin{pmatrix} \cos 5t & \sin 5t \\ -\sin 5t & -\cos 5t \end{pmatrix}$

(d) $e^{5t}\begin{pmatrix} \cos 5t & \sin 5t \\ -\sin 5t & \cos 5t \end{pmatrix}$

vii) The indicial equation of $t^2y'' + \alpha(t)y' + \beta(t)y = 0$ is

(a) $\mathfrak{z}(\mathfrak{z}-1)+\alpha\,\mathfrak{z} + \beta_0=0$ (b) $\mathfrak{z}2 + \mathfrak{z} + 1 = 0$

(c) $\mathfrak{z}2 + + \alpha\,\mathfrak{z} + \beta 0 = 0$ (d) $\mathfrak{z}(\mathfrak{z}-1)+ \mathfrak{z} + \beta_0=0$

viii) An nxn Matrix has .... (a) n distinct Eigen values

(b) at most n distinct Eigen values (c) n linearly independent Eigen vectors (d) no Eigen values.

ix) $\lambda(e^{ct}f(t))(\mathfrak{z}) =$ ____ (a) $f(\mathfrak{z}-c)$ (b) $(\lambda f)(\mathfrak{z}-c)$ (c)
(c) $(\lambda f)(\mathfrak{z})- c$

x) $(\lambda f'')(\mathfrak{z}) =$ ____ (a) $\mathfrak{z}\,\lambda f(f)\mathfrak{z} - f'(0) - f(0)$

(b) $\mathfrak{z}2\lambda\,(f)(\mathfrak{z}) - \mathfrak{z}f(0) - f'(0)$ (c) $\mathfrak{z}2\lambda\,(f)(\mathfrak{z}) - f(0) - \mathfrak{z}f'(0)$

(d) $\lambda f(\mathfrak{z}) - f(0) - f'(0)$

Q:2 Attempt any Two question. 14

a. State, without proof, the existence and uniqueness thereon for the solution of an Ivp of linear system of differential equations.

Determine whether the Ivp: $y' = \begin{pmatrix} 1 & -t & 0 \\ \frac{1}{t^2-1} & 0 & -1 \\ 2 & \frac{1}{t^2+1} & 3 \end{pmatrix} + \begin{pmatrix} e^t \\ \cos t \\ e^{-t} \end{pmatrix}$

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The motion of the particles two body problem is the conic one focus at the c.m.

$y(10) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ has unique solution and find the largest interval of existence of the solution.

b. State and prove variation of constants formula for second order scalar non-homogeneous equations.

c. Solve the Ivp: $y' = \begin{bmatrix} 1 & \\ 0 & \end{bmatrix} y + \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix}$, $y(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$= \int_{t_1}^{t_2} \frac{\psi_1(s)\psi_2(t) - \psi_2(s)\psi_1(t)}{\omega(\psi_1,\psi_2)(s)} g(s)\,ds$

Q:3 Answer the following. 14

a) Prove that the Ivp: $y'' + p(t)y' + q(t)y = r(t)$; $y(t_0) = \eta_1$, $y'(t_0) = \eta_2$ is equivalent to the Ivp: $y' = \begin{bmatrix} 0 & 1 \\ -q(t) & -p(t) \end{bmatrix} + \begin{bmatrix} 0 \\ r(t) \end{bmatrix}$, $y(t_0) = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$ where p,q $= I \to \mathbb{R}$ are continuous, $t_0 \in I$ and $\eta_1, \eta_2 \in \mathbb{R}$.

b) Find the Eigen value of $\begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$ and a fundamental matrix of $Y' = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix} y$ on $(-\infty, \infty)$

OR

a) Define exponential of a nxn Matrix and find a fundamental matrix of $y' = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} y$ on $(-\infty, \infty)$

b) If $\alpha = 2m$ where m is a non-negative integer then prove that $(1-t^2)y'' - 2ty' + \alpha(\alpha+1)y = 0$; $y(0)=1$, $y'(0)=0$ has a solution which is a polynomial of degree 2m.

Q.4. Answer any two : 14

a) Show that $ty'' + ty' + ty = 0$ has only one solution of the form $|t| \sum_{k=0}^{\infty} C_k t^k$, $c_0 = 1$ in any excluded *nbhd.* of 0.

b) Define Gamma function. State and prove the recursion formula for Gamma function.

c) State without proof, the first shifting thereon for Laplace transform. Using it find the Laplace transform of $t^n e^{ct}$, $C \in \mathbb{R}$, $n \in \{0, 1, 2, \ldots\}$.

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Q:5 Answer any two questions: 14

a) Define singular point, regular point of $9_0(t)\,y^n + 9_1(t)\,y^{n-1} + \ldots + 9_{n-1}(t)y^1 + 9_n(t)\,y = 0$. Locate and classify all singular points of $(t-1)^3\,y^{11} + 2\,(t-1)^2\,y^1 - ty = 0$.

b) Prove that a solution matrix $\varphi(t)$ $y^1 = A(t)y$ on I is a fundamental matrix iff $\det \varphi(t) \neq 0,\ \forall \in \in I$

c) Solve $y^{11} + 9y = \cos 2t$, $y(0)=1$, $y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform

d) Solve $y^{11} + y = 0$, $y=0$, $y^1=0$ when $t = 0$ using Laplace tranform.

003-016104 Seat No. ____________

M. Sc. (Sem. I) (CBCS) Examination

December – 2011

Maths. : CMT-1004

(Ordinary Differential Equations)

(New Course)

Faculty Code : 003

Subject Code : 016104

Time : 2.30 Hours] [Total Marks : 70

1 Attempt any seven questions : 2×7=14

(i) The order of $y^3 - t^5 (y')^4 + t(y'')^2 + y'''$ is ______

(a) 5 (b) 3

(c) 4 (d) 2

(ii) $y'' + y' - y = t$, $y(0) = 0$, $y'(0) = 1$ is equivalent to

(a) $y' = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) $y' = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(c) $y' = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(d) $y' = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(iii) If $A(t)$ is an $n \times n$ matrix continuous on I and $\Phi(t)$, $\psi(t)$ are fundamental matrices of $y' = A(t)Y$ on an interval I then

(a) $\psi(t) = C\Phi(t)$, $\forall t \in I$ for some constant $n \times n$ matrix C

(b) $\psi(t) = \Phi(t)C$, $\forall t \in I$ for some non-singular constant $n \times n$ matrix C

(c) $\Phi(t) = C\psi(t)$, $\forall t \in I$ for some constant $n \times n$ matrix C

(d) $\Phi(t) = \psi(t)$, $\forall t \in I$ for some constant $n \times n$ matrix C

(iv) If ψ_1, ψ_2 are two linearly independent solutions of $y'' + p(t)y' + q(t)y = 0$ on I and $t_0 \in I$ then the solution of $y'' + p(t)y' + q(t)y = r(t)$, $y(t_0) = 0$, $y'(t_0) = 0$ is

(a) $\int_{t_0}^{t} [\psi_1(s) - \psi_2(s)] r(s) ds$

(b) $\int_{-t_0}^{t} \frac{[\psi_2(t) - \psi_1(s)]}{W(\psi_1, \psi_2)(t)} ds$

(c) $\int_{t_0}^{t} \frac{\psi(s) \quad \psi_2(s)}{W(\psi_1, \psi_2)(s)} ds$

(d) $\int_{t_0}^{t} \frac{\psi_2(t)\psi_1(s) - \psi_1(s)(t)\psi_2(s)}{W(\psi_1, \psi_2)(s)} r(s) ds$

(v) For $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$, $\exp(tA) =$ __________

(a) $\exp t \begin{pmatrix} 2 & t \\ 0 & 2 \end{pmatrix}$

(b) $\exp(2t) \begin{pmatrix} t & 1 \\ 0 & 1 \end{pmatrix}$

(c) $\exp(2t) \begin{pmatrix} 0 & 1 \\ 1 & t \end{pmatrix}$

(d) $\exp(2t) \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$

(vi) For $(t-1)^3 y'' + 2(t-1)^2 y' - 7ty = 0$.

(a) 0 is a singular poinc

(b) ~~0 is a regular~~ singular point

(c) 1 is a regular singular point

(d) 1 is an irregular singular point

(vii) The indicial equation of $t^2 y'' - ty' + ty = 0$ is __________

(a) $z(z-1) - z + 1 = 0$

(b) $z^2 = 0$

(c) $z(z-1) + z - 1 = 0$

(d) $z(z-1) = 0$

(viii) $ty'' + ty' - y = 0$ has exactly __________ solution(s) of the ~~form $|t|^z \sum_{k=0}^{\infty} c_k t^k$, $c_0 = 1$ in an excluded nbhd of 0~~

(a) 2

(b) 1

(c) n_0

(d) 3

(ix) If $f \in \exists r$ and $Lf = F$ then

$L^{-1}(F^n(z))(t) =$ ______, $\forall t \in [0, \infty)$

(a) ~~$t^n f(t)$~~

(b) ~~$(-1)^n t^n f(t)$~~

(c) $-f(t)$

(d) $-t(f(t))''$

(x) For $C \in I\!R$, $L(\cosh Ct)(z) =$ __________, $\forall z \in \mathbb{C}$ st $R_e\, z > |c|$

(a) $\frac{z}{z^2 + c^2}$ (b) $\frac{c}{z^2 + c^2}$

(c) $\frac{z}{z^2 - c^2}$ (d) $\frac{c}{z^2 - c^2}$

2 Answer any two questions : $2 \times 7 = 14$

(a) Let $A(t)$ be an $n \times n$ matrix continuous on I. State and prove the necessary and sufficient condition for a solution matrix of $y' = A(t)y$ on I to be a fundamental matrix.

(b) True or False ? Justify :

(i) If $\Phi(t)$ is a solution matrix of $y' = A(t)y$ on I then $\det \Phi(t_0) \neq 0$, for some $t_0 \in I \Rightarrow \det \Phi(t) = 0, \forall t \in I$.

(ii) The columns of an $n \times n$ matrix $A(t)$ on I are linearly independent $\not\Rightarrow \det A(t) \neq 0, \forall t \in I$.

(c) If $\Phi(t)$ is a fundamental matrix $y' = A(t)y$ on I then prove that $\Phi(t)C$ is also a fundamental matrix, $\forall$ non-singular $n \times n$ matrix C. Give an example to show that $C\Phi(t)$ need not be even solution matrix of $Y' = A(t)Y$ on I.

3 Answer the following :

(a) Define the Wronskian $W(f_1, f_2, \ldots, f_n)$ of $(n-1)$-times differentiable functions $f_1, f_2, \ldots, f_n$ on I for some $n \geq 2$. If $p_1, p_2, \ldots, p_n$ are continuous on I then prove that n solutions $\psi_1, \psi_2, \ldots, \psi_n$ of $y^n + p_1(t)y^{n-1} + \ldots + p_n(t)y = 0$ are linearly independent iff $W(\psi_1, \psi_2, \ldots, \psi_n)(t) \neq 0, \forall t \in I$.

(b) Find the solution of the $Ivp: y' = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} y + \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$, $y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ on $(-\infty, \infty)$.

OR

3 (a) If A is a constant $n \times n$ matrix then prove that $\exp(tA)$ is the fundamental matrix of $y' = Ay$ on $(-\infty, \infty)$. Find $\exp(tA)$ for $A = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$. 7

(b) Find a particular solution and general solution of $y'' + y = \tan t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that $-\cos t \log|\sec t + \tan t|$ is a solution of $y'' + y = \tan t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. 7

4 Answer any two of the following : 2×7=14

(a) If $\alpha = 2m$ then prove the

$Ivp: (1-t^2)y'' - 2ty' + \alpha(\alpha+1)y = 0, \; y(0) = 0, \; y'(0) = 1$

has power series solution which converges for $|t| < 1$.

(b) (i) Locate and classify all the singular points of $t^2 y'' + ty' + (\alpha^2 - t^2)y = 0$ where α is a non-zero constant. 3

(ii) If q is a constant then determine the form of general solution and the region of validity of the general solution of $ty'' + (1-t)y' + qy = 0$. 4

(c) If $f \in \exists$ is differentiable and $f' \in \exists$ then prove that $L(f'(t))(z) = zL(f)(z) - f(0)$. Deduce that if $f \in \exists$ is n times differentiable and $f^1, f^2, \ldots, f^n \in \exists$ then

$$L(f^n(t))(z) = z^n L(f)(z) - \sum_{j=0}^{n-1} z^{n-1-j} f^j(0).$$

5 Answer any two of the following : 2×7=14

(a) Define the sequence of successive approximations to the solution of $y' = f(t, y)$; $y(t_0) = y_0$ where $f(t, y)$ is a given function. Construct the sequence of approximations to $y' = y$, $y(0) = 1$.

(b) State and prove Gronwals' inequality.

(c) Find $L^{-1}\left(\frac{3z+7}{z^2 - 2z - 3}\right)$.

(d) Solve $y'' + 25y = 10\cos 5t$, $y(0) = 2$, $y'(0) = 0$ using Laplace transform

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003-016104 / B-74 Seat No. ____________

M.Sc. (CBCS) (Sem. I) Examination

November / December – 2010

Ordinary Differential Equations

(Model-A) (New Course)

Faculty Code : 003

Subject Code : 016104

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

1 Answer any seven questions : [2×7=14]

(i) For $y' = A(t)y$,

(A) an n×n matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t) \neq o$, $\forall t \in I$.

(B) An n×n matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) \neq o$, for some $t_o \in I$.

(C) A solution matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) \neq 0$, for some $t_o \in I$.

(D) A solution matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) = 0$, for some $t_o \in I$.

(ii) If $\Phi(t)$ is a fundamental matrix of $y' = A(t)y$ on I then :

(A) $\Phi(t)C$ is also a fundamental matrix of $y' = A(t)y$ on I, $\forall$ n×n matrix C.

(B) $\Phi(t)C$ is also a fundamental matrix of $y' = A(t)y$ on I, $\forall$ non-singular matrix n×n matrix C.

(C) $C\Phi(t)$ is also a fundamental matrix of $y' = A(t)y$ on I; for all n×n matrix C.

(D) $C\Phi(t)$ is also a fundamental matrix of $y' = A(t)y$ on I, for all non singular n×n matrix C.

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(iii) If $\Phi(t)$ is a fundamental matrix of $y' = A(t)y$ on I then the solution of $y' = A(t)y + S(t)$, $y(t_0) = 0$ is

(A) $\int_{t_0}^{t} \Phi^{-1}(s) S(s)\,ds$

(B) $\Phi(t) \int_{t_0}^{t} s(s)\,dr$

(C) $\Phi(t) \int_{t_0}^{t} \Phi^{1}(r) S(s)\,dr$

(D) $\Phi(t) \int_{t_0}^{t} \Phi^{-1}(s) S(s)\,ds$

(iv) For $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$, $\exp(tA) =$ ________.

(A) $e^{3t}\begin{pmatrix} \cos 5t & \sin 5t \\ -\sin 5t & \cos 5t \end{pmatrix}$

(B) $e^{5t}\begin{pmatrix} \cos 3t & \sin 3t \\ -\sin 3t & \cos 3t \end{pmatrix}$

(C) $e^{5t}\begin{pmatrix} \cos 3t & \sin 5t \\ -\sin 5t & \cos 3t \end{pmatrix}$

(D) $e^{3t}\begin{pmatrix} \cos 5t & -\sin 3t \\ \sin 3t & \cos 5t \end{pmatrix}$

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003-016104 / B-74 Seat No. ____________

M.Sc. (CBCS) (Sem. I) Examination

November / December – 2010

Ordinary Differential Equations

(Model-A) (New Course)

Faculty Code : 003

Subject Code : 016104

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

1 Answer any seven questions : [2×7=14]

(i) For $y' = A(t)y$,

(A) an n×n matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t) \neq o$, $\forall t \in I$.

(B) An n×n matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) \neq o$, for some $t_o \in I$.

(C) A solution matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) \neq 0$, for some $t_o \in I$.

(D) A solution matrix $\Phi(t)$ on an interval I is a fundamental matrix iff det $\Phi(t_o) = 0$, for some $t_o \in I$.

(ii) If $\Phi(t)$ is a fundamental matrix of $y' = A(t)y$ on I then :

(A) $\Phi(t)C$ is also a fundamental matrix of $y' = A(t)y$ on I, ∀ n×n matrix C.

(B) $\Phi(t)C$ is also a fundamental matrix of $y' = A(t)y$ on I, ∀ non-singular matrix n×n matrix C.

(C) $C\Phi(t)$ is also a fundamental matrix of $y' = A(t)y$ on I, for all n×n matrix C.

(D) $C\Phi(t)$ is also a fundamental matrix of $y' = A(t)y$ on I, for all non singular n×n matrix C.

(iii) If $\Phi(t)$ is a fundamental matrix of $y' = A(t)y$ on I then the solution of $y' = A(t)y + S(t)$, $y(t_0) = 0$ is

(A) $\int_{t_0}^{t} \Phi^{-1}(s).S(s)ds$

(B) $\Phi(t)\int_{t_0}^{t} S(s)dr$

(C) $\Phi(t)\int_{t_0}^{t} \Phi^{1}(r).S(s)dr$

(D) $\Phi(t)\int_{t_0}^{t} \Phi^{-1}(s).S(s)ds$

(iv) For $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$, exp(tA) = ________.

(A) $e^{3t}\begin{pmatrix} \cos 5t & \sin 5t \\ -\sin 5t & \cos 5t \end{pmatrix}$

(B) $e^{5t}\begin{pmatrix} \cos 3t & \sin 3t \\ -\sin 3t & \cos 3t \end{pmatrix}$

(C) $e^{5t}\begin{pmatrix} \cos 3t & \sin 5t \\ -\sin 5t & \cos 3t \end{pmatrix}$

(D) $e^{3t}\begin{pmatrix} \cos 5t & -\sin 3t \\ \sin 3t & \cos 5t \end{pmatrix}$

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(v) If $a_o(t)$, $a_1(t)$, $a_2(t)$ are analytic at t_o then t_o is a singular point of $a_o(t)y'' + a_1(t)y' + a_2(t)y = 0$ if :

(A) none of $a_o(t)$, $a_1(t)$, $a_2(t)$ is zero at t_o

(B) $a_o(t_o) = o$ but not all $a_1(t_o)$, $a_2(t_o)$ are zero

(C) $a_o(t_o) = 0$

(D) all of $a_o(t)$, $a_1(t)$, $a_2(t)$ are zero at t_o

(vi) If a_1, a_2 are none-zero constants then for the Euler equation $(t-1)^2 y'' + (t-1)a_1 y' + a_2 y = 0$

(A) 1 is a regular point

(B) 1 is a regular singular point

(C) 1 is an irregular singular point

(D) 1 is an ordinary point

(vii) If $\alpha(t) = \sum_{k=0}^{\alpha} \alpha_k t^k$, $\beta(t) = \sum_{k=0}^{\alpha} \beta_k t^k$, $\forall |k| < r$ for none $r > 0$

then the indicial equation of $t^2 y^{11} + t\alpha(t)y^1 + \beta(t)y = 0$ is :

(A) $z^2 + \alpha_o \beta + \beta_o = 0$

(B) $Z + \alpha_o(z - 1) + \beta_o = 0$

(C) $Z^2 + \beta_o Z + \alpha_o = 0$

(D) $Z(Z - 1) + \alpha_o Z + \beta_o = 0$

(viii) If α, β are analytic at o and Z_1, Z_2 are the roots of the indicial quation of $t^2y'' + t\alpha(t)y' + \beta(t)y = 0$ with $R_eZ_1 \geq R_eZ_2$ then $t^2y'' + t\alpha(t)y'\beta(t)y = 0$ has two lineriy independent solutions of the term $|t|^z \sum_{k=0}^{\alpha} C_k t^k$, $C_0 = 1$ in an excluded nbhd of o if:

(A) Z_1, Z_2 are distinct

(B) $Z_1 = Z_2$

(C) $Z_1 - Z_2$ is not equal to 0, 1, 2,....

(D) $Z_1 - Z_2$ is positive integer.

(ix) If L(f)=F and $C \in \mathbb{C}$ then $L(e^{ct} f(t))$ = ________

(A) F (Z+C)

(B) F (B-C)

(C) F(Z)+C

(D) F(Z)-C

(x) For $C \in \mathbb{C}$, $L^{-1}\left(\frac{C}{Z^2 + C^2}\right)(t) =$ ________

(A) cosct

(B) sinct

(C) sinhct

(D) coshct

35

2. Answer any two questions : 2×7=14

(i) State and prove variation of constant formula for the solution of the IVP $y^1 = A(t)y + S(t)$, $y(t_o) = 0$ on an interval I containing t_o.

(ii) If $\alpha = 2m$, where m is a non-negative integer then prove that the solution of the IVP $(1-t^2)y^{11} - 2ty^1 + \alpha(\alpha+1)y = 0$; $y(o) = 1$; $y^1(o) = 0$ is a polynomial of degree 2m.

(iii) State, without proof, Gronwall's inequality. Hence OR otherwise prove that the solution of the $IVP = y^1 = f(t;y)$, $y(t_o) = y_o$ has a unique solution where $f: R \to R$ is continuous, bounded, $\frac{df}{dy}$ is continuous and bounded and $R = \{(t,y) \mid |t-t| < a, |y - y_o| < b\}$.

3. Answer the following :

(i) (a) If A is a constant n×n matrix and $v_1, v_2, ..., v_n$ are linearly independent eigen vectors corresponding to the eigen values $\lambda_1, \lambda_2, ..., \lambda_n$ of A then prove that : 5

$\Phi(t) = [e^{\lambda_1 t} v_1, e^{\lambda_2 t} v_2, ..., e^{\lambda_n t} v_n]$ is a fundamental matrix for $y^1 = Ay$ on $(-\alpha, \alpha)$.

(b) With usual notations, if $f \in \exists_1$ i... times differentiable and $f^1, f^{11} \ldots \in \exists_1$ then prove that :

$$L(f^n(t))(z) = Z^n(L(f)(z) - \sum_{j=0}^{n-1} Z^{n-1-j} f^j(0)$$

(c) Locate and classify the singular points of :

$(t-1)^3 y^{11} + 2(t-1)^2 y^1 - 7ty = 0$.

OR

3 (i) Solve the $IVP : y^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$, $y(o) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(ii) Find $L^{-1}\left(\dfrac{3Z+7}{Z^2-2Z-3}\right)$

4 Answer any two of the following :

(i) If p(t), q(t) are continuous on some interval I and $t_o \in I$ then prove that for every $y_o \in \mathbb{R}$, the IVP: $y^1 + p(t)y = q(t)$, $y(t_o) = y_o$ has a unique solution on I.

(ii) Solve the IVP : $y'' + 9y = \cos 2t$, $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform.

(iii) Solve that the equation $ty'' + ty' - y = 0$ has only one solution of the form $|t|^z \sum_{k=0}^{\alpha} c_k t^k$; $c_o = 1$ in an excluded nbhd of 0.

5 Answer any two of the following :

(i) Define the Wronskian $w(f_1, f_2, ... f_n)$ on n-1 times differentiable functions $f_1, f_2, ... f_n$. If $p_1, p_2, ... p_n : I \to \mathbb{R}$ are contineous then prove that n solutions $\psi_1, \psi_2, ... \psi_n$ of $y^n + p_1(t) y^{n-1} + ... + p_n(t) y = 0$ on I are linearly independent iff $w(\psi_1, \psi_2, ... \psi_n)(t) \neq 0$; $\forall t \in I$. 7

(ii) Find a fundamental matrix of $y' = Ay$ on $(-\infty, \infty)$ where $A = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$ 7

(iii) Solve the IVP : $y'' - 2ty' + 2ny = 0$, $n = 2m$, an even integer, $y(o) = \frac{(-1)^m (2m)!}{m!}$, $y'(o) = 0$. 7

(iv) Solve $y'' + y = t$, $y'(o) = 1$, $y(\pi) = 0$ using Laplace transform. 7

PCF-003-1161004 Seat No. 15044

M. Sc. (Mathematics) (Sem. I) (CBCS) Examination

December - 2018

CMT - 1004 : Theory of Ordinary Differential Equation

(Old and New Course)

38

Faculty Code : 003
Subject Code : 1161004

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Answer all the questions.
(2) There are five questions.
(3) Figures to the right indicate full marks.

1 Answer all questions : 7×2=14

(1) Find general solution of $y''' + 3y'' + 3y' + y = 0$ on $\mathbb{R}$

(2) (a) Define Gamma Function and
(b) State Bessel's Equation.

(3) Prove that e^{3t} and te^{3t} are two Linearly Independent solutions of $y'' + 6y' + 9y = 0$ on $(-\infty, \infty)$.

(4) Define : (1) Fundamental Matrix
(2) Irregular Singular Point.

(5) Let A be a $n * n$ matrix then Show that A has atmost n distinct Eigen values and A has atmost n L.I Eigen vectors.

(6) If y_1, y_2 are solutions of

$\left(1-x^2\right)y'' - 2xy' + p(p-1)y = 0$ with the initial

conditions $y_1(0) = (0), y_1'(0) = -1, y_2(0) = 1, y_2'(0) = 0$

then find $w(y_1, y_2)\left(\frac{1}{2}\right)$.

(7) State First Shifting Theorem and find $L\left(e^{at}\right)(z)$.

(8) State Second Fundamental Theorem of calculus and Find Gamma (1)

(9) Let A and B be $n*n$ matrix and $AB = BA$ then $\exp(A+B) = \exp(A)*\exp(B)$

(10) State the condition of the solution of an Initial value Problem of a system of 1st order linear differential equation.

2 Answer any two : 2×7=14

(1) State and prove Gronwell's Inequaiity.

(2) Prove that if $\alpha = 2m+1$ where m is a non-negative integer then the solution ϕ of the Legendre's equation with $y(0)=0$ and $y'(0)=1$ is a polynomial of degree $2m+1$. Compute this polynomial for $m = 0,1,2$.

(3) (a) Construct the successive approximation $\phi_0, \phi_1, \phi_2, \phi_3$ to a solution of $y' = \cos y$ with $y(0) = 0$.

(b) State and prove Variation of constant formulae for scalar linear first order homogenous differential equation.

3 All are compulsory : 2×7=14

(1) Find the solution of the initial value problem $y' = ty$ with $y(0) = 1$ and $y'(0) = 0$.

(2) Let A be a constant 2×2 complex matrix then prove that there exists a constant 2×2 non-singular matrix T such that T^{-1} AT has the form $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$

OR

3 All are compulsory : 2×7=14

(1) Find the particular solution of $y'' + y = \tan t$ on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$; $y(0) = 0$ and $y'(0) = 0$.

(2) Prove that if $P_1, P_2, P_3, \ldots P_n : I \to \mathbb{R}$ are continuous functions then the solutions $\varphi_1, \varphi_2, \varphi_3, \ldots \varphi_n$ of second order scaler linear differential equations are linearly independent if and only if $w(\varphi_1, \varphi_2, \varphi_3, \ldots \varphi_n)(t) \neq 0; \forall t \in I$.

40

4 Answer the following questions : 2×7=14

(1) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \forall t \in -(-\infty, \infty)$ and find exp $(tA); \forall t \in (-\infty, \infty)$.

(2) Prove that Eigen vectors corresponding to the distinct Eigen values of $n * n$ matrix are linearly independent in $\mathbb{R}^n$ or $\mathbb{C}^n$.

5 Answer any **two** : 2×7=14

(1) Find : (a) $L(\sinh ct)(z)$ and (b) $L(Cosat)(z)$.

(2) Define Convolution. Further show that if $f \in H$ and $\frac{f(t)}{t} \in H$ then $L\left(\frac{f(t)}{t}\right)(z) = \int_z^{\infty} (Lf(w))\, dw$ for which $img(w)$ is bounded and $\mathrm{Re}(w) \to \infty$.

(3) (a) State and prove change of scale property.

(4) Solve $y'' - 3y' + 2y = 4e^{2t}$ with $y = -3$ and $y' = 5$ when $t = 0$ using Laplace Transform.

Seat No. ________________

F8AB-003-1161006

M. Sc. (Sem. I) Examination

December - 2022

Mathematics : EMT-1001

(Classical Mechanics - I)

Faculty Code : 003
Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours / Total Marks : 70

Instructions : (1) There are total five questions.
(2) Each question carries equal marks.
(3) All the questions are compulsory.

1 Attempt the following : (any seven) **14**

(1) Define : Radius vector and Acceleration.

(2) Define : Moment of force.

(3) Define with example : Non-Holonomic constraints.

(4) Define with example : Scaleronomuous constraints.

(5) When a system is said to be a conservative ?

(6) Define with example : Degrees of freedom.

(7) Define : Configuration space.

(8) Find the degrees of freedom of fixed fulcrum and bob of a simple pendulum.

(9) Define central force.

(10) State only the Kepler's third law of planetary motion.

2 Attempt the following : **14**

(a) State and prove angular momentum conservation theorem for the mechanics of system of particles.

OR

Discuss in detail the conservation of total energy for a system of particles.

(b) Discuss in detail the problem of Atwood machine.

OR

Derive the Lagrange's equations of motion for a single particle in space with mass m in

(i) Cartesian co-ordinates

(ii) Plane polar coordinates

3 Attempt the following : 14

(a) Find the minimum surface of revolution about y-axis.

(b) Derive the Lagrange's equations of motion for general system.

OR

(b) A particle falls a distance y_o in a time $t_o = \sqrt{{2y_o}/{g}}$.

If the distance $y = at + bt^2$ then show that the integral

$\int_0^{t_o} L dt$ has an extremum for real values of coefficients

only when $a = 0$ and $b = \frac{g}{2}$.

4 Attempt the following : 14

(a) Derive the equations of motion and find the first integrals for two bodies central force motion.

(b) Show that the shortest distance between two points in plane is a straight line.

5 Attempt the following : (any two) 14

(a) Derive the orthogonal matrix of transformation in two dimensional co-ordinate system.

(b) Define cyclic coordinate and show that if V being independent of velocities and L is not an explicit function of time then total energy is conserved.

(c) Define Euler angles and obtain the transformation matrix A from space axes to body axes. Also derive A^{-1}.

(d) Define Coriolis force and discuss any two effects of it.

SBW-003-1161006 Seat No. ________

M. Sc. (Sem. I) Examination

February / March - 2022

Mathematics : EMT - 1001

(Classical Mechanics - I)

Faculty Code : 003

Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Attempt the following : **14**

(1) Define : Velocity, Acceleration and Linear Momentum.
(2) Define : Configuration space.
(3) Define with example : Non-Holonomic constraints.
(4) Define with example : Scaleronomuous constraints.
(5) When a system is said to be a conservative ?
(6) Define : Degrees of freedom and count the number of degrees of freedom of a fixed fulcrum of a simple pendulum.
(7) State the problems arising due to constraints.

2 Attempt the following : **14**

(1) Define : Monogenic system. Is the monogenic system conservative ? Justify your answer.
(2) State only the Hamilton's variational principle.
(3) State only the Kepler's first law of planetary motion.
(4) Find the degrees of freedom for dumbbell and bob of a simple pendulum.
(5) Define : angular momentum.
(6) Define: central force.
(7) Define : Torque on the motion of a particle.

3 Attempt the following : **14**

(a) State and prove linear momentum conservation theorem for a single particle.
(b) Discuss in detail the conservation of total energy for a system of particles.

4 Attempt the following : **14**

(a) For the problem of Atwood machine show that :

$$\ddot{x} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right) g .$$

(b) Derive the Lagrange's equations of motion for conservative Holonomic system.

5 Attempt the following : **14**

(a) Derive the Lagrange's equations of motion for a single particle in space with mass m in

(i) Cartesian co-ordinates

(ii) Plane polar co-ordinates

(b) Find the minimum surface of revolution about y-axis.

6 Attempt the following : **14**

(a) Show that central force motion of two bodies about their C.M. can always be reduced to an equivalent one body problem.

(b) Derive the equations of motion and find the first integrals for two bodies central force motion.

7 Attempt the following : **14**

(a) Find the shortest distance between two points in plane.

(b) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop.

8 Attempt the following : **14**

(a) Discuss in detail the use of direction cosines to describe the independent co-ordinates relative to the rigid body motion.

(b) Define cyclic co-ordinate and show that if V being independent of velocities and L is not an explicit function of time then total energy is conserved.

9 Attempt the following : **14**

(a) Derive : Kepler's third law of planetary motion.

(b) Derive the orthogonal matrix of transformation in XY-plane.

10 Attempt the following : **14**

(a) Derive the orthogonal transformation in terms of Cayley-Klein parameters.

(b) Define Euler angles and obtain the transformation matrix A from space axes to body axes.

BA-003-1161006 Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

March - 2021

EMT - 1001 : Mathematics

(Classical Mechanics - I)

Faculty Code : 003
Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Attempt the following : 14

(1) Define : Linear momentum.
(2) State only the Linear momentum conservation theorem for a single particle.
(3) Define with example : Non-Holonomic constraints.
(4) Define with example : Rheonomuous constraints.
(5) Define with example : Degrees of freedom.
(6) What is monogenic system ?
(7) Define : Configuration space.

2 Attempt the following : 14

(1) Define : Cyclic co-ordinate.
(2) What will be the shape of orbit of the planet mercury about the Sun ?
(3) State only the Kepler's first Law of planetary motion.
(4) Define Central Force.
(5) Define moment of force.
(6) State the equation of constraints acting on the rigid bodies.
(7) Define generalized momentum with respect to the coordinate x.

3 Attempt the followings : 14

(a) Discuss in detail the Brachestochrone problem.

(b) State and prove angular momentum conservation theorem for a single particle.

4 Attempt the following : 14

(a) Explain in detail the conservation of total energy for a system of particles.

(b) State and prove linear momentum conservation theorem for a system of particles.

5 Attempt the following : 14

(a) Explain in detail principle of virtual work and derive the D'Alembert's principle.

(b) Using D'Alemberts principle derive the Lagrange's equations of motion for general system.

6 Attempt the following : 14

(a) If the total mass of the system is concentrated about C.M. and moving with it then show that the total K.E. of the system is K.E. at the C.M. plus K.E. about C.M.

(b) Obtain the equations of the motion for a particle in space with reference to Cartesian as well as polar coordinate systems.

7 Attempt the following : 14

(a) Discuss in detail the problem of Atwood machine and show that the tension of rope appears nowhere in the equation of motion.

(b) Derive Lagrange's equation of motion using Hamilton's variational principle.

8 Attempt the following : 14

(a) Show that the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one body problem.

(b) Discuss in detail the techniques of calculus of variations.

9 Attempt the following : **14**

(a) A particle of mass m moves under a central force then show that :

(i) Its orbit is a plane curve.

(ii) Its areal vector sweeps out equal area in equal time.

(b) Determine the nature of orbit of a particle moving under an attractive the force $F = {}^{-k}\!/_{r^2}$ (where $k =$ constant). Also derive the Kepler's third Law of planetary motion.

10 Attempt the following : **14**

(a) Define Euler angles and obtain the transformation matrix from space axes to body axes.

(b) Define Cayley-Klein parameters and obtain the orthogonal matrix of transformation in terms of Cayley-Klein parameters.

JBH-003-1161006 Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

December – 2019

Mathematics : EMT-1001

(Classical Mechanic-I)

(Old & New Course)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.

(2) Attempt all the questions.

(3) Each question carries equal marks.

1 Attempt any seven : **14**

1. Define : Linear momentum and Angular momentum of a particle.
2. State minimum two differences between Holonomic constraints and non-Holonomic constraints.
3. Define with example: Scaleronomous constraints.
4. When a system is said to be a conservative?
5. Define: moment of force.
6. Define with example: Degrees of freedom.
7. Define: Configuration space.
8. Define: Cyclic co-ordinates.
9. State only the Hamilton's variational principle.
10. State only the Kepler's first law of planetary motion.

2 Attempt the following : **14**

(a) Derive the Lagrange's equations of motion for general system.

OR

(a) State and prove Angular momentum conservation theorem for a system of particles.

(b) Discuss in detail the conservation of total energy for a system of particles.

3 Attempt the following : **14**

(a) Derive the Lagrange's equations of motion using Hamilton's variational principle.

OR

(a) Discuss in detail the problem of Atwood machine and show that the tension of rope appears nowhere in the expression of acceleration.

(b) Find the shortest distance between two points in plane.

4 Attempt the following : **14**

(a) Derive the matrix of orthogonal transformation in terms of Cayley-Klein parameters.

(b) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop.

5 Attempt any two : **14**

(a) Derive the equations of motion and the first integrals for two bodies central force problem.

(b) Discuss in detail the use of direction cosines to describe the independent co-ordinates relative to the rigid body motion.

(c) Define Euler angles and obtain the transformation matrix from space axes to body axes.

(d) Define Coriolis force and discuss any one effect of the same.

(e) A particle falls a distance y_o in a time $t_0 = \sqrt{2y_0/g}$.

If the distance $y = at + bt^2$ then show that the integral $\int_0^{t_0} L dt$ has an extremum for real values of coefficients only when $a = 0$ and $b = \frac{g}{2}$.

PCG-003-1161006 Seat No. 15044

M. Sc. (Sem. I) (CBCS) Examination

December - 2018

CMT - 1001 : Mathematics

(Calssical Mathematics)

(New / Old Course)

Faculty Code : 003
Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) There are five questions.
(3) Figures on right side indicate the marks.

1 Attempt any **seven** : 14

(1) Define : Linear momentum.
(2) Define : torque or moment of force.
(3) Define with example : Holonomic constraints.
(4) Define with example : Scaleronomuous constraints.
(5) When a system is said to be conservative?
(6) Define with example : Degrees of freedom.
(7) Define : Configuration space.
(8) Define : Monogenic system.
(9) State only the Hamilton's variational principle.
(10) State only the Kepler's third law of planetary motion.

2 Attempt the followings : 14

(a) State and prove linear momentum conservation theorem for a system of particles.

OR

(a) Discuss in detail the conservation of total energy for a system of particles.

(b) Derive the Lagrange's equations of motion for conservative Holonomic system.

3 Attempt the followings : 14

(a) Derive the Langrange's equation of motion for a single particle in space with mass m in

(i) Cartesian co-ordinates

(ii) Plane polar co-ordinates

OR

(a) Discuss in detail the problem of Atwood machine.

(b) Find the shortest distance between two points in plane.

4 Attempt the followings : **14**

(a) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop.

(b) A particle falls a distance y_0 in a time $t_0 = \sqrt{2y_0/g}$. If the distance $y = at + bt^2$ then show that the integral $\int_0^{t_0} L\,dt$ has an extremum for real values of coefficients only when $a = 0$ and $b = \frac{g}{2}$.

5 Attempt any **two** : **14**

(a) Derive the equations of motion and find the first integrals for two bodies central force problem.

(b) Discuss in detail the use of direction cosines to describe the independent co-ordinates relative to the rigid body motion.

(c) Define Euler angles and obtain the transformation matrix from space axes to body axes.

(d) Derive the orthogonal transformation in terms of Cayley-Klein parameters.

(e) Define cyclic co-ordinate and show that if V being independent of velocities and L is not an explicit function of time then total energy is conserved.

HEK-003-1161006 Seat No. 15008

M. Sc. (Sem. I) (CBCS) Examination

November / December – 2017

EMT - 1001 : Mathematics

(Classical Mechanics-I)

(New Course)

3

Faculty Code : 003

Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Figures on right side indicate the marks.

1 Attempt any seven : 14

(1) Define: Linear momentum.

(2) Define: torque or momentum of force.

(3) Define with example: Non Holonomic constraints.

(4) Define with example: Rheonomous constraints.

(5) When a system is said to be a conservative?

(6) Define with example: Degrees of freedom.

(7) Define: Configuration space.

(8) Define: Monogenic system.

(9) State only the Hamilton's variational principle.

(10) State only the Kepler's first law of planetary motion.

2 Attempt the followings : 14

(a) State and prove Angular momentum conservation theorem for a system of particles. ch-1 (10)

OR

(a) Discuss in detail the conservation of total energy for a system of particles. ch-1 (16)

(b) Derive the Lagrange's equations of motion for general system. ch-1 (28)

4

3 Attempt the followings : 14

(a) Derive the Lagrange's equations of motion using Hamilton's variational principle. ch-2 47

OR

(a) Discuss in detail the problem of Atwood machine. ch-1 37

(b) Find the shortest distance between two points in plane. ch.-2 62

4 Attempt the followings : 14

(a) A particle falls a distance y_0 in a time $t_0 = \sqrt{2y_0/g}$. If the distance $y = at + bt^2$ then show that the integral $\int_0^{t_0} L\,dt$ has an extremum for real values of coefficients only when $a = 0$ and $b = \frac{g}{2}$. ch-2 71

(b) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop. ch-2 (43)

5 Attempt any two : 14

(a) Derive the equations of motion and find the first integrals for two bodies central force problem. ch-3 (88)

(b) Discuss in detail the use of direction cosines to describe the independent co-ordinates relative to the rigid body motion.

(c) Define Euler angles and obtain the transformation matrix from space axes to body axes. ch-4 C

(d) Derive the orthogonal transformation in terms of Cayley-Klein parameters.

MCB-003-1161006

Seat No. ____________

M. Sc. (Sem. I) (CBCS) Examination

December - 2016

EMT - 1001 : Mathematics

(Classical Mechanics - I) (New Course)

Faculty Code : 003

Subject Code : 1161006

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (i) Attempt all the questions.
(ii) Each question carry equal marks.

1 Attempt the following : (any seven)

(1) Define holonomic constraints.

(2) Define Rheonomous constraints.

(3) Define with example : Degrees of freedom.

(4) Define configuration space.

(5) Define monogenic system.

(6) State only the Hamilton's variational principle.

(7) State only the Kepler's first law of planetary motion.

(8) Define cyclic co-ordinate.

(9) State only the Kepler's third law of planetary motion.

(10) Determine the degrees of freedom of a dumb-bell.

2 Attempt the following : (any two)

(a) Explain in detail the conservation of total energy for a system of particles.

(b) Explain in detail the principle of virtual work and device D'Alembert's principle.

(c) Derive the equations of motion for a single particle in space in

(i) Cartesian co-ordinates

(ii) Plane polar co-ordinates.

3 Attempt the following :

(a) Derive the Lagrange's equations of motion for a general system.

(b) Discuss in detail the problem of Atwood machine.

OR

3 (a) Discuss in detail the techniques of calculus of variations.

(b) Find the shortest distance between two points in a plane.

4 Attempt the following :

(a) Derive the equations of motion and first integrals in the problem of two body central force motion.

(b) State and prove Euler's theorem for the motion of a rigid body.

5 Attempt any two :

(1) Define Euler angles and obtain the transformation matrix from space axes to body axes.

(2) Discuss in detail the infinitesimal rotations and derive the formula $dr = rx\,d\Omega$.

(3) Establish the formula

$$\left(\frac{d}{dt}\right)_s = \left(\frac{d}{dt}\right)_r + wx$$

where notations are being usual.

(4) Explain coriolis force and discuss any two effects of it.

 Seat No. ____________

M. Sc. (Mathmematics) (Sem. I) (CBCS) Examination

December - 2015

EMT - 1001 : Classical Mechanics - I

Faculty Code : 003
Subject Code : 016106

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.
(2) Each question carries equal marks.
(3) There are five questions.

1 Choose the appropriate alternative/alternatives : (any seven)

(1) The angular momentum of a particle is defined as
(A) p = mV (B) L = r × P
(C) N = r × F (D) F = ma

(2) The linear momentum is conserved if
(A) F = 0 (B) N = 0
(C) T = 0 (D) None of these

(3) The shortest distance between two points in a plane is
(A) Parabola (B) Ellipse
(C) Straight line (D) Circle

(4) The kinetic energy of the system is defined as
(A) p = mV (B) V = mgh
(C) F = ma (D) $T = \frac{1}{2}mV^2$

(5) The angular momentum of a particle is conserved if
(A) T = 0 (B) N = 0
(C) F = 0 (D) None of these

(6) Any coordinate q_i is cyclic if
(A) $\frac{\partial L}{\partial a_i} = 0$
(B) Lagrangian does not contain q_i
(C) L = 0
(D) None of these

(7) The nature of orbit of planet venus around sun is

(A) Ellipse (B) Circle

(C) Parabola (D) None of these

(8) According to Kepler's law the areal vector sweeps out

(A) half area in double time

(B) double area in half time

(C) one fourth area in half time

(D) equal area in equal time

(9) The number of degrees of freedom of dumb-bell is

(A) 2 (B) 3

(C) 5 (D) 6

(10) The number of degrees of freedom of a fulcrum of simple pendulum is

(A) 1 (B) 0

(C) 2 (D) 3

2 Attempt any two :

(a) State and prove linear momentum conservation theorem for a system of particles.

(b) If the total mass ofthe system is concentrated about C.M. and moving with it then show that the total K.E. of the system is K.E. at the C.M. + K.E. about C.M.

(c) Find the equation of motion for a bead sliding on a uniformly rotated wire in a force free space.

3 Attempt the followings :

(a) Discuss in detail the problem of Atwood Machine.

OR

(a) Discuss in detail the Brachestochrone problem.

(b) Derive Lagrange's equations of motion for general system.

4 Attempt the followings :

(a) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop.

(b) State Hamilton's variational principle and find the minimum surface of revolution about Y-axis.

OR

(b) A particle falls a distance Y_0 in a time $t_0 = \sqrt{\frac{2y_0}{g}}$. If the distance y at any time t is $y = at + bt^2$ then show that the integral $\int_0^{t_0} L\, dt$ is extremum for real values of the coefficients only when $a = 0$ and $b = g/2$.

5 Attempt any two :

(a) Define cyclic coordinates and show that the generalized momentum conjugate to a cyclic coordinate is conserved. Using this result derive that if component of applied torque vanishes then the corresponding component of angular momentum is conserved.

(b) Show that the central force motion of two bodies about their C.M. can always be reduced to an equivalent one body problem.

(c) A particle of mass m moves under a central force then show that

(i) Its orbit is a plane curve

(ii) Its areal vector sweeps out equal area in equal time.

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XD-127

003-016106

M.Sc. (Maths) (Sem.-I) Examination

December-2014

Mathematics

EMT – 1001 : Classical Mechanics - I

Faculty Code : 003
Subject Code : 016106

Time : 2½ Hours] **[Total Marks : 70**

Instructions : (1) Attempt all the question
(2) Each question carries equal marks.

1. Choose the appropriate alternative/alternatives (any seven) :

(1) The linear momentum is defined as
(a) $L = r \times p$ (b) $p = mv$
(c) $N = r \times F$ (d) $L = T - V$

(2) The angular momentum of a particle is conserved if
(a) $L = 0$ (b) $\dot{p} = 0$
(c) $N = 0$ (d) $F = 0$

(3) The shortest distance between two points in a plane is
(a) Circle (b) Parabola
(c) Straight line (d) None of these

(4) The Lagrangian L is defined as
(a) $T + V$ (b) $r \times F$
(c) $T - V$ (d) T^2

(5) Any physical quantity q is conserved provided
(a) $\dot{q} = 0$ (b) $q = 0$
(c) derivative does not exist (d) None of these

(6) Any co-ordinate q_j is cyclic provides
(a) Lagrangian does not contain q_j
(b) $\frac{\partial L}{\partial q_j} = 0$
(c) $L = 0$
(d) $q_j = 0$

(7) According to Kepler's law the areal vector sweeps out
(a) double area in half time
(b) half area in double time
(c) equal area in equal time
(d) None of these

(8) Holonamic constraints are
(a) Zero
(b) expressible in terms of algebraic equations
(c) can't be express in terms of algebraic equations ⇒ Non- holonamic
(d) None of these

(9) Rheonomons constraints are
(a) dependent on time
(b) independent of time ⇒ scalronomous.
(c) constant in time
(d) None of these

(10) The nature of orbit of planet Jupiter around Sun is
(a) Circle
(b) Parabola
(c) Ellipse
(d) Hyperbola

2. Attempt any two :
(a) State and prove angular momentum conservation theorem for a system of particles.
(b) Find the equation of a bead sliding on a uniformly rotates wire.
(c) Explain in brief the conservation of total energy for the system of particles.

3. Attempt the followings :
(a) Find the minimum surface of revolution about y-axis.
(b) Discuss in detail the Branchestochrone problem.

OR

(a) Find the shortest distance between two point in a plane.
(b) A hoop rolling without slipping down an inclined plane then find the force of friction acting on the hoop.

4. Attempt any two :
(a) Derive Lagrange's equations of motion for general system.
(b) Find the equations of motion in plane polar co-ordinates for a single particle in space.
(c) Show that the two body Central force motion about their C.M. can always be reduced to an equivalent are body problem.

5. Attempt any two :

(a) State Hamilton's variational principle and using it derive Lagrange's equations of motion.

(b) Show that generalized momentum conjugate to a cyclic co-ordinate is conserved. Using this result derive that if component of applied torque vanishes, then the corresponding component of angular momentum is conserved

(c) A particle of mass m moves under a Central force, then show that

(i) Its orbit is plane curve.

(ii) Its areal vector sweeps out equal area in equal time.

(d) If the mass of the body is concentrated about C.M., then show that the total angular momentum of system is equal to the angular momentum of C.M. plus angular momentum about C.M.

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003-016106

M.Sc. (CBCS) – Mathematics (Sem.-I) Examination

November-2013

MATHEMATICS

EMT-1001 : CLASSICAL MECHANICS - I

13

Faculty Code : 003

Subject Code : 016106

Time : 2 ½ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

1. Choose the appropriate alternative/alternatives (any seven) :

(1) The linear momentum of a particle is conserved when

(a) N = 0 (b) F = 0

(c) P = 0 (d) None of these

dp/dt

(2) The Lagrangian L equals to

(a) T – V (b) T + V

(c) TV (d) None of these

(3) Holonomic constraints are

(a) always zero

(b) always negative

(c) expressible in terms of algebraic equations

(d) dependent on time

(4) The moment of force is defined as

(a) N = rF (b) N = r/F

(c) $N = r^2 \times F^2$ (d) N = r × F

(5) The shortest distance between two points in a plane is

(a) Ellipse (b) Great circle

(c) Straight line (d) Undefined

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(6) The term aphelion refers to

(a) closest approach to the Sun

(b) fastest approach to the Sun

(c) average distance from the axis

(d) None of these

(7) The spring equinox occurs on

(a) 23rd March
(b) 22nd December
(c) 21st June
(d) 23rd September

(8) The path of the motion of planet Venus around the Sun is

(a) Parabola
(b) Ellipse
(c) Hyperbola
(d) Circle

(9) According to Kepler's third law of planetary motion

(a) $\tau^2 \propto a^3$
(b) $\tau^2 \propto a^2$
(c) $\tau^2 \propto a^{3/2}$
(d) [illegible]

(10) Any co-ordinate q_j is cyclic if

(a) L is constant
(b) $L = 0$
(c) L does not contain q_j
(d) $\frac{\partial L}{\partial q_j} = 0$

2. Attempt any two :

(a) State and prove angular momentum conservation theorem for single particle.

(b) State D'Alembert's principle and using it derive Lagrange's equations of motion for general system.

(c) Explain in brief the conservation of total energy for the system of particles.

3. Attempt the followings :

(a) Discuss in detail the problem of Atwood machine and derive the relation $\ddot{x} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right) g$.

(b) Find the minimum surface of revolution about y-axis.

OR

(a) Find the equation of motion for a bead sliding on a uniformly rotated wire in free space.

(b) Discuss in detail the Brachestochrone problem.

4. Attempt any two :

(a) Discuss in detail the techniques of Calculus of variation.

(b) Show that the Central Force motion of two bodies about their C.M. can always be reduced to an equivalent one body problem.

(c) Derive the equations of motion and first integrals for the two body central force motion.

5. Attempt any two :

(a) Define :

(i) Scleronomous constraints

(ii) Degrees of freedom

(iii) Holonomic constraints

(iv) Cyclic coordinate

(b) A particle of mass m moves under a central force then show that

(i) Its orbit is a plane curve.

(ii) Its area vector sweeps out equal area in equal time.

(c) Obtain Lagrange's equations of motion for a simple pendulum.

(d) A particle falls a distance y_0 in a time $t_0 = \sqrt{\frac{2y_0}{g}}$. If the distance y is defined at time t is $y = at + bt^2$ then show that $\int_0^{t_0} L\, dt$ is an extremum for real values of the coefficients only when $a = 0$ and $b = \frac{g}{2}$.

SC-003016106

SC-003016106 Seat No.__________

M.Sc. Mathematics Semester-I (CBCS) Examination

October / November - 2012

EMT-1001 : CLASSICAL MECHANICS-1

Time : 2 ½ Hours Total Marks : 70

Instructions:

1) Attempt all the questions.
2) Each question carries equal marks.

Q:1 Fill in the blanks : (Each carries two marks)

1) The linear momentum p of the particle is as :

A) $\dot{p} = 0$, (b) $p = mv$ (c) $p = m/v$ (d) $p = \frac{1}{2} mv^2$

2) The total angular momentum of the system is conserved when: [illegible] moment [illegible] zero [illegible] (d) None of these

3) A co-ordinate q_i is cyclic when

a) $\frac{\partial L}{\partial q_i} = 0$ (b) $\frac{\partial L}{\partial q_i} \neq 0$ (c) $\frac{\partial L}{\partial \dot{q}_i} = 0$ (d) $L = 0$

4) According to principle of virtual work

a) $F_i = 9$ (b) F_i = constant (c) $\dot{p}_i = 0$ (d) $F_i - \dot{p}_i = 0$

5) According to Hamilton's variational principle.

a) $\int_{t1}^{t2} L\,dt$ has a stationary value

b) $\int_{t1}^{t2} L\,dt = 0$

c) $\int_{t1}^{t2} F\,dt = 0$

d) None of these.

6) Kinetic energy T in plane polar co-ordinates is defined as:

a) $T = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2)$ (b) $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2)$

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c) T = ½ m $\dot{x}^2$ d) none of these

7) The potential energy V is defined as :

(a) v = mgh (b) gvh (c) v = $^h/_2$ (d) none of these .

8) The earth moves around the sun in elliptical orbit then

a) The moon is at one of the foci

b) The sun is at one of the foci

c) The sun and the moon are at the same foci.

d) Can't be predicted .

9) According two kepper's third law

a) $\tau \alpha a^3$ b) $\tau \alpha a^1$ c) $\tau^{3/2}$ a^2 d) $\tau = 0$

10) The shortest distance between two point in a plane is a

(a) Straight line b) Ellipse c) Circle d) 0

Q:2 Attempt any Two

a. Define:

(i) Holonimic constraints.

(ii) Scleronomous constraints.

(iii) [illegible]

(iv) Monogenic system.

(v) Configuration space.

(vi) Generlized co-ordinates

(vii) torque.

b. Explain in detail the conservation of energy for a particle.

c. If the mass of the body is concentrated about C.M. then show that the total angular momentum of the system is equal to the angular momentum about C.M.

Q:3 Attempt the following:

a. A hoop krolling with out rooling without slipping dwon an inclined plane then find the force of friction acting on acting on the hoop.

b. Discuss in detail the Brachestochrone problem.

OR

i) Find the minimum surface of revolution about y-axis.

b) For the problem of atwood machine show that $\ddot{x} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$.

Q:4 **Attempt any two :**

a) A particle of mass m moves under a central force then show that.

(i) Its orbit is a plane curve.

(ii) Its areal vector sweeps out equal area in equal time.

b) State Hamilton's variational principle and discuss in detail the techniques of calculus of variations.

c) Show that the generalized momentum conjugase to a cyclic co-ordinate is conserved. Using this result deduce that if component of totall applied force vanishes the corresponding.

Q:5 **Attempt any two :**

a) The potential energy of a linear harmonic oscillator is $v = \frac{1}{2} kx^2$ then find the equation of motion using Hamilton's principle.

b) Obtain equations of motion for simple pendulum.

c) A particle falls a distanced y_0 in a time $t_0 = \sqrt{\frac{2y_0}{g}}$. If the distance y at any time t is $y = at + bt^2$ then show that the integral $\int_0^{t_0} L\,dt$ is an extremum for real values of the coefficients only when $a = 0$ and [illegible]

d) Find the Lagrangian and the equation of motion for a bead sliding on a uniformly rotated wire in free space.

003-016106 Seat No.________

M. Sc. (Sem. I) (C.B.C.S.) Examination

December - 2011

Mathematics : MT-1001

(Classical Mechanics - I)

Faculty Code : 003
Subject Code : 016106

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all questions.
(2) Each question carries equal marks.

1 Choose the appropriate alternatives : (any seven)

(1) The moment of force is defined as

(A) $N = r \times F$

(B) $p = mv$

(C) $\ell = r\theta$

(D) none of these

(2) The quantity q is a conserved quantity provided

(A) time derivative of q is constant

(B) time derivative of q is zero

(C) derivative does not exist

(D) none of these

(3) The linear momentum of a particle is conserved when

(A) G = 0

(B) a = 0

(C) F = 0

(D) none of these

(4) The angular momentum of particle is conserved when

(A) F = 0

(B) g = 0

(C) p = 0

(D) none of these

(5) The Lagrangian L equal to

(A) K.E. + P.E.

(B) Energy of the system

(C) K.E. − P.E.

(D) K.E./(P.E.)^2

(6) Any co-ordinate q_j is cyclic if

(A) Lagrangian is constant

(B) Lagrangian does not contain

(C) Lagrangian is zero

(D) None of these

(7) Rhenomous constraints are

(A) dependent on time

(B) independent of time

(C) may or may not depend on time

(D) contains cyclic co-ordinates

(8) The shortest distance between two points in a plane is

(A) Circle

(B) Ellipse

(C) Parabola

(D) Straight line

(9) According to Kepler's law the areal vectors sweeps out

(A) double area in constant time

(B) equal area in equal time

(C) no area initially

(D) none of these

(10) The nature of orbit of planet mercury around the sun is

(A) Circle

(B) Parabola

(C) Ellipse

(D) Hyperbola

2 Attempt any two.

(a) State and prove linear momentum conservation theorem for system of particles.

(b) Find the equations of motion for a bead sliding on a uniformly rotated wire in free space.

(c) Discuss the problem of Atwood machine and derive that

$$\ddot{x} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

3 Attempt the followings :

(a) Find the shortest distance between two points in a plane.

(b) Discuss in detail the brachistochrone problem.

3 Attempt the followings :

(a) Find the minimum surface of revolution about y-axis.

(b) A hoop rolling without slipping down an inclined plane then find the force of friction on the hoop.

4 Attempt any two :

(a) Explain in brief the conservation of total energy for the system of particles.

(b) If the mass of the body is constrained to move about C.M then show that the total angular momentum of the system is equal to the angular momentum of C.M. plus angular momentum about C.M.

(c) Show that the two body central force motion about their C.M. can always be reduced to an equivalent one body problem.

5 Attempt any two :

(a) Define :

(i) Configuration space

(ii) Degrees of freedom

(iii) Cyclic co-ordinate

(iv) Holonomic constraints

(v) Scaleronomous constraints

(vi) State Hamilton's variational principle

(b) Discuss in detail the techniques of calculus of variation.

(c) Show that the generalized momentum conjugate to a cyclic co-ordinate is conserved, Using this result deduce that if component of applied torque vanishes the corresponding component of angular momentum is conserved.

(d) Derive the Lagrange's equation of motion for general system.

003-016106/ACC-317 Seat No. ______

M. Sc. Examination

November / December - 2010

Mathematics : MMT-1001

(Classical Mechanics - I)

Faculty Code : 003

Subject Code : 016106

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) Attempt all the questions.

(2) Each question carries equal marks.

1 Choose the appropriate alternatives : (Attempt any seven)

(1) The linear momentum is defined as

(A) $p = mv$

(B) $F = ma$

(C) $A = \frac{1}{2}r^2\theta$

(D) None of this

(2) The angular momentum is defined as

(A) $F = ma$

(B) $P = mv$

(C) $L = r \times p$

(D) None of this

(3) The quantity q is a conserved quantity provided

(A) $\frac{dq}{dt} = 0$

(B) $dq = dq$

(C) $\frac{dq}{dt} = 1$

(D) None of this

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(4) The linear momentum of a single particle is conserved when

(A) $P = 0$

(B) $V = 0$

(C) $F = 0$

(D) $g = 0$

(5) The angular momentum for the system of particles is conserved when :

(A) Total force is zero

(B) Total torque is zero

(C) Velocity is zero

(D) None of this

(6) Any co-ordinate q_j: is cyclic if

(A) If lagrangian contains q_j

(B) If lagrangian does not contain q_j

(C) If force is constant

(D) None of this

(7) The lagrangian of the system is defined as

(A) The sum of K.E. and P.E.

(B) The difference of K.E. and P.E.

(C) The product of K.E. and P.E.

(D) None of this

L = T − V

(8) The shortest distance between two points in a plane is

(A) St. line

(B) Circle

(C) Ellipse

(D) Parabola

(9) Non-Holonomic constraints are

(A) Expressible in terms of algebric equations

(B) Not expressible in terms of algebric equations

(C) Derivable from the potential

(D) None of this

(10) The nature of orbit of [illegible] around the sun is

(A) Circular.

(B) Elliptical

(C) Parabolic

(D) Hexagenal

2 Attempt any two :

(a) State and prove [illegible] momentum conservation theorem for the system of [illegible].

(b) Find the lagrangian [illegible] the equation of motion for a bead sliding on a uniformly [illegible]ated wire in free space.

(c) For the problem of [illegible]ood machine derive the relation

$$\ddot{x}=\left(\frac{M_1-M_2}{M_1+M_2}\right)g$$

3 Attempt the following :

(a) Discuss in detail the brachistochrone problem.

(b) Obtain the equations of motion for a particle in free space in :

(i) Cartesian co-ordinates

(ii) Plane polar co-ordinates.

OR

3 Attempt the following :

(a) A hoop rolling without slipping down inclined plane then find the force of friction acting on the hoop.

(b) Find the minimum surface of revolution about y-axis.

4 Attempt any two :

(a) If the mass of the body is constrained to move about C.M. then show that the total angular momentum of the system is equal to the angular momentum of C.M. plus angular momentun about C.M.

(b) Show that two body central force motion about their C.M. can always be reduced to an equivalent one body problem.

(c) Explain in detail the conservation of total energy for the system of particles.

5 Attempt any two

(a) Derive Lagrange's equations of motion for conservative holonomic system.

(b) A particle of mass m moves under the influence of central force then show that :

(i) Its orbit is a plane curve.

(ii) Its areal vector sweeps out equal area in equal time.

(c) Define :

(i) Degrees of freedom

(ii) Configuration space

(iii) Cyclic co-ordinate.

More over state Hamilton's variational principle and using it derive Lagrange's equation of motion.

(d) Prove that generlized momentum conjugate to a cyclic co-ordinate is conserved. Using this result deduce that if component of the applied torque vanishes the corresponding component of angular momentum is constant.