Shree H.N. Shukla College of Science
M.Sc. (Mathematics) Sem-3
IMP questions of functional analysis

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1. Every finite dimensional n.l.s over $\mathbb{K}$ is Banach space.
2. Show that (C[a,b], $\|\cdot\|_\infty$) is Banach space.
3. Show that (l^p, $\|\cdot\|_p$) is Banach space.
4. State and prove Riesz lemma.
5. State and prove Riesz-Reprasentation theorem for bounden sesquilinear mapping on Hilbert Space.
6. Let X and Y are norm linear spaces and Y is Banach Sapce then Show that B(X,Y) is Banach Space.
7. State and prove Schwarz inequality.
8. State and Prove Bessel's inequality.
9. State and Prove Hahn-Banach theorem for norm linear space.
10. An orthonormal set M in Hilbert space H is total in H if and only if for all x ϵ H the Perseval relation holds.
11. Let T be a bounded linear operator then
    (a) $(x_n) \to x \Rightarrow Tx_n \to Tx$ Where $x_n, x \in \mathcal{D}(T)$.
    (b) The null space $\mathcal{N}(T)$ is closed.
12. Let X be finite dimensional norm linear space then every linear operator on X is bounded.
13. Let X and Y be a vector spaces both real or both complex. Let T: $\mathcal{D}(T) \to Y$ be a linear operator with domain $\mathcal{D}(T) \subseteq X$ and range $\mathcal{R}(T) \subseteq Y$.Then $T^{-1}: \mathcal{D}(T) \to \mathcal{R}(T)$ is exist if and only if Tx=0 ⇒ x=0.
14. Let X and Y be a norm linear spaces and T: $\mathcal{D}(T) \to Y$ be linear operator , where $\mathcal{D}(T) \subseteq X$. Then T is continues iff T is bounded.
15. Let X and Y are norm linear space. Let $\|\cdot\|$:B(X,Y) → $\mathbb{R}$, $\|T\| = \sup_{\substack{x\in X \\ x\neq 0}} \frac{\|Tx\|}{\|x\|}$,T ∈ B(X,Y) then Show that $\|\cdot\|$ is norm on B(X,Y).
16. Prove that every orthonormal set in inner product space is linearly independent.
17. Show that $(S+T)^*=S^*+T^*$, Where S: $H_1 \to H_2$ and T: $H_1 \to H_2$ are bounded linear operators and H_1 and H_2 be a Hilbert spaces.
18. Show that $(ST)^* = T^* S^*$, Where S: H → H and T: H → H are bounded linear operators and H be a Hilbert space.
19. Show that $(\alpha T)^* = \bar{\alpha} T^*$ Where T: $H_1 \to H_2$ are bounded linear operators and H_1 and H_2 be a Hilbert spaces.
20. Show that $< T^*y, x > = <$ y , $Tx >$ Where T: $H_1 \to H_2$ are bounded linear operators and H_1 and H_2 be a Hilbert spaces.
21. Show that $(T^*)^* = T$ Where T: $H_1 \to H_2$ are bounded linear operators and H_1 and H_2 be a Hilbert spaces.

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22. Show that $\|T^*T\| = \|TT^*\| = \|T\|^2$ Where T: $H_1 \to H_2$ are bounded linear operators and H_1 and H_2 be a Hilbert spaces.

23. $< , >$ is inner product space on vector space X. $x_n \to x$ and $y_n \to y$ in X than $<x_n, y_n> \to <x,y>$ ($<,>$ is continues function on X x X).

24. Prove that $(\mathbb{R}^n, \|\cdot\|)$ is Banach space. Where $\|\cdot\|$ is Euclidean norm on $\mathbb{R}^n$.

25. Given example of incomplete metric space.

26. Definition: Weak convergence , Strong convergence ,Reflexive space, canonical mapping, isometry .