March 2025 sem-4

Seat No.

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 4 MARCH 2025 (Regular) MARCH - 2025

LINEAR ALGEBRA

Faculty Code : 003

Subject Code : 16SKMSMA-CO-04-00001

Time : 2:30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 If V be n-dimensional vector space over F, if $T \epsilon A_f V$ has the matrix $M_1(T)$ in the basis $\{v_1, v_2, \dots, v_n\}$ and $M_2(T)$ in the basis $\{w_1, w_2, \dots, w_n\}$ over F Then there is an element $C \epsilon F_n$ suchthat $M_2(T) = C M_1(T) C^{-1}$

2 Let $dim_f V = n$ and $A \epsilon A_F(V)$, Let $p(x) \epsilon F(x)$ be minimal polynomial of T over F, If $h(x) \epsilon F(x)$ is such that h(T)= Zero linear transformation from V into V Then $P(x)/q(x)$ in $F(x)$

3 Define Minimal Polynomial with an example

4 Define T-Invariant Subspace with an example

5 Define Index of Nilpotence, find index of nilpotence for $T: R^3 \rightarrow R^3$ defined as $T(x_1, x_2, x_3) = (0, x_3, x_1), \forall (x_1, x_2, x_3) \epsilon R^3$

6 Let V be n-dimensional vector space over F, Let $T_1, T_2 \epsilon A_f(v)$ be such that there exist basis B_1, B_2 of V over F satisfying the condition "The Matix of T_1 in B_1 = The Matix of T_2 in B_2 " Then $T_1 and T_2$ are similar

7 Let $T: R^3 \rightarrow R^3$ given by $T(e_1) = (0,1,1)$, $T(e_2) = (0,0,0)$, $T(e_3) = (0,0,0)$ Verify that T is nilpotent and also determine invariant of T

8 Define Companian Matrix

9 State and prove Cayley Hamilton Theorem for Characteristic polynomial

10 State and prove Polarization Identity

Q.2 Answer the following (Any Two) 14

1 Let V be finite dimensional space over F Then T is invertible if and only if the constant term of minimal polynomial is not zero

2 Let V be finite dimensional vector space over F then $A_f V$ and F_n are isomorphic as algebra over F

3 $V = P_2[x] = \{a + bx + cx^2 : a, b, c \in R\}$, Define $T: V \to V$ by $T(a + bx + cx^2 = b + 2x, \forall a + bx + cx^2 \in P_2[x]$, Prove that there exist $C \in R^3$ such that C $M_1(T)C^{-1}$ = $M_2(T)$, Where $M_1(T)$= is the matrix of T in the basis $\{1, x, x^2\}$, $M_2(T)$= is the matrix of T in the basis $\{1, 1+x, 1+x^2\}$

Q.3 Answer the following 14

Let V be finite dimensional vector space over F and dim(v)=n, let $T \in A_f V$, If all the characteristic roots of T lies in F Then T satisfies a polynomial of degree n over F

Let V be finite dimensional vector space over F, let $T \in A_f V$, let $v_1, v_2, \dots, v_r$ be linear subspace of V such that the following conditions holds

i) $V = V_1 \oplus V_2 \oplus \dots \oplus V_r$ ii) $T(V_i) \subseteq V_i$ iii) $dim(V_i) = n_i$ for each i = 1, 2, …, r

Let B_i for i = 1, 2, …, r basis of V_i over F, let $A_i \in F_{n_i}$ be the matrix of $T/_{V_i}$ in the basis, B_i over F, for i = 1, 2, …, r Then there exist a basis B of over F such that the matrix of T in the basis B is

$$\begin{bmatrix} A_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & A_2 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & A_r \end{bmatrix}$$

OR

Answer the following 14

State and prove Theorem: Rational Canonical Form

Let V, W be finite dimensional vector space over field F, let $\Psi: V \to W$ be an isomorphism of V on to W, let $S \in A_f(v), T \in A_f(v)$ such that $\Psi(S(v)) = T(\Psi(v))\ for\ all\ v \in V$ Then S and T have the same elementary divisors

Q.4 Answer the following questions (Any Two) 14

Let n≥1, let $A, B \in F_n, \alpha \in F$ then (i) tr(A+B) = tr(A)+tr(B) (ii) tr(AB) = tr(BA)

(iii) tr(A) = tr(B) (iv) tr(αA) = αtr(A) whenever $A, B \in F_n$ are similar

State and prove Jacobsen's Lemma

Q.5 Answer the following (Any Two) 14

1 Define bilinear form, Let V be vector space over field F, let $L_1, L_2: V \to F$ be linear functionals, Let $F: V \times V \to F$ be defined as $F(u, v) = L_1(u)L_2(v),\ for\ any\ (u, v) \in V \times V$ Check weather mapping F is bilinear or not

2 Let V be finite dimensional vector space over a field F, $dim_f V = n$, let $F: V \times V \to F$ be bilinear form, let $B_1 = \{v_1, v_2, \dots, v_n\}$ and $B_2 = \{w_1, w_2, \dots, w_n\}$ be any two basis of V over F Then there exist an invertible matrix $C \epsilon F_n$ such that $[f]_{B_2} = C[f]_{B_1}C^t$ where C^t is transpose of C

3 Let V be finite dimensional vector space over a field F, $dim_f V = n$, Then $L(V, V, F) \cong F_n$ as vector space over F and $dim_f L(V, V, F) = n^2$

4 Define symmetric bilinear form, let V be a finite dimensional vector space over F, let $dim_f V = n$, let $F: V \times V \to F$ be bilinear form on V then f is symmetric if and only if $[f]_B$ is symmetric.